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On downside risk predictability through liquidity and trading activity: a quantile regression approach

Antonio Rubia and Lidia Sanchis-Marco*

Abstract

Most downside risk models implicitly assume that returns are a sufficient statistic with which to forecast the daily conditional distribution of a portfolio. In this paper, we address this question empirically and analyze if the variables that proxy for market liquidity and trading conditions convey valid information to forecast the quantiles of the conditional distribution of several representative market portfolios. Using quantile regression techniques, we report evidence of predictability that can be exploited to improve Value at Risk forecasts. Including trading- and spread-related variables improves considerably the forecasting performance.

Keywords: Value at Risk, Basel, Liquidity, Trading Activity.

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1 Introduction

Implementing risk control and monitoring systems requires quantitative procedures to capture the level of underlying uncertainty and make accurate predictions. The Basel Committee on Banking Supervision (BCBS) has contributed greatly to the popularization of certain international standards, known as Basel I and II Accords, in the financial services industry. This regulatory setting entitles eligible financial institutions to use internal models based on the Value-at-Risk (VaR) measure for meeting market risk capital requirements. It is agreed that, without the efforts made to comply with the BCBS standards, the financial industry would likely be facing a deeper crisis. The depth of the economic turmoil has shown the necessity for new standards to achieve greater banking sector resilience and the need to improve the existing procedures for quantifying market risk.[1] The present paper is motivated by this concern.

The existing literature has suggested a number of procedures for forecasting downside risk, mainly VaR, which largely differ in their degree of sophistication: From the simple EWMA approach popularized by RiskMetrics to the more advanced probabilistic settings based on the Extreme Value Theory; see Manganelli and Engle (2004) and McNeil *et al.* (2005) for a review. Remarkably, a number of empirical studies have revealed that most of these methods do not seem to perform successfully in practice under standard backtesting techniques (*e.g.*, Kuester *et al.*, 2006), which underlines the practical complexity that lies behind the simple notion of VaR. Why is accurate VaR forecasting so elusive? Whereas most of the previous literature has attempted to address this question on the grounds of model misspecification, in this paper we adopt an alternative view within the framework of model risk and analyze the important role played by the set of conditioning information. In spite of the large methodological differences, the existing methodologies to model market risk share a common characteristic: They all rely exclusively on the information conveyed by historical returns. Naturally, this may turn out to be unnecessarily restrictive for practical purposes, since the conditional loss function of a portfolio may exhibit non-trivial links with the state variables that characterize the market environment and trading conditions and which may help forecasting bursts in volatility and illiquidity, particularly, in times of stress.[2]

[1] The BCBS is currently carrying out a reform program that aims to enhance Basel II towards a new regulatory setting on the basis of higher supervisory standards (Basel III), and which is expected to be completed by the end of 2010. The BCBS has issued several consultative documents in the interim outlining the main features of the new regulatory setting. It is interesting to note that, despite its limitations, the VaR paradigm is still a pillar in the internal model approach for market risk adopted in Basel III, although further refinements, such as stressed VaR and stress tests, will also be required to provide a complementary risk perspective.

[2] The implicit belief that returns subsume all the relevant information to forecast downside risk may be originated in a conservative interpretation of the Efficient Market Hypothesis. This

In this paper, we address empirically the premise that certain state variables that are observable in the market trading process exhibit predictive power to forecast the tail of the conditional loss distribution and, consequently, are useful for risk management. Although predictability is not necessarily limited to the set of variables we analyze, our main focus is on bid-ask spread and volume-related measures. Our study is motivated by previous findings and theoretical considerations in the asset pricing and market microstructure literature which have underlined the link between returns and market liquidity, activity, and private information arrivals. Like returns, liquidity- and volume-related variables are available on the trading-basis and are highly sensitive to information flow. Like volatility, these variables are believed to reflect collective expectations, environmental conditions and market sentiments which have a major influence on investor decisions. In contrast to returns and volatility, however, trade-related variables seem to have been ignored in the literature devoted to downside risk modelling, even though there exists previous evidence that supports the predictive power of volume and liquidity variables on volatility (Suominen, 2001; Bollerslev and Melvin, 1994). The main aim of this paper, therefore, is to analyze empirically whether downside risk forecasts can be enhanced by using this information or if, on the contrary, the predictability of the conditional loss distribution is limited to past returns and their volatility, as implicitly assumed in most of the empirical models used in practice.

More specifically, we study the performance of several liquidity- and volume-related predictors in day-ahead VaR forecasting at different quantiles using daily log returns from volume- and value-weighted market portfolios, book-to-market (B/M), and size-sorted portfolios from the US Stock Exchange in the period 01/1988 through 12/2002.[3] As in the literature concerned with return predictability, the simplest way to appraise forecastability is to use simple least-squares analysis in predictive linear regressions; see, for instance, Cochrane (2005). However, conditional quantiles are unobservable and have to be estimated or, at best, modelled as a latent process, which makes such an approach infeasible. Fortunately, the Quantile Regression theory (Koenker and Bassett, 1978) allows us to analyze formally predictability in the conditional percentiles without departing significantly from the intuitive spirit that characterizes predictive regressions. Using quantile regression we can directly model the tail of the conditional distribution of returns through a functional form that relates the VaR time-varying dynamics to its own

forbids the systematic predictability of returns on the basis of the available information, *i.e.*, posits an orthogonal condition on the first-order conditional moment. However, it remains silent about higher-order moments, such as conditional volatility, or other distributional features of returns, such as conditional percentiles. Furthermore, financial markets largely depart in practice from the complete-market and symmetric-information hypotheses that underlie a number of theoretical models in the asset pricing literature. In the presence of asymmetric or imperfect information and other frictions, even the conditional mean of returns may be predictable.

[3] Three main reasons prompted us to consider this specific sample: *i*) market-portfolio data allow us to eliminate the idiosyncratic noise that may affect the main conclusions on individual stocks, *ii*) the sample period is particularly interesting for risk management as it includes a stress scenario originating in the burst of the technological bubble in 2000, and *iii*) we can analyze the aggregate measures of liquidity and volume that have been used previously in several studies; see Chordia *et al.* (2001). We thank Prof. A. Subrahmanyam for making these data available.

past as well as to lagged predictors without making an explicit assumption on the distribution of returns, building on the so-called CAViaR model setting in Engle and Manganelli (2004). A restricted version of this model, which considers returns-related information solely, can be taken as a natural benchmark to address statistically the incremental value of liquidity- and volume-related variables. Even more importantly, these models can be used to construct VaR forecasts, which allows us to resort to standard backtesting and statistical techniques (*e.g.*, Christoffersen, 1998; Piazza *et al.*, 2009) to analyze the actual out-of-sample performance. Our main empirical conclusions largely support the suitability of the liquidity- and volume-related variables in forecasting daily VaR.

This paper belongs to the stream of literature that has focused on quantile regressions for VaR modelling and forecasting; see, among others, Taylor (1999, 2000, 2008), Kouretas and Zarangas (2005), Bao et al. (2006) and Kuester et al. (2006). The distinctive feature of our study is the analysis of the predictive ability of the observable variables that the proxy for liquidity and trading-activity in day-ahead VaR modelling. To the best of our knowledge, no previous paper has focused on this important aspect, although some recent studies can be related to this analysis. Cenesizoglu and Timmerman (2008) analyze the predictability of the distribution of monthly returns on a set of state variables that are believed to predict the equity premium, such as valuation and corporate ratios, bond yields, and different measures of default and market risk. Some of these variables are shown to be helpful in predicting different quantiles, particularly, at the right tail of the distribution, which may lead to more efficient portfolio selection and option trading. Adrian and Brunnermeir (2009) model weekly VaR dynamics in the banking industry using a similar set of state variables aiming to characterize the determinants of CoVaR dynamics. Our paper contributes to this literature by providing novel evidence on *i*) the predictability of the left tail of the conditional distribution of daily returns on the basis of volume and liquidity measures, and *ii*) the suitability of these variables for downside risk forecasting. This study is also related to the previous studies focused on the analysis of return predictability and the links between volatility and the main variables related to the trading flow, such as bid-ask spreads and trading-based measures; see, among others, Clark (1973), Tauchen and Pitts (1983), Stoll (1989) and Kalimipalli and Warga (2002).

The remaining part of the paper is organized as follows. Section 2 reviews the basic elements in VaR modelling and briefly introduces the quantile regression approach. Section 3 describes the main features in the data set analyzed and carries out the empirical analysis. The main analysis focuses on the volume-weighted market portfolio, using both (quantile) predictive regressions and backtesting analysis techniques. For the sake of completeness, we also analyze predictability for different characteristic portfolios, namely, value-weighted, book-to-market (B/M) and size-sorted market portfolios. Finally, Section 4 summarizes and concludes.

2 Modelling and forecasting downside risk: Value at Risk

Let r_t, $t = 1, ..., T$, be the daily log-return time-series of a portfolio, and let $\mathcal{F}_t$ be the natural filtration that includes all the available information at time t, such as any measurable transformation on the past observations of r_t as well as any other observable variable. For simplicity, but no loss of generality, we assume in the sequel that returns behave as a stationary martingale difference sequence such that $E(r_t|\mathcal{F}_{t-1}) = 0$, with bounded moments $E\left(|r_t|^{\delta}\right) < \infty$ for some $\delta > 2$ large enough.[4] For a probability $\lambda \in (0,1)$, we define the $(1-\lambda)\%$ VaR of a financial asset or portfolio as the maximum loss over a horizon of h days which is expected at the $(1-\lambda)\%$ confidence level given $\mathcal{F}_t$, *i.e.*, the λ-quantile of the conditional loss distribution of the portfolio. Formally, we can denote:

$$\begin{aligned} VaR_{\lambda,t+h} &= -\left\{\inf x \in \mathbb{R} : \Pr\left(r_{t,h} \leq x|\mathcal{F}_t\right) \geq \lambda\right\} \\ &= -\left\{Q_{\lambda,t}\left(h\right)\right\} \end{aligned} \tag{1}$$

with $Q_{\lambda,t}(h)$ defined implicitly, and $r_{t,h} = \sum_{j=1,h} r_{t+j}$ denoting the h-period return.

In market risk management, h typically ranges from 1 to 10 days and $\lambda = \{0.01,\ 0.05\}$. For instance, the Basel market risk framework stresses the use of the 1% conditional percentile to determine capital adequacy, while traders in a bank are often constrained by the rule that the 95% VaR of their position should not exceed a given bound on a daily basis (McNeil *et al.*, 2005). The extant literature in financial econometrics has suggested very different procedures with which to model and forecast VaR dynamics. In the following subsection, we discuss the main characteristics of the quantile regression approach. Appendix A sketches the main features of several alternative procedures that shall be used together with quantile regression in the empirical analysis in Section 4.

2.1 Quantile regression: CAViaR models

Following Koenker and Bassett (1978) and Basset and Koenker (1982), the expected conditional quantile could be written as a $\mathcal{F}_t$-measurable linear function of n explanatory variables, $X_t = (x_{1t}, ..., x_{nt})'$, and an $(n \times 1)$ vector of unknown coefficients β_λ that depends on the λ-quantile, namely, $Q_{\lambda,t}(h) = X_t'\beta_\lambda$. Particularizing for a one-day holding period, $h = 1$, this formulation is equivalent to assume the quantile regression model

$$r_{t+1} = X_t'\beta_\lambda + u_{t+1,\lambda},\ t = 1, ..., T \tag{2}$$

[4]This condition is not strictly necessary but simplifies exposition considerably. It comes with no loss of generality because it is customary to demean (daily) returns previously to ensure that the resultant series behaves as a martingale difference sequence; see, for instance, Taylor (2008). In Section 4, we filter out the predictable pattern in the conditional mean of the different portfolio returns by fitting an AR(1) model prior to apply the VaR methods.

where $u_{t,\lambda}$ is an error term satisfying $E(u_{t,\lambda}|X_{t-1}) = 0$. As in the standard regression model, this general specification does not impose any particular restriction on the distribution of the data.

Model (2) is highly reminiscent of the standard linear regression setting. Whereas the least-squares analysis attempts to characterize the conditional mean of the distribution as a function of the regressors, $E(r_{t+1}|X_t)$, the quantile regression allows us to model directly the dynamics of the conditional λ-quantile of the distribution. A well-known particular case arises for $\lambda = 1/2$ (also known as least absolute deviation regression model), which is intended to provide estimates of the slope coefficients for the conditional median of the process. In this case, the relevant coefficients can be estimated consistently by minimizing the sum of the absolute values of the residuals. More generally, given an arbitrary value of $\lambda \in (0,1)$, the unknown β_λ vector of parameters in (2) can be estimated consistently as:

$$\widehat{\beta}_\lambda : \arg \min_{\beta_\lambda \in \mathbb{R}^n} \left\{ \sum_{t=1}^{T} \lambda \left| u_{t+1,\lambda} \right| \mathbb{I}_{(r_{t+1} \geq X_t' \beta_\lambda)} + \sum_{t=1}^{T} (1-\lambda) \left| u_{t+1,\lambda} \right| \mathbb{I}_{(r_{t+1} < X_t' \beta_\lambda)} \right\} \quad (3)$$

where $\mathbb{I}_{(\cdot)}$ is an indicator function taking value one if the condition stated in the subscript holds true.

Taylor (1999) used quantile regressions to forecast VaR at different horizons with explanatory variables defined on both a proxy for volatility and the particular horizon involved. More generally, Engle and Manganelli (2004) proposed a class of quantile regression models which are specifically intended to infer VaR dynamics. The distinctive feature is that the conditional quantile is seen as a latent autoregressive process which may also depend on a number of covariates and that may exhibit nonlinearities in parameters, the so-called Conditional Autoregressive Value at Risk (CAViaR).

Thus, recalling the VaR definition, and following Engle and Manganelli (2004), we consider the following simple CAViaR-type specification throughout the paper:

$$VaR_{\lambda,t} = \beta_{\lambda,0} + \beta_{\lambda,1} VaR_{\lambda,t-1} + \beta_{\lambda,2} |r_{t-1}| + \beta_\lambda^* \; f(X_{t-1}) \quad (4)$$

with $\beta_\lambda = \left(\beta_{\lambda,0}, \beta_{\lambda,1}, \beta_{\lambda,2}, \beta_\lambda^*\right)'$, and X_t being a certain predictive variable, which we consider as being different than returns.[5] The term $f(\cdot)$ denotes a suitable transformation of the original data. For instance, we set $f(X_t) = |\log(X_t)| \equiv X_t^*$ in our analysis.[6]

The functional form of this model attempts to exploit parsimoniously the statistical information conveyed by the past of the conditional quantile, as well as

[5] The model could readily be generalized to account for asymmetries or higher-order lags. However, Kuester *et al.* (2006) showed the good performance of this parsimonious specification in relation to more sophisticated alternatives.

[6] The logarithmic transformation smoothes the variable and reduces the statistical problems related to outlying observations and heteroskedasticity. In our analysis, all the predictive variables (see Section 3.1) are strictly positive, which enables this transformation. Taking the absolute value of the resultant regressor is not strictly necessary, but since in our analysis the log-transformation yields either strictly positive or negative series, it facilitates the homogeneous discussion of results.

that in the X_t^* variable. The main purpose of the autoregressive structure is to ensure that the dynamics of the conditional quantile change smoothly over time. Since VaR dynamics are highly persistent, the lag of the dependent process could also be seen as an instrumental variable that proxies for the true latent process. Similarly, the variable $|r_{t-1}|$ is a natural proxy for the unobservable volatility of returns. Since it introduces a source of (stochastic) short-term variation related to the arrival of news in the market, this process is expected to be a major driver in any market risk measure. At this point, the similarities between the basic structure of the CAViaR model under the restriction $\beta_\lambda^* = 0$ (so-called Symmetric Absolute Value CAViaR model, Restricted-CAViaR henceforth) and the class of GARCH models widely used to characterize volatility are fully evident.

In addition to the GARCH-type architecture, the VaR dynamics in the CAViaR setting could be driven by the lagged values of a set of state variables. The existing literature has not yet discussed which variables should be included in such an analysis. For daily market data, the most obvious candidates seem to be the variables that proxy for liquidity and other trade conditions in the market. The main problem in this regard is that market liquidity is a difficult magnitude to measure and one which is not easy to relate to a single aspect of the market. The central strategy we adopt consists of individually analyzing a number of trade-based and order-based market variables (see Section 3.1 for details) which are widely accepted to be related to liquidity in order to detect predictability. Note that, although the results in a parametric modelling may be sensitive to the choice of a particular proxy of liquidity or another, we should expect a robust picture to emerge when considering a wide range of proxying variables. The model resulting from extending the basic Restricted-CAViaR specification with the lags of a single predictor can be seen as a low-order individual autoregressive distributed lag model for the conditional quantile. This class of models is known to be particularly useful in the forecasting analysis; see, for instance, Rapach and Strauss (2009), and allows us to examine how a model extended with a variety of individual proxies of liquidity performs relative to the restricted model.

Finally, under suitable regularity conditions and as the sample size is allowed to grow unbounded, it can be shown (cf. Engle and Manganelli 2004, Thms. 1 and 2) that:

$$\sqrt{T}(\widehat{\beta}_\lambda - \beta_\lambda) \xrightarrow{d} \mathcal{N}(0, V_\lambda) \tag{5}$$

i.e., $\widehat{\beta}_\lambda$ is a $\sqrt{T}$-consistent estimate of the unknown vector β_λ, and the (suitably re-scaled) estimation bias is asymptotically distributed as a normal distribution with zero mean and finite covariance matrix V_λ. In order to consistently estimate this matrix, Engle and Manganelli (2004) propose a robust estimator that combines kernel density estimation with the heteroskedasticity-consistent covariance matrix estimator of White (1980); see Engle and Manganelli (2004, Thm. 3) for details. We shall use this approach to perform formal inference on the significance of the β_λ^* estimate.

3 Empirical analysis

3.1 Data

We consider the returns of different representative market portfolios in order to characterize the VaR dynamics. In particular, our basic data set comprises continuously compounded returns from the volume- and value-weighted portfolios in the US market over the period 01-04-1988 to 12-31-2002, totaling 3,782 daily observations. These data are available from CRSP and shall be used in our main analysis in Section 4.2. In addition, we analyze daily log returns on the B/M-sorted and size-sorted portfolios in Section 4.3. These data are available at Kenneth French's website.

Along with daily portfolio log returns, we observe daily data of several potentially predictive variables throughout the period analyzed and which are constructed from individual firm bid-ask spreads and volume data; see Chordia *et al.* (2001) for details. These variables can be classified as:

i) Trading-related variables: Trading Volume (V) measured in thousands of shares; Number of Trades (NT) calculated as the sum of sell and buy trades; Number of Sell trades (NS); Number of Shares Sold in thousands (NSS) and Traded Volume in Dollars (TVD).

ii) Liquidity variables: Quoted Spread (QS) measured as the dollar difference between ask and bid prices; Effective Spread (ES) given by the signed difference between trade price and bid-ask midpoint (MP); Relative Quoted Spread (RQS) defined as the ratio QS/MP, and Relative Effective Spread (RES) defined as the ratio ES/MP.[7]

Table 1 displays in Panel A the usual descriptive statistics for the demeaned time-series of the different portfolio returns analyzed in the paper, and all the explanatory variables (in logarithms) used in our analysis in Panel B. Returns exhibit the characteristic stylized features in daily samples: Excess of kurtosis, mild degree of skewness and negligible autocorrelation. The most salient feature of the predictors in the trade-based and the order-based measures is the strong degree of persistence as measured by the first-order autocorrelation coefficient. Returns are contemporaneously correlated to all the variables analyzed (correlations are not shown for restricteding space but are available upon request). In particular, returns are positively correlated with the variables in the volume group (the average correlations are around 39%) and negatively correlated to liquidity-related variables (the average correlations are around -25%). As usual, the variables within each group are strongly correlated among themselves, and largely and negatively correlated with the variables in the other group. Cross-correlations range from -79%, for TVD and QS, to -88%, for TVD and QS.

[7] In addition to these variables, we considered alternative variables which did not led to qualitative changes over those reported in the next section. For instance, considering the logarithm transformation of the volume or the unexpected volume –measured as the residuals from an AR(1) model– does not seem to have a major change in the out-of-sample ability of the model. These results are not presented to save space but are available from the authors upon request.

Table 1: Descriptive Statistics.

Panel A: Returns								
r_t	Mean	Median	Max.	Min.	Var	Skew.	Kurt.	$\rho_{(1)}$
$r_{t,Vol}$	0.00	0.01	5.54	-6.70	0.98	-0.20	7.78	-0.06
$r_{t,Value}$	0.02	0.06	4.97	-6.11	0.49	-0.35	10.84	-0.07
$r_{1t},_{B/M}$	0.01	0.02	6.65	-7.85	1.22	-0.11	7.29	-0.07
$r_{2t},_{B/M}$	0.00	0.02	4.81	-6.32	0.67	-0.43	8.36	-0.05
$r_{1t,size}$	0.00	0.05	6.02	-7.56	0.66	-0.71	11.06	-0.05
$r_{2t,size}$	0.00	0.01	5.73	-6.89	1.05	-0.16	7.39	-0.06
Panel B: Predictive variables								
X_t^*	Mean	Median	Max.	Min.	Var	Skew.	Kurt.	$\rho_{(1)}$
V	7.39	7.21	9.69	5.52	0.73	0.36	1.86	0.96
NT	6.85	6.69	8.76	5.07	0.65	0.30	1.64	0.98
NS	6.09	5.94	8.02	4.24	0.65	0.29	1.67	0.98
NSS	6.51	6.33	8.80	4.50	0.73	0.35	1.86	0.96
TVD	11.10	11.04	13.00	9.13	0.76	0.17	1.66	0.96
QS	-1.89	-1.74	-1.20	-3.40	0.21	-1.64	4.79	0.99
ES	-2.29	-2.11	-1.50	-3.80	0.20	-1.53	4.48	0.99
RQS	-5.39	-5.39	-4.80	-6.90	0.20	-1.02	3.23	0.99
RES	-5.96	-5.76	-5.20	-7.20	0.20	-0.96	3.08	0.99

This table shows in Panel A the descriptive statistics (mean, median, maximum, minimum, variance, skewness and kurtosis) of the demeaned returns (previously multiplied by 100) for the volume and value weighted market portfolio, B/M and Size sorted portfolios corresponding to Low30 (r_{1t}) and High30 (r_{2t}). Panel B shows the descriptive analysis for the explanatory variables involved in the analysis (in logarithms). The last column indicates the first-order autocorrelation of the variables. The variables included are V (trading Volume); NT (Number of Trades); NS (Number of Sell trades); NSS (Number of Shares Sold in thousands); TVD (Traded Volume in Dollars); QS (Quoted Spread); ES (Effective Spread); RQS (Relative Quoted Spread) and RES (Relative Effective Spread).

3.2 Market portfolio analysis

Throughout the following subsections we analyze the ability of the predictive variables to forecast day-ahead VaR for the returns of the volume-weighted market portfolio, both through a predictive regression that involves the whole sample and an out-of-sample predictive analysis in a rolling-window approach. The focus on day-ahead estimation is consistent with the holding period considered for internal risk control by most financial firms (Taylor, 2008). Owing to their empirical relevance in risk management, we are particularly interested in the quantiles $\lambda \in \{0.01,\ 0.05\}$, but shall also analyze a wider range of probabilities given by $\Theta_\lambda = \{0.01,\ 0.025,\ 0.05, 0.075\}$ to characterize predictability along the left tail of the distribution. It should be mentioned that in this section we also considered the analysis on the returns of the value-weighted market portfolio, which are not presented for saving space as the qualitative evidence is completely analogous to that discussed below. The complete analysis is available from authors upon request.

The daily returns of the volume-weighted market portfolio over the total period analyzed (see Figure 1 below) include different episodes in terms of activity and volatility. The beginning of the sample corresponds to the period that followed the market crash in October 1987. After the extraordinary crash, the volatility of the market decreased progressively and reverted to normal levels. In 1998, Long-Term Capital Management failure in the hedge-fund industry led to a massive bailout by other major banks and investment houses that, in turn, generated an excess of volatility in the market and which preceded the burst of the dot-com firms in 2000. Finally, the data from 2000 onwards show the large excess of volatility that characterized the market after the burst of the technological bubble. This particular period, depicted by the grey line in Figure 1, will be analyzed in detail in the out-of-sample analysis (back test) later on.

3.2.1 Predictive regression analysis

We first analyze predictability through predictive-like quantile regressions. More specifically, we estimate the unrestricted CAViaR model

$$VaR_{\lambda,t} = \beta_{\lambda,0} + \beta_{\lambda,1} VaR_{\lambda,t-1} + \beta_{\lambda,2}|r_{t-1}| + \beta^*_\lambda \ |\log(X_{t-1})\,| \qquad (6)$$

for any of the conditioning variables X_t described in Section 3.1, using the entire sample, from 01-04-1988 to 12-31-2002, with $3,782$ observations. Our main interest is to test the suitability of these unrestricted models against a restricted CAViaR specifications based solely on returns. Clearly, a rejection of the null hypothesis $\mathrm{H}_0 : \beta^*_\lambda = 0$ for a specific model and quantile provides formal evidence on the suitability of the posited variable as a predictor of the conditional distribution. The objective function in the quantile regression minimization problem is optimized through the Simulated Annealing optimization algorithm (Goffe, Ferier and Rogers, 1994), while the asymptotic covariance matrix is estimated using a robust sandwich-type estimator based on a k-nearest neighbor kernel, as described

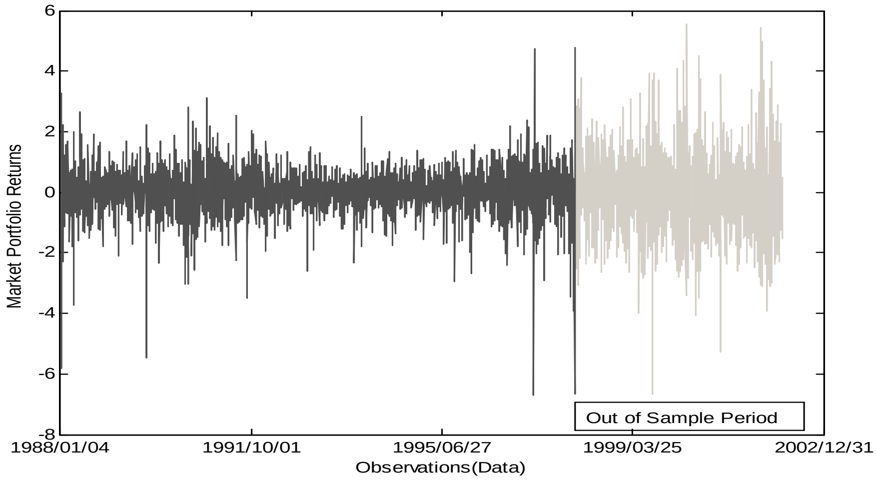

Figure 1: Returns of the volume weighted market portfolio.

in Engle and Manganelli (2004).[8]

Table 2 reports the estimated coefficients and the one-sided robust p-values for the all the variables for $\lambda \in \{0.01, 0.05\}$. Following Engle and Manganelli (2004), we set the bandwidth-type parameter in the robust estimation of the covariance matrix to $k = 40$ and $k = 60$ for the 1% and 5% quantiles, respectively. Several features are worth commenting upon. Firstly, the resulting estimates show the strong degree of persistence in the VaR dynamics as measured by the estimate of the autoregressive coefficient, $\widehat{\beta}_{\lambda,1}$. The degree of persistence is weaker for $\lambda = 0.01$ because extreme percentiles are more likely to be driven by jumps (outlying observations) in data generating process of returns, which are expected to exhibit a more random, irregular behavior. Secondly, and accordingly with the previous result, the average influence of the volatility process on the VaR estimates, as measured by the coefficient $\widehat{\beta}_{\lambda,2}$, becomes more important as the size of the quantile decreases. These results completely agree with the qualitative evidence discussed, among others, in Engle and Manganelli (2004). Turning our attention to the empirical relevance of the different predictors, all the variables analyzed have a positive effect on the conditional distribution of returns. Increments in the volume- or spread-related variables tend to generate larger levels of VaR, as might be expected from the theoretical considerations and previous evidence discussed in Section 1. The analysis on the significance of the estimated coefficients offers, however, mixed results. Whereas the null hypothesis of no predictability is strongly rejected for any of the variables analyzed for $\lambda = 0.05$, it cannot be rejected at any of the usual confidence levels for the 1% percentile.

There are two possible explanations for this finding. On the one hand, it is possible that a variable has predictive power at a given probability yet not at another. Thus, the statistical analysis would indicate a heterogeneous ability in market liquidity to predict the tail of the return distribution, with extreme percentiles being harder to foresee. Since the outlying observations that characterize the left tail of the returns distribution are related to the jump-component of its data generating process, and since this component is likely to be driven by a non-regular process, this point seems plausible: outliers and other extreme movements may be too volatile and irregular to be predicted systematically by covariates other than market volatility itself. On the other hand, the discrepancies may be the consequence of statistical distortions, particularly, for small percentiles such as $\lambda = 0.01$. In fact, there are two major sources of potential biases in this context: (small-sample) biases stemming from the robust estimation of the covariance matrix V_{λ}, and biases resulting from dealing with a target probability which is close to the boundary.

[8]Simulated annealing is a local random-search search algorithm that can accept values that increase the objective function (rather than lower it) with a probability that decreases as the number of iterations increases. The main purpose is to prevent the search process from becoming trapped in local optima, which in addition provides low sensitivity to the choice of the initial values. To minimize the possibility of getting convergence to local optima, the optimization process was repeated $1,000$ times over the whole sample.

Table 2: Inference results from predictive quantile regressions.

	$\lambda = 5\%$				$\lambda = 1\%$			
X_t^*	$\hat{\beta}_{\lambda,0}$	$\hat{\beta}_{\lambda,1}$	$\hat{\beta}_{\lambda,2}$	$\hat{\beta}_{\lambda}^*$	$\hat{\beta}_{\lambda,0}$	$\hat{\beta}_{\lambda,1}$	$\hat{\beta}_{\lambda,2}$	$\hat{\beta}_{\lambda}^*$
V	-0.04	0.96	0.05	0.01	0.02	0.92	0.11	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.37)	(0.00)	(0.06)	(0.16)
NT	-0.03	0.96	0.05	0.01	0.03	0.92	0.11	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.25)	(0.00)	(0.04)	(0.13)
NS	-0.02	0.96	0.05	0.01	0.03	0.92	0.11	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.24)	(0.00)	(0.04)	(0.16)
NSS	-0.03	0.96	0.05	0.01	0.03	0.92	0.11	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.27)	(0.00)	(0.06)	(0.19)
TVD	-0.06	0.96	0.05	0.01	-0.01	0.92	0.12	0.01
	(0.00)	(0.00)	(0.00)	(0.00)	(0.41)	(0.00)	(0.05)	(0.12)
QS	-0.01	0.97	0.05	0.01	0.07	0.92	0.14	0.01
	(0.01)	(0.00)	(0.00)	(0.00)	(0.12)	(0.00)	(0.08)	(0.32)
ES	-0.00	0.97	0.05	0.00	0.08	0.91	0.15	0.01
	(0.40)	(0.00)	(0.00)	(0.01)	(0.14)	(0.00)	(0.11)	(0.31)
RQS	-0.04	0.97	0.05	0.01	0.04	0.92	0.14	0.01
	(0.01)	(0.00)	(0.00)	(0.00)	(0.32)	(0.00)	(0.08)	(0.36)
RES	-0.08	0.96	0.05	0.02	-0.03	0.89	0.15	0.03
	(0.05)	(0.00)	(0.00)	(0.05)	(0.34)	(0.00)	(0.13)	(0.18)

This table shows the estimated parameters and robust p-values (in brackets) for the entire sample and the quantile regression model (4),

$$VaR_{\lambda,t} = \beta_{\lambda,0} + \beta_{\lambda,1} VaR_{t-1} + \beta_{\lambda,2}|r_{t-1}| + \beta_{\lambda}^* \ |\log(X_{t-1}^*)|),$$

given $\lambda = 0.05$ and $\lambda = 0.01$. The first column shows the volume-related and liquidity variables X_t^* analyzed.

The theoretical arguments that support consistency in the robust estimation of V_λ hold asymptotically. As with other robust estimates of the covariance matrix, the correct choice of the bandwidth-type parameter in the kernel-type estimation involved plays a critical role, particularly, in finite samples. Furthermore, when λ is close to the boundary limits, the asymptotic variance increases because of the small density of the observations in that region. The conjunction of these factors may cause significant biases in the estimates of the covariance matrix and meaningful distortions in the subsequent hypotheses testing. In order to analyze the sensitivity of the results to the estimation of V_λ, we estimated the CAViaR models for $\lambda \in \Theta_\lambda$ with the bandwidth-type parameter taking values over a wide range, $k \in \{10,\ 30,\ 50,\ 70,\ 90\}$. Table 3 summarizes the main results from this analysis, showing the estimates of the β_λ and $\beta_{\lambda,2}$ coefficients and their robust p-values given k. For the remaining parameters, there was no qualitative changes and, therefore, we do not present results in order to save space, although these are available upon request. Table 3 reveals several interesting findings. For relatively large values of λ in the tail of the distribution, the empirical evidence strongly supports predictability for all the variables, independently of the value of k, which allows us to conclude that predictable relationships do exist. Nevertheless, as λ decreases towards the zero limit, the statistical evidence of predictability weakens and eventually vanishes. Overall, the evidence of predictability seems to be more evident for the variables in the volume-related group, in which the null $\mathrm{H}_0 : \beta_\lambda^* = 0$ is rejected for most variables, and (marginally) rejected even for $\lambda = 0.01$. The overall analysis also reveals that the main statistical conclusions in this analysis do not seem to be particularly sensitive to the choice of the bandwidth-type parameter k.

The second potential source of biases in this context is related to the existence of statistical distortions in the quantile regression methodology when dealing with quantiles close to the boundaries. This well-known problem originates from the fact that conventional large sample theory does not apply sufficiently far in the tails, a behavior which worsens in finite samples and which may lead to unsound inference; see, for instance, Chernozhukov (2005) for a detailed discussion. As a result, inference may be misleading. Note, for instance, that the significance analysis of $\widehat{\beta}_{\lambda,2}$ in Table 3 leads to puzzling conclusions. Whereas the estimates of the coefficient related to market volatility largely increases as λ is smaller, standard errors increase reducing the degree of statistical significance and causing $\mathrm{H}_0 : \beta_{\lambda,2} = 0$ not to be rejected for some nominal levels. This suggests that VaR dynamics may not be driven by volatility, precisely in the context values in which total market volatility is the most likely driver of process. This counterintuitive feature seems a statistical artifact rather than a true rejection of significance.

Owing to the statistical difficulties in dealing with extremes percentiles, empirical studies often avoid the analysis for values of λ close to boundaries; see, for instance, Cenesizoglu and Timmerman (2008). In our context, however, we have alternative methods with which to study the forecasting ability of the predictive variables even for such percentiles. Since the ultimate purpose of VaR modelling is to generate accurate market risk forecasts, the most important question is whether

Table 3: Sensitivity analysis of p-values to different k-bandwidth.

λ		V	NT	NS	NSS	TVD	QS	ES	RQS	RES	$\lvert r_{t-1}\rvert$
7.5%	$\hat{\beta}_\lambda$	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.02**	**0.00**	**0.01**	**0.02**	**0.06**
	10	(0.00)	(0.00)	(0.00)	(0.00)	(0.24)	(0.00)	(0.02)	(0.00)	(0.13)	(0.00)
	30	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.04)	(0.07)	(0.02)	(0.00)
k	50	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.06)	(0.06)	(0.00)
	70	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.03)	(0.07)	(0.04)	(0.00)
	90	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.04)	(0.02)	(0.00)
5.0%	$\hat{\beta}_\lambda$	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.00**	**0.01**	**0.02**	**0.05**
	10	(0.02)	(0.00)	(0.01)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.16)	(0.00)
	30	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.04)	(0.00)	(0.00)	(0.06)	(0.00)
k	50	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.04)	(0.00)
	70	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.02)	(0.00)	(0.07)	(0.00)
	90	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)	(0.05)	(0.00)
2.5%	$\hat{\beta}_\lambda$	**0.02**	**0.02**	**0.02**	**0.03**	**0.02**	**0.01**	**0.00**	**0.00**	**0.03**	**0.07**
	10	(0.04)	(0.14)	(0.43)	(0.00)	(0.00)	(0.04)	(0.34)	(0.39)	(0.46)	(0.01)
	30	(0.03)	(0.17)	(0.10)	(0.04)	(0.07)	(0.10)	(0.29)	(0.36)	(0.08)	(0.11)
k	50	(0.02)	(0.08)	(0.06)	(0.02)	(0.04)	(0.12)	(0.26)	(0.35)	(0.11)	(0.09)
	70	(0.03)	(0.03)	(0.04)	(0.02)	(0.02)	(0.11)	(0.27)	(0.35)	(0.06)	(0.07)
	90	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.07)	(0.28)	(0.35)	(0.04)	(0.07)
1.0%	$\hat{\beta}_\lambda$	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.03**	**0.14**
	10	(0.04)	(0.02)	(0.05)	(0.11)	(0.03)	(0.24)	(0.03)	(0.30)	(0.00)	(0.03)
	30	(0.15)	(0.16)	(0.12)	(0.15)	(0.14)	(0.34)	(0.33)	(0.38)	(0.24)	(0.05)
k	50	(0.12)	(0.12)	(0.11)	(0.14)	(0.11)	(0.33)	(0.28)	(0.38)	(0.15)	(0.03)
	70	(0.14)	(0.15)	(0.14)	(0.16)	(0.10)	(0.28)	(0.24)	(0.30)	(0.16)	(0.03)
	90	(0.11)	(0.13)	(0.11)	(0.12)	(0.10)	(0.28)	(0.24)	(0.32)	(0.15)	(0.03)

This table shows the estimated coefficients $\hat{\beta}_{\lambda,2}$ and $\hat{\beta}^*_\lambda$ and robust p-values (in brackets and in bold) of the test for individual significance from model (4) and the entire sample when the robust asymptotic covariance matrix is estimated with a kernel with values of $k \in \{10,\ 30,\ 50,\ 70,\ 90\}$ in the covariance matrix estimation process for a larger set of quantiles. The $\hat{\beta}_{\lambda,2}$ estimates (last column) are from model (4) with $X_t^* = V$.

the covariate-extended risk models have real out-of-sample predictability or not. Therefore, the backtesting analysis provides us with a more appropriate framework with which to compare the relative performance of the models, and to attempt to disentangle the predictive ability of the model for any quantile. This issue is studied carefully in the following subsection.

3.2.2 Out-of-sample analysis: backtesting analysis

In this section we analyze the day-ahead out-of-sample performance of the covariate-extended CAViaR models relative to the restricted Restricted-CAViaR for $\lambda \in \Theta_{\lambda}$. In addition, we also analyze the relative performance of the covariate-extended model against several established methods that are based solely on returns, such as the EWMA model (RiskMetrics), the Gaussian GARCH(1,1) model, and a model that combines GARCH estimation with a block-maxima approach in the Extreme Value Theory. The main characteristics of these alternative procedures are sketched in Appendix A; see McNeil *et al.* (2005) for further details.

Following Engle and Manganelli (2004) and Alexander and Sheedy (2008) we consider a rolling-window estimation period formed with the most recent $2,700$ observations available at any day to initialize the risk models and generate VaR forecasts for the probabilities $\lambda \in \Theta_{\lambda}$. This allows us to construct a sequence of $N = 1,000$ one-day ahead daily VaR predictions that can be compared with realized returns.[9] Whereas backtesting is a crucial element in the validation process of an internal risk model, a definitive methodology has yet to be determined. In the absence of a regulatory setting, the existing literature has adopted certain statistical standards that we briefly discuss in the sequel. Appendix B provides a more detailed discussion.

The most popular backtesting procedures analyze the statistical properties of an indicator variable (known as exception or VaR violation) that signals the occurrence of a realized return exceeding the expected VaR level, *i.e.*, a random process, here denoted as $H_{\lambda,t}$, that takes value one if $r_t < -VaR_{\lambda,}$ and zero otherwise, $t = 1, ..., N$. A desirable property is that the realized number of exceptions should represent approximately a $\lambda\%$ of the total out-of-sample period. This property is known in risk management as *reliability* and suggests the testable condition $\text{H}_{0,U} : E\left(H_{\lambda,t}\right) = \lambda$. Also, the dynamic properties of $H_{\lambda,t}$ should resemble those of an independent and identically distributed process, which combined with the reliability property renders the joint property of perfect conditional coverage, namely, $\text{H}_{0,C} : E\left(H_{\lambda,t}|\mathcal{F}_{t-1}\right) = \lambda$.

Christoffersen (1988) proposed a sequence of likelihood-ratio tests to check the properties of *i*) correct unconditional coverage, *ii*) (first-order serial) independence, and *iii*) correct conditional coverage in the exception variable given *i*) and *ii*); see Appendix B for details. These tests are widely used in practice and, therefore, constitute our first backtesting approach. In the sequel, we shall refer to these procedures as LR_{UC}, LR_{IND} and LR_{CC}, respectively. Similarly, Engle and Manganelli (2004) proposed another exception-based testing procedure to test for

[9] The average estimates of the parameter estimates over the out-of-sample period show the same patterns discussed before and, hence, are not presented in order to save space, although these results are available upon request.

the crucial $H_{0,C}$ hypothesis, the so-called dynamic quantile test (henceforth DQ). This test is widely believed to have better properties than LR_{CC} because it has power to detect higher-order dependences as it explodes a richer set of information. Consequently, we shall use the DQ procedure in our analysis. Finally, Piazza *et al.* (2009) have recently proposed an encompassing VaR quantile-regression test intended to test $H_{0,C} : E(H_{\lambda,t}|\mathcal{F}_{t-1}) = \lambda$. The distinctive characteristic is that the test is not defined on the exception variable, but on the time-series of predictions itself. This procedure, denoted as VQR in the sequel, uses a linear quantile regression to address if the VaR model is correctly specified. It should be noted that the test is asymptotically equivalent to DQ, yet, as claimed by the authors, may have better properties in finite samples. Inference on the basis of quantile regressions, on the other hand, may suffer from statistical problems when dealing with extreme quantiles in finite samples, as discussed previously.

We first analyze the backtesting results for the risk models based solely on returns: VaR-GARCH, VaR-EWMA, EVT-BM (extreme value theory) and Restricted-CAViaR models. Table 4 reports the main outcomes from this analysis, displaying the empirical frequency of exceptions $\widehat{\lambda}_H = \sum_{t=1}^{N} H_{\lambda,t}/N$, the test statistics of the LR_{UC}, LR_{IND}, LR_{CC}, DQ and VQR testing procedures as well as their respective p-values. For all these models, the empirical unconditional coverage for the percentiles $\lambda \geq 0.05$ tend to be greater than the nominal level, with $\widehat{\lambda}_H$ significantly departing from λ in most cases. Consequently, the hypotheses of perfect conditional and unconditional coverages tend to be rejected. Similar evidence has been reported previously, for instance, in Taylor (2008) for GARCH and CAViaR-type models; see also Piazza *et al.* (2009).

For quantiles $\lambda < 0.05$, the distortions in the unconditional coverage are considerably reduced for all but the EWMA model and, therefore, the $H_{0,U}$ hypothesis tends not to be rejected. The overall evidence for $H_{0,C}$, however, is mixed and depends on the particular test applied. Whereas the conservative LR_{CC} test accepts the hypothesis of perfect coverage, the DQ and VQR tests, which are considered as much more powerful testing procedures, tend to reject the null hypothesis of correct conditional coverage. Among the different returns-based VaR procedures analyzed, the EVT method seems to yield the best performance but, overall, none of these procedures seems able to pass convincingly the backtesting analysis. Remarkably, the VQR test rejects the correct performance of *all* the returns-based models for *any* of the conditional quantiles analyzed.

Next, we turn our attention to the backtesting results from the volume- and spread-extended CAViaR models. Table 5 displays the main backtesting results. The most remarkable feature is that, whereas the standard Restricted-CAViaR specification and other returns-based approaches exhibit large biases, the inclusion of the proxies for market liquidity and trading activity conditions improves the out-of-sample results considerably. The estimated VaR dynamics are shifted

Table 4: Backtesting VaR analysis for benchmark models. Volume weighted market portfolio.

MODEL	λ	Exc.	LR_{UC}	LR_{IND}	LR_{CC}	DQ	VQR
EWMA	7.5%	8.9%	2.68(0.10)	0.67(0.41)	3.38(0.18)	23.45(0.00)	14.11(0.00)
	5.0%	5.5%	0.51(0.47)	0.00(0.99)	0.52(0.77)	11.40(0.07)	23.54(0.00)
	2.5%	1.5%	4.78(0.02)	0.46(0.49)	5.21(0.07)	18.48(0.00)	84.25(0.00)
	1.0%	0.5%	3.09(0.08)	0.05(0.82)	3.13(0.21)	3.45(0.74)	177.84(0.00)
GARCH(1,1)	7.5%	11.3%	18.22(0.00)	0.29(0.59)	18.59(0.00)	44.21(0.00)	20.85(0.00)
	5.0%	7.4%	10.63(0.00)	0.46(0.49)	11.15(0.00)	29.44(0.00)	31.05(0.00)
	2.5%	2.8%	0.35(0.55)	0.08(0.78)	0.44(0.80)	22.51(0.00)	60.93(0.00)
	1.0%	0.9%	0.10(0.75)	0.16(0.69)	0.27(0.87)	11.40(0.07)	23.54(0.00)
EVT-BM	7.5%	10.4%	10.85(0.00)	0.58(0.45)	11.49(0.00)	18.24(0.00)	14.91(0.00)
	5.0%	6.1%	2.36(0.12)	0.51(0.48)	2.89(0.23)	9.31(0.15)	10.10(0.01)
	2.5%	2.4%	0.04(0.83)	0.31(0.58)	0.35(0.84)	3.32(0.76)	46.30(0.00)
	1.0%	0.5%	3.10(0.08)	0.04(0.84)	3.13(0.21)	3.64(0.72)	74.36(0.00)
SAV-CAViaR	7.5%	10.1%	8.86(0.00)	0.46(0.49)	8.74(0.01)	28.14(0.00)	9.53(0.01)
	5.0%	7.4%	10.63(0.00)	0.50(0.48)	11.19(0.00)	28.91(0.00)	9.63(0.01)
	2.5%	3.2%	1.85(0.17)	0.00(0.99)	1.86(0.39)	10.86(0.09)	11.77(0.00)
	1.0%	1.3%	0.83(0.36)	0.31(0.57)	1.15(0.56)	14.05(0.02)	27.28(0.00)

This table shows the Backtesting analysis for the one-day forecasts of the VaR given the EMWA, Gaussian GARCH, EVT and the Restricted-CAViaR (SAV-CAViaR) model using the volume weighted market portfolio. The second column shows the estimated ratio of empirical exceptions. LR_{UC}, LR_{IND}, LR_{CC}, DQ and VQR denote the values of the test statistics for unconditional coverage, independence, conditional coverage, DQ test and VQR test, respectively, (see Appendix B for details), whereas the p-values of the respective test statistics are exhibit in brackets.

Table 5: Backtesting VaR analysis for volume and liquidity extended CAViaR models. Volume weighted market portfolio. See details in Table 4.

X_t^*	λ	Exc.	LR_{UC}	LR_{IND}	LR_{CC}	DQ	VQR
V	7.5%	8.1%	0.51(0.48)	0.06(0.80)	0.43(0.81)	8.03(0.24)	1.72(0.42)
	5.0%	5.5%	0.51(0.47)	0.36(0.55)	0.88(0.64)	7.98(0.23)	1.69(0.43)
	2.5%	2.2%	0.38(0.53)	0.94(0.33)	1.32(0.51)	6.02(0.42)	5.36(0.07)
	1.0%	0.7%	1.02(0.31)	0.08(0.77)	1.09(0.58)	1.38(0.96)	29.78(0.00)
NT	7.5%	8.1%	0.51(0.48)	0.06(0.80)	0.43(0.81)	7.50(0.27)	1.10(0.58)
	5.0%	6.0%	1.98(0.16)	0.61(0.43)	2.61(0.27)	10.92(0.09)	3.15(0.21)
	2.5%	2.3%	0.17(0.68)	1.03(0.30)	1.20(0.55)	4.02(0.67)	2.13(0.34)
	1.0%	0.9%	0.10(0.75)	0.14(0.70)	0.25(0.88)	2.30(0.88)	33.69(0.00)
NS	7.5%	8.2%	0.69(0.41)	0.03(0.85)	0.55(0.76)	7.06(0.31)	1.42(0.49)
	5.0%	5.5%	0.51(0.47)	0.36(0.55)	0.88(0.64)	7.63(0.26)	1.56(0.46)
	2.5%	2.0%	1.10(0.29)	0.78(0.38)	1.87(0.39)	5.24(0.51)	5.29(0.07)
	1.0%	0.6%	1.89(0.17)	0.06(0.81)	1.94(0.38)	1.99(0.92)	32.95(0.00)
NSS	7.5%	8.0%	0.35(0.55)	0.01(0.91)	0.25(0.88)	6.29(0.39)	1.72(0.42)
	5.0%	5.6%	0.73(0.39)	0.28(0.59)	1.03(0.60)	9.53(0.16)	3.39(0.18)
	2.5%	1.9%	1.61(0.20)	0.70(0.40)	2.29(0.32)	5.63(0.46)	7.44(0.02)
	1.0%	0.6%	1.89(0.17)	0.06(0.81)	1.94(0.38)	2.15(0.90)	53.83(0.00)
TVD	7.5%	8.3%	0.89(0.34)	0.01(0.91)	0.71(0.69)	6.96(0.32)	2.50(0.29)
	5.0%	6.1%	2.39(0.12)	1.44(0.23)	3.86(0.14)	13.58(0.03)	3.97(0.14)
	2.5%	2.8%	0.36(0.55)	0.08(0.78)	0.44(0.80)	4.47(0.61)	2.13(0.34)
	1.0%	1.2%	0.38(0.54)	2.45(0.12)	2.83(0.24)	21.14(0.00)	47.69(0.00)
QS	7.5%	8.6%	0.51(0.48)	0.06(0.80)	0.43(0.81)	8.21(0.22)	5.53(0.06)
	5.0%	5.3%	0.18(0.67)	0.55(0.46)	0.75(0.69)	10.34(0.11)	5.39(0.07)
	2.5%	2.1%	0.69(0.40)	0.86(0.35)	1.54(0.46)	8.57(0.19)	21.89(0.00)
	1.0%	1.0%	0.00(1.00)	0.18(0.67)	0.18(0.91)	14.68(0.02)	95.41(0.00)
ES	7.5%	8.0%	0.35(0.55)	0.10(0.75)	0.34(0.84)	9.02(0.17)	5.26(0.07)
	5.0%	5.0%	0.00(1.00)	0.92(0.34)	0.92(0.63)	8.89(0.17)	4.56(0.10)
	2.5%	2.2%	0.38(0.53)	0.94(0.33)	1.32(0.51)	7.96(0.24)	23.53(0.00)
	1.0%	0.9%	0.10(0.75)	0.14(0.70)	0.25(0.88)	13.84(0.03)	37.52(0.00)
RQS	7.5%	8.6%	1.67(0.19)	0.09(0.75)	1.50(0.47)	0.08(0.08)	3.93(0.14)
	5.0%	5.5%	0.51(0.47)	0.36(0.55)	0.88(0.64)	12.46(0.05)	2.68(0.26)
	2.5%	2.4%	0.04(0.84)	1.13(0.28)	1.17(0.55)	13.43(0.03)	12.78(0.00)
	1.0%	0.7%	1.02(0.31)	0.08(0.28)	1.09(0.58)	1.56(0.95)	32.18(0.00)
RES	7.5%	7.9%	0.23(0.63)	0.00(0.97)	0.13(0.93)	9.02(0.17)	5.46(0.07)
	5.0%	5.5%	0.51(0.47)	0.36(0.55)	0.88(0.64)	10.94(0.09)	1.44(0.46)
	2.5%	2.4%	0.04(0.84)	1.13(0.29)	1.17(0.56)	7.53(0.27)	4.71(0.10)
	1.0%	1.0%	0.00(1.00)	0.18(0.67)	0.18(0.91)	12.59(0.05)	38.64(0.00)

(see, Figure 2 and comments below for a discussion) in the correct direction such that most of the empirical departures from the theoretical coverage are removed. The empirical exception rates tend to stabilize around the nominal levels without generating dependence or clustering in the exceptions. As a result, all covariate-extended CAViaR models are able to amply pass the three backtesting analyses of Christoffersen (1988) at any of the usual confidence levels. Similarly, the DQ test tends to largely support the suitability of the extended models, showing sizeable statistical gains with respect to the restricted case in the vast majority of cases analyzed. Finally, and in sharp contrast to the results from returns-based methodologies, the VQR test yields supportive evidence for correct conditional coverage in the extended quantile regression setting, particularly, for the set of variables in the volume group and for quantiles larger than 1%. For the 1% quantile, however, the VQR rejects the null hypothesis of correct coverage. In view of the overall success, particularly for variables such as Number of Trades, for which the remaining testing procedures strongly support the correct coverage hypothesis. Under the VQR metric, therefore, none of the different models and methodologies used in our analysis are able to pass the back tests when addressing the 1% percentile. [10]

Coupled with the evidence presented in the predictive-regression analysis, the overall evidence in the out-of-sample results allows us to conclude that the degree of liquidity and the trading conditions that characterize the market, as proxyed by the different variables considered and, particularly, by those related to volume characteristics, are predictors of the day-ahead conditional distribution of daily returns and improve the out-of-sample performance of the VaR forecasts. For extreme quantiles, such as 1%, the statistical evidence in our analysis is less conclusive: while the testing procedures based on quantile-regression inference do not support predictability for this percentile, the remaining back tests analyses do.

It is interesting to discuss in greater detail the differences between the forecasts from the restricted and unrestricted CAViaR models. To this end, Figure 2 displays the one-day 95% VaR forecasts from the restricted Restricted-CAViaR model (solid line) against the unrestricted CAViaR model extended with either Relative Quoted Spread (RQS) or Volume (black and grey dashed lines respectively). As shown in Table 4, the actual proportion of VaR exceptions from the Restricted-CAViaR model over the sample is much higher than the expected 5%, and consequently the model is biased towards underestimating the actual level of market risk in our sample. As depicted in Figure 2, the inclusion of the proxies for market liquidity and trading activity generates an upward shift in the dynamics of the predicted VaR process and introduces further variability in the forecasts with respect to the predictions from the restricted model. As a result, the gap between

[10] This may be symptomatic of size distortions. Piazza *et al.* (2009) analyze the small-sample properties of the test through Monte Carlo simulation. Under experimental conditions, it is shown that the VQR tends to reject the null model more frequently than expected, particularly, for small quantiles such as $\lambda = 0.01$. On real data, the true characteristics of the unknown data generating process may interfere with the asymptotic properties of the test and introduce further distortions.

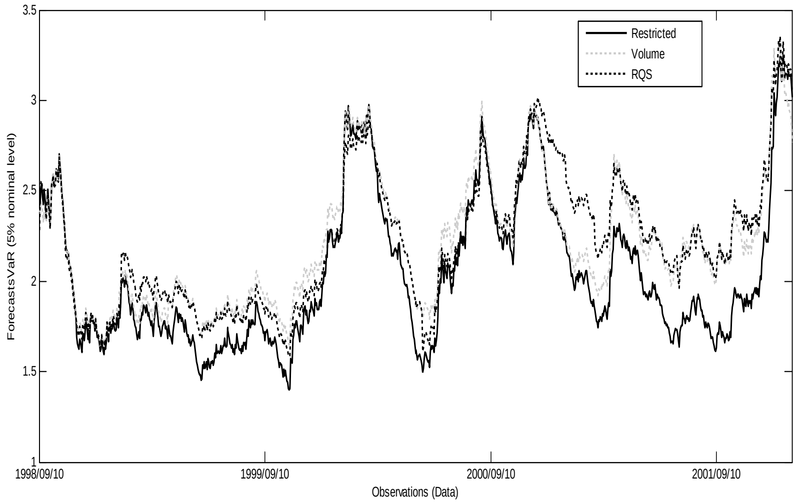

Figure 2: Forecasts VaR at nominal level 5% of Restricted-CAViaR model (solid line) and Unrestricted CAViaR model extended with volume and Relative Quoted Spread (RQS) variable (grey and black dashed lines respectively).

the expected and the actual proportion of exceptions is reduced, which improves the unconditional performance of the model, as discussed previously. Furthermore, and as is revealed from the back test analysis, this improvement is achieved without generating clusters of patches of exceptions. In view of overwhelming statistical support given by all the backtesting procedures and the quantile predictive regression, we therefore must conclude that the VaR predictions from the risk models extended with the variables that proxy for liquidity and, particularly, volume, are able to track the dynamics followed by the true VaR process more closely than a purely-returns based model.

3.3 B/M and size portfolios

It is interesting to analyze the quantile-predictability of other representative portfolios. Fama and French (1992) found that B/M and size characteristics seem to capture most of the cross-section of average stock returns. Also, it is usual that investors consider risk profiles based on growth/value and size to make financial decisions. Hence, it deserves interest to analyze predictability at different quantiles for B/M-sorted (Low30 and High30) and Size-sorted (Low30 and High30) portfolios. The results based on predictive quantile regressions are similar to those discussed previously. We therefore discuss the main results for the out-of-sample analysis, since some meaningful differences worthy of comment arise in this case.

Tables 6.1 and 6.2 report the main outcomes from the backtesting analysis of the CAViaR models extended with trade- and order-related variables, respectively, for the Low30 and High30 B/M portfolios given $\lambda \in \Theta_\lambda$[11]. There are meaningful differences across these portfolios. The LR_{UC} and LR_{CC} tests tend to indicate a correct performance for all the covariate-extended models for both portfolios. However, the DQ and, particularly, the VQR test, show a much more conservative picture about the overall predictability of the Low30 portfolio, particularly, for spread-related variables. In contrast, the overall evidence of predictability for the High30 B/M portfolio is much stronger. All the back tests, including VQR, suggest that the spread-related variables (particularly, effective and relative effective spread) tend to predict correctly the distribution of the return at the quantiles analyzed. Therefore, the extent of empirical predictability varies attending to growth/value profiles of the portfolios involved, which in turn favours certain variables over others. As in the case of the market portfolio, the VQR test rejects the correct performance of all the returns-based risk models for all the quantiles $\lambda \in \Theta_\lambda$ involved, both in the Low30 and High30 portfolios (results not reported for saving space). Therefore, the overall evidence suggests that models including state variables that capture different aspects of the trading process outperform models based on returns exclusively.

[11] The results of the independence test LR_{IND} are not presented in this section in order to save space, although these results are available upon request.

Table 6.1: Backtesting VaR analysis for volume extended CAViaR models. B/M portfolio (Low 30 and High30). See details in Table 4.

		B/M (Low30)					B/M (High30)				
X_t^*	λ	Exc.	LR_{UC}	LR_{CC}	DQ	VQR	Exc.	LR_{UC}	LR_{CC}	DQ	VQR
V	7.5%	8.7%	1.98(0.15)	1.72(0.42)	11.92(0.06)	2.09(0.35)	8.8%	2.31(0.12)	2.86(0.23)	4.36(0.63)	3.37(0.19)
	5.0%	5.5%	0.51(0.47)	0.52(0.77)	8.05(0.23)	2.11(0.35)	6.9%	6.83(0.01)	8.02(0.01)	12.09(0.06)	12.83(0.00)
	2.5%	2.3%	0.16(0.68)	0.56(0.75)	5.44(0.49)	6.09(0.05)	3.6%	4.37(0.04)	4.46(0.10)	14.77(0.02)	9.87(0.01)
	1.0%	1.0%	0.00(1.00)	3.17(0.20)	14.32(0.03)	15.30(0.00)	1.3%	0.83(0.36)	1.17(0.55)	9.05(0.17)	4.51(0.10)
NT	7.5%	8.8%	2.31(0.12)	2.06(0.35)	11.92(0.06)	2.09(0.35)	8.9%	2.68(0.10)	3.07(0.21)	4.10(0.66)	4.99(0.08)
	5.0%	5.9%	1.61(0.20)	1.69(0.42)	15.57(0.02)	2.38(0.30)	6.8%	6.17(0.01)	7.51(0.02)	11.96(0.06)	13.26(0.00)
	2.5%	2.5%	0.04(0.83)	2.47(0.28)	7.98(0.24)	3.71(0.16)	3.1%	1.38(0.24)	1.39(0.49)	7.77(0.25)	1.82(0.40)
	1.0%	1.0%	0.00(1.00)	3.17(0.20)	12.86(0.05)	23.05(0.00)	1.3%	0.83(0.36)	2.83(0.24)	19.17(0.00)	0.38(0.83)
NS	7.5%	8.6%	1.67(0.19)	1.41(0.49)	14.61(0.02)	2.06(0.36)	8.8%	2.32(0.13)	2.86(0.23)	4.16(0.65)	3.85(0.15)
	5.0%	5.4%	0.33(0.56)	0.34(0.84)	8.01(0.24)	1.23(0.54)	6.8%	6.16(0.01)	7.51(0.02)	12.08(0.06)	12.74(0.00)
	2.5%	2.4%	0.04(0.83)	2.47(0.28)	8.27(0.22)	2.30(0.30)	3.2%	1.85(0.17)	1.86(0.39)	7.81(0.25)	11.13(0.00)
	1.0%	0.7%	1.01(0.31)	5.73(0.05)	19.33(0.00)	45.13(0.00)	1.2%	0.38(0.54)	2.67(0.26)	18.49(0.01)	0.38(0.83)
NSS	7.5%	8.4%	1.12(0.28)	0.91(0.63)	11.36(0.08)	2.51(0.29)	8.9%	2.67(0.10)	3.07(0.21)	4.12(0.66)	3.73(0.15)
	5.0%	5.6%	0.73(0.39)	0.74(0.68)	10.82(0.09)	2.20(0.33)	6.7%	5.52(0.02)	6.14(0.04)	8.76(0.19)	15.51(0.00)
	2.5%	2.2%	0.38(0.53)	0.87(0.64)	4.74(0.58)	4.30(0.12)	3.2%	1.84(0.19)	1.86(0.39)	7.70(0.26)	10.47(0.01)
	1.0%	0.9%	0.10(0.74)	3.72(0.05)	15.46(0.02)	14.45(0.00)	1.5%	2.18(0.13)	2.62(0.26)	17.72(0.00)	15.79(0.00)
TVD	7.5%	8.9%	2.67(0.10)	2.43(0.29)	14.04(0.03)	3.40(0.18)	9.0%	3.06(0.08)	3.31(0.19)	4.21(0.65)	4.09(0.13)
	5.0%	6.4%	3.80(0.05)	4.08(0.12)	15.60(0.02)	3.20(0.20)	6.8%	6.16(0.01)	7.51(0.02)	12.33(0.05)	11.12(0.00)
	2.5%	2.7%	0.16(0.68)	1.88(0.38)	6.42(0.38)	5.46(0.07)	3.4%	2.99(0.08)	3.61(0.16)	10.19(0.12)	6.80(0.03)
	1.0%	1.4%	1.43(0.23)	3.31(0.19)	11.66(0.07)	26.72(0.00)	1.0%	0.00(1.00)	0.18(0.91)	0.43(0.99)	0.06(0.97)

Table 6.2: Backtesting VaR analysis for spread extended CAViaR models. B/M portfolio (Low 30 and High30). See details in Table 4.

		B/M (Low30)					B/M (High30)				
X_t^*	λ	Exc.	LR_{UC}	LR_{CC}	DQ	VQR	Exc.	LR_{UC}	LR_{CC}	DQ	VQR
QS	7.5%	8.9%	2.67(0.10)	2.43(0.29)	14.19(0.03)	4.89(0.09)	8.6%	1.67(0.19)	1.89(0.38)	2.69(0.84)	3.54(0.17)
	5.0%	5.5%	0.51(0.47)	0.52(0.77)	16.95(0.01)	12.49(0.00)	6.1%	2.38(0.12)	2.91(0.23)	7.51(0.27)	4.89(0.09)
	2.5%	2.1%	0.69(0.40)	1.58(0.45)	12.10(0.06)	80.92(0.00)	2.8%	0.35(0.55)	0.43(0.80)	17.05(0.01)	1.11(0.57)
	1.0%	0.7%	1.01(0.31)	1.09(0.57)	3.29(0.77)	41.48(0.00)	0.8%	0.43(0.51)	0.55(0.75)	12.28(0.06)	0.14(0.93)
ES	7.5%	8.4%	1.12(0.28)	0.91(0.63)	15.74(0.02)	5.34(0.07)	8.2%	0.68(0.40)	1.49(0.47)	2.83(0.83)	2.56(0.28)
	5.0%	5.4%	0.32(0.56)	0.34(0.84)	0.05(0.05)	10.63(0.00)	5.7%	0.98(0.32)	1.99(0.36)	6.58(0.36)	3.92(0.14)
	2.5%	2.5%	0.00(1.00)	0.23(0.88)	5.55(0.05)	29.39(0.00)	2.4%	0.04(0.83)	0.34(0.83)	4.76(0.57)	2.62(0.27)
	1.0%	0.9%	0.10(0.74)	0.24(0.88)	2.38(0.88)	22.26(0.00)	0.9%	0.10(0.74)	0.26(0.87)	20.00(0.00)	0.18(0.91)
RQS	7.5%	9.1%	3.47(0.06)	3.28(0.19)	15.45(0.01)	5.31(0.07)	8.9%	2.67(0.10)	3.07(0.21)	4.40(0.62)	5.45(0.07)
	5.0%	5.3%	0.18(0.66)	0.74(0.68)	14.77(0.02)	22.93(0.00)	6.3%	3.29(0.06)	3.65(0.16)	7.16(0.31)	7.67(0.02)
	2.5%	2.5%	0.00(1.00)	0.23(0.88)	9.81(0.13)	55.98(0.00)	2.9%	0.62(0.42)	0.67(0.71)	16.10(0.01)	3.41(0.18)
	1.0%	1.0%	0.00(1.00)	0.18(0.91)	2.23(0.89)	12.81(0.00)	1.0%	0.00(1.00)	0.18(0.91)	21.14(0.00)	6.25(0.04)
RES	7.5%	8.4%	1.12(0.28)	0.91(0.63)	10.99(0.08)	2.91(0.23)	8.8%	2.31(0.12)	2.87(0.23)	4.09(0.66)	3.67(0.16)
	5.0%	5.6%	0.73(0.39)	1.02(0.59)	14.19(0.02)	22.63(0.00)	6.3%	3.29(0.06)	4.44(0.11)	7.06(0.32)	8.77(0.01)
	2.5%	2.5%	0.00(1.00)	0.23(0.88)	9.20(0.16)	14.33(0.00)	2.4%	0.04(0.83)	0.34(0.84)	11.77(0.07)	4.37(0.11)
	1.0%	1.4%	1.43(0.23)	3.31(0.19)	18.04(0.01)	45.92(0.00)	1.3%	0.83(0.36)	1.15(0.56)	19.21(0.00)	2.53(0.28)

Table 7.1: Backtesting VaR analysis for volume extended CAViaR models. Size portfolio (Low30 and High30). See details in Table 4.

		SIZE (Low30)					SIZE (High30)				
X_t^*	λ	Exc.	LR_{UC}	LR_{CC}	DQ	VQR	Exc.	LR_{UC}	LR_{CC}	DQ	VQR
V	7.5%	7.2%	0.13(0.71)	0.29(0.86)	3.77(0.71)	1.41(0.49)	8.7%	1.98(0.15)	1.72(0.42)	9.98(0.13)	2.17(0.34)
	5.0%	3.8%	3.29(0.06)	3.50(0.17)	8.75(0.19)	1.90(0.39)	5.9%	1.61(0.20)	2.09(0.35)	9.99(0.12)	2.50(0.29)
	2.5%	2.3%	0.16(0.68)	1.24(0.53)	2.00(0.92)	1.90(0.39)	2.6%	0.04(0.84)	1.98(0.37)	3.94(0.68)	1.28(0.53)
	1.0%	1.4%	1.43(0.23)	1.84(0.39)	2.84(0.83)	6.13(0.05)	0.7%	1.01(0.31)	1.09(0.57)	2.80(0.83)	11.96(0.00)
NT	7.5%	7.4%	0.50(0.47)	0.51(0.77)	7.10(7.11)	2.65(0.27)	8.7%	1.98(0.15)	1.72(0.42)	9.54(0.15)	1.56(0.46)
	5.0%	3.9%	2.74(0.09)	2.90(0.23)	7.79(0.25)	2.75(0.25)	6.0%	1.98(0.15)	1.71(0.42)	8.53(0.20)	2.56(0.28)
	2.5%	2.20%	0.38(0.53)	1.36(0.50)	2.25(0.89)	6.01(0.05)	2.6%	0.04(0.84)	1.98(0.37)	4.76(0.57)	0.78(0.68)
	1.0%	1.2%	0.37(0.53)	0.67(0.71)	1.19(0.98)	3.99(0.14)	0.7%	1.01(0.31)	1.09(0.57)	2.71(0.84)	14.94(0.00)
NS	7.5%	7.3%	0.05(0.80)	0.17(0.91)	4.25(0.64)	4.72(0.09)	8.8%	2.31(0.12)	2.06(0.35)	9.59(0.14)	1.90(0.39)
	5.0%	3.8%	3.29(0.06)	3.50(0.17)	8.72(0.19)	2.80(0.25)	5.5%	0.51(0.47)	1.73(0.42)	8.99(0.17)	2.50(0.29)
	2.5%	2.2%	0.38(0.53)	1.36(0.50)	2.26(0.90)	2.80(0.25)	2.4%	0.04(0.83)	0.34(0.83)	2.66(0.85)	2.03(0.36)
	1.0%	1.2%	0.37(0.53)	0.67(0.71)	1.16(0.98)	7.95(0.02)	0.6%	1.88(0.16)	1.93(0.37)	2.48(0.87)	13.09(0.00)
NSS	7.5%	7.3%	0.05(0.80)	0.17(0.91)	3.36(0.76)	5.89(0.05)	8.6%	1.67(0.19)	1.41(0.49)	9.31(0.16)	1.32(0.52)
	5.0%	3.8%	3.29(0.06)	0.62(0.17)	8.75(0.19)	3.26(0.20)	5.8%	1.28(0.25)	1.92(0.38)	9.94(0.13)	1.74(0.42)
	2.5%	2.1%	0.69(0.40)	1.58(0.45)	2.68(0.85)	6.16(0.05)	2.1%	0.69(0.40)	1.54(0.46)	3.67(0.72)	1.15(0.57)
	1.0%	1.3%	1.17(0.55)	1.17(0.55)	1.93(0.92)	1.92(0.38)	0.6%	1.88(0.16)	1.93(0.37)	2.63(0.85)	15.20(0.00)
TVD	7.5%	7.5%	0.00(1.00)	0.04(0.97)	5.15(0.52)	5.10(0.08)	9.2%	3.90(0.04)	3.57(0.16)	12.04(0.06)	3.37(0.19)
	5.0%	3.6%	4.55(0.03)	4.92(0.08)	9.62(0.14)	2.29(0.32)	6.3%	3.29(0.06)	4.04(0.13)	12.25(0.06)	3.56(0.17)
	2.5%	2.3%	0.16(0.68)	1.24(0.53)	2.00(0.92)	7.17(0.03)	3.3%	2.38(0.12)	3.12(0.20)	6.53(0.37)	2.22(0.33)
	1.0%	1.2%	0.37(0.53)	0.67(0.71)	1.16(0.98)	1.01(0.60)	1.0%	0.00(1.00)	3.17(0.20)	12.47(0.05)	10.82(0.00)

Table 7.2: Backtesting VaR analysis for spreads extended CAViaR models. Size portfolio (Low30 and High30). See details in Table 4.

		SIZE (Low30)					SIZE (High30)				
X_t^*	λ	Exc.	LR_{UC}	LR_{CC}	DQ	VQR	Exc.	LR_{UC}	LR_{CC}	DQ	VQR
QS	7.5%	7.6%	0.01(0.90)	0.03(0.98)	8.14(0.23)	3.31(0.19)	8.4%	1.12(0.28)	0.91(0.63)	13.92(0.03)	6.45(0.04)
	5.0%	4.1%	1.81(0.17)	1.85(0.39)	10.36(0.11)	8.05(0.02)	5.7%	0.98(0.32)	3.12(0.21)	15.12(0.02)	4.52(0.10)
	2.5%	2.4%	0.04(0.83)	1.22(0.54)	5.43(0.49)	7.06(0.03)	2.1%	0.69(0.40)	1.54(0.46)	4.91(0.56)	7.87(0.02)
	1.0%	1.5%	2.18(0.13)	2.65(0.26)	4.18(0.65)	3.01(0.22)	0.6%	1.88(0.16)	1.95(0.37)	3.47(0.75)	76.58(0.00)
ES	7.5%	7.2%	0.13(0.71)	0.29(0.86)	9.51(0.15)	3.37(0.19)	8.2%	0.68(0.40)	0.55(0.75)	12.67(0.05)	6.45(0.04)
	5.0%	3.8%	3.29(0.06)	3.47(0.17)	9.61(0.14)	3.34(0.19)	5.4%	0.32(0.56)	1.77(0.41)	12.76(0.05)	4.12(0.13)
	2.5%	2.3%	0.16(0.68)	1.24(0.53)	5.81(0.44)	9.02(0.01)	2.1%	0.69(0.40)	1.29(0.52)	4.22(0.65)	10.49(0.01)
	1.0%	1.5%	2.18(0.13)	2.65(0.26)	4.36(0.63)	2.14(0.34)	0.7%	0.00(1.00)	1.10(0.57)	4.90(0.56)	12.61(0.00)
RQS	7.5%	7.6%	0.01(0.90)	0.34(0.84)	10.27(0.11)	2.58(0.27)	8.5%	1.38(0.23)	1.14(0.56)	11.38(0.08)	2.58(0.27)
	5.0%	4.1%	1.81(0.17)	1.85(0.39)	9.45(0.15)	11.62(0.00)	5.8%	1.28(0.25)	1.92(0.38)	17.27(0.01)	4.23(0.12)
	2.5%	2.3%	0.16(0.68)	1.24(0.53)	2.03(0.92)	4.21(0.12)	2.5%	0.00(1.00)	1.23(0.54)	4.13(0.66)	2.28(0.32)
	1.0%	1.4%	1.43(0.23)	1.84(0.39)	7.30(0.29)	2.62(0.27)	0.8%	0.43(0.51)	0.54(0.76)	1.86(0.93)	13.29(0.00)
RES	7.5%	7.3%	0.05(0.80)	0.17(0.91)	7.55(0.27)	3.46(0.18)	6.9%	0.53(0.46)	0.90(0.63)	11.31(0.08)	2.36(0.31)
	5.0%	3.8%	3.29(0.06)	3.47(0.17)	8.84(0.18)	9.69(0.01)	5.7%	0.98(0.32)	0.98(0.60)	14.84(0.02)	8.76(0.01)
	2.5%	2.1%	0.69(0.40)	1.58(0.45)	2.66(0.85)	15.37(0.00)	2.2%	0.38(0.38)	0.87(0.64)	5.28(0.51)	3.52(0.17)
	1.0%	1.4%	1.43(0.23)	1.84(0.39)	2.89(0.82)	2.21(0.33)	0.5%	3.09(0.07)	3.12(0.20)	2.97(0.81)	70.59(0.00)

Finally, Tables 7.1 and 7.2 show the main outcomes for the backtesting analysis on the Low30 and High30 Size-sorted portfolios given the variables in the volume and liquidity groups, respectively. The results in this analysis are, in general, more similar to those discussed for the volume-weighted market portfolio. The backtesting analysis reveals a great performance of both volume- and liquidity-extended risk models to forecast the VaR of the Low30 portfolio at any of the quantiles. The best results are observed for volume-related variable, for which all the back tests, including the VQR, tend to accept the suitability of the model. For the High30 portfolio, however, the results are more conservative and in line with those already discussed for the volume-weighted portfolio: volume and liquidity proxies seem to make a good job for most quantiles under the different tests, with the overall evidence being more ambiguous for the 1% percentile in the VQR test. This suggests that the conditional distribution of small capitalization stocks is predictable, particularly, when including volume-related variables. It should be noted that the overall performance of returns-based procedure (not presented) seems better for small capitalization assets, with the extreme value theory models yielding the best performance throughout. Nevertheless, the performance is inferior to the extended models and, as in the previous cases, none of the returns-based models are able to pass the VQR test.

4 Concluding remarks

In this paper, we have analyzed the predictability of the tail of the conditional distribution of market returns using different variables that are widely related to trading activity and market liquidity. Our methodological approach has mainly built on the quantile regression methodology and, particularly, the CAViaR quantile regression setting proposed in Engle and Manganelli (2004). This strategy allows us to study in a simple and direct way the forecasting ability of a number of covariate-extended models in relation to a restricted model that relies solely on returns, using both predictive regressions and backtesting analysis.

The extent of statistical evidence supporting predictability may vary depending mainly on the size of the target quantile. However, the overall evidence strongly suggests the existence of predictability of the conditional distribution on the basis of trading activity and liquidity variables. This evidence is particularly strong for the set of volume-related variables in the market portfolio and, more generally, is fairly robust against the consideration of different representative market portfolios, different proxy variables of liquidity and trading activity, and different testing procedures. Consequently, the main conclusion from this paper is that the day-ahead conditional distribution of returns is predictable when using observable market information which is not necessarily limited to returns and, therefore, such information can be used in downside risk modelling.

Finally, although our analysis has focused on the analysis of quantiles and, therefore, allows us to obtain direct conclusions for the VaR methodology, the main conclusions from our analysis may be extrapolated to other quantile-based downside risk measures, among which expected shortfall (conditional VaR) is the most representative. If quantiles are predictable, trivial transformations such as the mean of quantiles should be predictable as well. The formal analysis of this interesting issue is left for future research.

Appendix A: VaR models

In Section 4 we compare the performance of CAViaR model with other standard procedures to compute VaR. These include the EWMA, GARCH and Extreme Value Theory methods. The common setting in these parametric models assumes that (conditionally demeaned) returns obey dynamics given by

$$r_t = \sigma_t \eta_t, \quad \eta_t | \mathcal{F}_{t-1} \sim iid\mathcal{N}(0,1) \tag{A.1}$$

where σ_t denotes the conditional volatility of the process. We briefly discuss the main settings of these approaches in the sequel.

A. VaR EWMA

RiskMetrics popularized the Exponential Weighting Moving Average (EWMA) scheme as an easy way to model the volatility process. The latent volatility dynamics are assumed to obey the recursive dynamics:

$$\widehat{\sigma}_t^2 = \xi \ \widehat{\sigma}_{t-1}^2 + (1-\xi) r_{t-1}^2, \quad t = 1, ..., T \tag{A.2}$$

with the initial condition $\widehat{\sigma}_0^2 = r_0^2 = E\left(r_t^2\right)$. The smoothing parameter $0 \leq \xi \leq 1$ can be estimated, although RiskMetrics advises the setting $\xi = 0.95$ for data recorded on a daily basis. Then, the one-day ahead forecast given $\mathcal{F}_T$ is simply given by $\widehat{\sigma}_{T+1|T}^2 = \xi \ \widehat{\sigma}_T^2 + (1-\xi) r_T^2$.

RiskMetrics further assumes the particularly strong assumption that the innovations η_t are conditionally normal distributed, from which the one-period ahead VaR forecast would be given by $-\mathcal{Z}_\lambda \widehat{\sigma}_{T+1|T}$, with $\mathcal{Z}_\lambda$ denoting the λ-quantile of the standard normal distribution. In order to ensure robustness against departures from normality, we proceed in a slightly different way. Let $\widehat{\eta}_t = r_t / \widehat{\sigma}_t$ be the estimated innovations given the estimates of the EWMA volatility process, and let $Q_\lambda(\widehat{\eta}_t)$ be the unconditional λ-quantile of the empirical distribution of $\widehat{\eta}_t$. Then, a 'robustified' VaR forecast that does not rely upon distributional assumption is given by:

$$VaR_{\lambda,t+1}\left(EWMA\right) = -Q_\lambda(\widehat{\eta}_t) \widehat{\sigma}_{T+1|T} \tag{A.3}$$

B. VaR GARCH

The simplest GARCH (1,1) model is the most popular approach to model and forecast market risk in practice due to its impressive performance and statistical properties (Hansen and Lunde, 2005). The standard GARCH(1,1) model assumes that daily returns obey dynamics given by:

$$\begin{aligned} r_t &= \sigma_t \eta_t, \quad \eta_t | \mathcal{F}_{t-1} \sim iid\mathcal{N}(0,1) \\ \sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{A.4}$$

with the restrictions $\omega > 0$, α, $\beta \geq 0$ ensuring that the conditional variance process is well-defined. Although financial returns are known to be non-normally distributed, the Gaussian assumption is particularly convenient because it ensures parameter consistency under certain regularity conditions even in the absence of normality. Parameters can thus be estimated by (quasi) maximum likelihood estimation, yielding a consistent estimate of the conditional variance process. The day-ahead forecast is then given by:

$$\widehat{\sigma}^2_{T+1|T} = \widehat{\omega} + \widehat{\alpha} r_T^2 + \widehat{\beta}\ \widehat{\sigma}_T^2 \tag{A.5}$$

As in the EWMA model discussed previously, given the GARCH estimates $\widehat{\sigma}_t$ and the resultant standardized innovations, $\widehat{\eta}_t = r_t/\widehat{\sigma}_t$, the 'robust' one-day VaR-GARCH forecast is determined as:

$$VaR_{\lambda,T+1}\,(GARCH) = -Q_\lambda(\widehat{\eta}_t)\widehat{\sigma}_{T+1|T}. \tag{A.6}$$

with $Q_\lambda(\widehat{\eta}_t)$ denoting the λ-quantile of the empirical distribution.

C. VaR Extreme Value Theory

This method can be seen as a parametric refinement of the previous approaches. Essentially, the procedure requires the characterization of the tail behavior of the set of i.i.d. innovations η_t in the return process. To circumvent the problem that η_t is not observable directly, the estimated residuals $\widehat{\eta}_t = r_t/\widehat{\sigma}_t$ can be used instead, with $\widehat{\sigma}_t$ determined according to some volatility model, such as those discussed previously. Since GARCH estimates tend to outperform any other procedure, we estimate the empirical process $\widehat{\eta}_t$ on the basis of the GARCH(1,1) estimates as discussed above.

The rest of the procedure is described as follows. Given the series $-\widehat{\eta}_t$, the total sample period is divided into $B = 740$ blocks of length $l = 5$ observations to record the maximum value of each block (*i.e.*, the maximum loss in the period), say m_b, $b = 1, ..., B$, in a time-series process. The Extreme Value Theory suggests fitting the Generalized Extreme Value distribution (GEV, also known as Fisher–Tippett distribution) to this series. The GEV arises as the limit distribution of properly normalized maxima of a sequence of i.i.d. random variables, and is characterized by the density function

$$f\left(z_b;\rho_1,\rho_2,\rho_3\right) = \left[\frac{1}{\rho_2}\left[1+\rho_3 z_b\right]\right]^{-1-1/\rho_3} \exp\left\{-\left[1+\rho_3 z_b\right]^{-1/\rho_3}\right\} \tag{A.7}$$

if $z_b > -1$, where $z_b = \left(m_b - \rho_1\right)/\rho_2$ denotes the standardized variable. The (unknown) parameters characterize the shape (ρ_1), scale (ρ_2) and location (ρ_3) of the distribution and can be estimated consistently by different methods, such as maximum-likelihood. The importance of this approach is that by inverting this distribution (with the unknown parameters replaced by their consistent estimates), we can go from the asymptotic GEV distribution of maxima to the distribution of the observations themselves and obtain a closed-form expression for the unconditional VaR of $\widehat{\eta}_t$ given λ, namely,

$$Q_\lambda(\widehat{\eta}_t) = \widehat{\rho}_3 - \frac{\widehat{\rho}_2}{\widehat{\rho}_1}\left[1 - \left\{-\log\left(1-\frac{1}{\lambda l}\right)\right\}^{-\widehat{\rho}_1}\right] \tag{A.8}$$

Finally, as in the EWMA and GARCH approaches, we generate the one-day ahead

VaR forecast as

$$VaR_{\lambda,T+1}\left(EVT\right) = -\widehat{\sigma}_{T+1|T}Q_{\lambda}(\widehat{\eta}_t) \tag{A.9}$$

with $\widehat{\sigma}_{T+1|T}$ determined as in $(A.5)$.

Appendix B: Backtesting analysis

I) Unconditional test, LR_{UC}.

The most basic assumption is that the market risk model provides a correct unconditional coverage, namely, $\mathrm{H}_{0,U} : E\left[H_{\lambda,t}\right] = \lambda$. The null hypothesis is rejected for large values of the likelihood-ratio test defined as

$$LR_{UC} = 2(N - N_\lambda)\left[\log(1 - \frac{N_\lambda}{N}) - \log(1-\lambda)\right] + 2N_\lambda\left[\log\frac{N_\lambda}{N} - \log\lambda\right] \sim \chi^2_{(1)} \tag{B.1}$$

where $\chi^2_{(1)}$ stands for a Chi-squared distribution with one degree of freedom, $N_\lambda \equiv \sum_{t=1,N} H_{t,\lambda}$ is the number of exceptions, and N is the total number of out-of-sample observations. Note that $N_\lambda/N = \widehat{\lambda}_H$ is simply the sample mean of $H_{\lambda,t}$, $i.e$, the sample equivalent of $E\left[H_{\lambda,t}\right]$.

II) Independence test, LR_{IND}.

If exceptions are serially correlated, the property of reliability conditional coverage will be defective even if the unconditional coverage is correct, because the risk of bankruptcy is higher. Christoffersen (1998) proposes the analysis of the first-order serial correlation in $H_{\lambda,t}$ through a binary first-order Markov chain with transition probability matrix

$$\Pi = \begin{pmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{01} \end{pmatrix}, \text{ with } \quad \pi_{ij} = \Pr(H_{\lambda,t} = j \mid H_{\lambda,t-1} = i), i, j \in \{0,1\} \tag{B.2}$$

The approximate joint likelihood conditional on the first observation is

$$\mathcal{L}(\Pi; H_{\lambda,t} \mid H_{\lambda,1}) = (1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}, \tag{B.3}$$

where n_{ij} represents the number of transitions from state i to state j. The maximum-likelihood estimators under the alternative hypothesis are $\widehat{\pi}_{01} = n_{01}/\left(n_{00} + n_{01}\right)$, and $\widehat{\pi}_{11} = n_{11}/\left(n_{10} + n_{11}\right)$. Under the null hypothesis of independence, we have $\pi_{01} = \pi_{11} = \pi_0$, with $\pi_0 = \lambda$, from which the conditional binomial joint likelihood is

$$\mathcal{L}(\pi_0; H_{\lambda,t} \mid H_{\lambda,1}) = (1-\pi_{01})^{n_{00}+n_{10}}\pi_{01}^{n_{01}+n_{11}}. \tag{B.4}$$

Note that π_0 can be estimated as $\widehat{\pi}_0 = N_\lambda/N$. The likelihood ratio test for the hypothesis of independence is given by

$$LR_{IND} = 2\left[\log\mathcal{L}(\hat{\Pi}; H_{\lambda,t} \mid H_1) - \log\mathcal{L}(\widehat{\pi}_0; H_{\lambda,t} \mid H_{\lambda,1})\right] \sim \chi^2_{(1)} \tag{B.5}$$

III) Conditional test, LR_{CC}.

Finally, we can study simultaneously whether the VaR violations are independent and occur with the correct probability, *i.e.*, $\mathrm{H}_{0,C} : E\left[H_{\lambda,t}|\mathcal{F}_{t-1}\right] = \lambda$. Because $\widehat{\pi}_0$ is unconstrained, the test in equation (B.5) does not impose the correct coverage. Christoffersen (1998) devised a joint test for independence and correct coverage (*i.e.*, correct conditional coverage) by combining the previous tests:

$$LR_{CC} = 2\left[\log \mathcal{L}(\hat{\Pi}; H_{\lambda,T} \mid H_1) - \log \mathcal{L}(\lambda; H_T \mid H_1)\right] \sim \chi^2_{(2)} \qquad \text{(B.6)}$$

This is equivalent to testing if the sequence of $H_{\lambda,t}$ is independent and the probabilities to observe an exception given the set of information is equal to the nominal level λ, namely, $\pi_{01} = \pi_{11} = \lambda$. Therefore, we can write

$$LR_{CC} = LR_{UC} + LR_{IND}, \qquad \text{(B.7)}$$

which provides the suitable test statistic to check whether $H_{\lambda,t}$ exhibits correct conditional coverage properties. Since the test involves two restrictions, the asymptotic convergence to a $\chi^2_{(2)}$ distribution.

IV) Dynamic Quantile test, DQ (Engle and Manganelli, 2004).

The LR_{CC} test does not have the power to detect higher-order dependences in $H_{\lambda,t}$. Engle and Manganelli (2004) introduced a test that accounts for a more general form of dependence in order to test $\mathrm{H}_{0,C} : E\left[H_{\lambda,t}|\mathcal{F}_{t-1}\right] = \lambda$. Define the time series:

$$Hit_{\lambda t} = H_{\lambda,t} - \lambda \qquad \text{(B.8)}$$

and let the matrix of instrumental variables

$$\mathbf{Z}_t = \left[Hit_{t-1}, Hit_{t-2}, ..., Hit_{t-p}, VaR_{\lambda,t-1}, ..., VaR_{\lambda,t-q}\right]. \qquad \text{(B.9)}$$

for some predetermined, fixed lag values $p, q \geq 1$. Note that $\mathrm{H}_{0,C} : E\left[H_{\lambda,t}|\mathcal{F}_{t-1}\right] = \lambda$ implies the martingale condition $E\left[Hit_{\lambda,t}|\mathcal{F}_{t-1}\right] = 0$, which in turn could tested as $E\left[Hit_{\lambda,t}|\mathbf{Z}_t\right] = 0$ for suitable values of p and q. Following Engle and Manganelli (2004), we set $p = 4, q = 1$ and test the martingale restriction through ordinary least-squares analysis in the auxiliary regression $Hit_t = \mathbf{Z}_t\beta + u_t$ by analyzing the joint restriction $\mathrm{H}_0 : \beta = 0$ through a standard F test, given by the test statistic

$$DQ = \frac{\widehat{\beta}'\mathbf{Z}_t'\mathbf{Z}_t\widehat{\beta}}{\lambda(1-\lambda)} \qquad \text{(B.10)}$$

with $\widehat{\beta}$ denoting the OLS estimate of β. Under the null hypothesis, $DQ \sim \chi^2_{(p+q)}$.

V) VaR Quantile Regression test, VQR (Piazza *et al.*, 2009).

Given the VaR forecasts $VaR_{\lambda,t}$, $t = 1, ..., N$, Piazza *et al.* (2009) focus on the encompassing quantile regression

$$r_t = \alpha_{0,\lambda} + \alpha_{1,\lambda} VaR_{\lambda,t} + \varepsilon_{t,\lambda}, \;\; t = 1, .., N \qquad \text{(B.11)}$$

where r_t denotes the realized returns over the out-of-sample period. The hypothesis that $VaR_{\lambda,t}$ is an optimal forecasts of the conditional λ-quantile of r_t can be tested through the joint restriction $H_0 : \alpha_{0,\lambda} = 0, \alpha_{1,\lambda} = 1$ or, equivalently, $H_0 : \beta_\lambda = 0$ with $\beta_\lambda = (\alpha_0, \alpha_1 - 1)'$ in the previous regression model.

Let $\widehat{\beta}_\lambda$ the quantile regression estimate of β_λ. Under standard regularity conditions, the asymptotic distribution of $\sqrt{N}\left(\widehat{\beta}_\lambda - \beta\right)$ is a normal with zero mean and finite variance Ω. Under the null, it follows that

$$VQR = T[\widehat{\beta}_\lambda' \Omega^{-1} \widehat{\beta}_\lambda] \sim \chi^2_{(2)} \tag{B.12}$$

where the covariance matrix can be estimated by usual methods.

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