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# Pricing executive stock options under employment shocks<sup>\*</sup>

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## Abstract

We obtain explicit expressions for the subjective, objective and market value of perpetual executive stock options (ESOs) under exogenous employment shocks driven by an independent Poisson process. Within this set-up, we obtain the executive's optimal exercise policy which allows us to analyze the determinants of both, the subjective valuation by executives and the objective valuation by firms. The perpetual ESO is compared with the more realistic finite maturity ESO finding that the approximation is reasonably good. We also use the objective valuation's results for accounting purposes. Further, we analyze the objective valuation distribution when there is uncertainty about the employment shock parameter. Finally, the role of ESOs in the design of executive's incentives is also discussed.

**Keywords:** ESO, Risk Aversion, Undiversification, Incentives, FAS 123R.

**JEL Classification:** G11, G13, G35, M52

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# 1 Introduction

The increasing relevance of executive stock options (ESOs) as a component of corporate compensation has led the International Accounting Standard Board (IASB) to issue the International Financial Reporting Standard 2 (IFRS 2) in February 2004. In March 2004, the Financial Accounting Standard Board (FASB) has also revised the Financial Accounting Standard 123 (FAS 123R) with a similar purpose, namely, to provide a fair value method for shared based compensation arrangement.<sup>1</sup> Computing the fair value of one ESO is not straightforward, this is so because ESOs are American call options modified to create the required incentives for aligning executive's goals with shareholders' interests. Thus, ESOs cannot be sold or transferred, although partial hedge is possible by trading correlated assets. In addition, they can only be exercised after ending the vesting period. Consequently, standard methods for valuing American options are not directly applicable and a growing literature has been searching for a solution to the issue of ESO valuation. As in Hull and White (2004), Leung and Sircar (2009) and FASB statement 123R, we also assume that the possible dilution effect is anticipated by the the market and already reflected in the stock price immediately after the ESO grant.

As any American call option, the value of an ESO depends on its payoff and this is clearly determined by the holder's exercise policy. In this regard, the literature has distinguished among three possible values for ESOs. First, there is a subjective ESO value given by what one ESO is worth to an executive. This value is quite different from the market price obtained from standard risk neutral valuation methods, as the Black-Scholes price for European options. This difference is clearly illustrated by a common finding in the empirical literature on the subject. As illustrated by the works of Huddart and Lang (1996) and Bettis et al. (2005), ESOs are typically exercised well before maturity and mostly right after the end of the vesting period. In general, this behavior is inconsistent with the exercise policy that a well diversified investor facing an unrestricted environment would follow, giving rise to the ESO market price. Finally, motivated by the recent accounting standards, the literature has also introduced the objective value or the shareholders' cost when issuing the option. Although companies do not face the same hedging restrictions than their executives do, the ESO cost is not equal to its

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<sup>1</sup>We will only concentrate on the FAS 123R since both standards establish rather the same purpose concerning the fair value.

market price because of the executive's exercise policy.

Thus, either the subjective or the objective value turns out to be dependent on the precise way in which the exercise policy is incorporated. In this regard, we can distinguish two broad lines of analysis. In the first line, the exercise policy is obtained from structural models in which executives maximize their expected utility subject to a given set of constraints. Lambert et al. (1991) is an early example. They establish the subjective ESO value as its certainty-equivalence. Subsequently, Huddart (1994) and Kulatilaka and Marcus (1994) use this framework to provide an estimate of this subjective value determining the executive's exercise rule on a binomial tree. However, they restrict the executive to hold her wealth only in the risk free asset. Hall and Murphy (2002) and Cai and Vijh (2005) allow a more general setting in which the executive's wealth can also be held in restricted stocks or some market portfolio, respectively. Tian (2004) also uses this approach to study the incentive effects of European-style ESOs. All of them are static analyses in which the executive maximizes the expected utility of terminal wealth. Kahl et al. (2003) and Ingersoll (2006) use a dynamic approach such that the executive takes decisions about his optimal consumption and portfolio composition.

In a second line of reduced form models, the exercise policy is described either by some exogenous random event or by some exogenous parameter (or both) that forces the early option exercise and hence, the option expected life. An early example might be Jennergren and Näslund (1993), who introduce an exogenous and independent Poisson process whose first arrival forces the early exercise of (vested) options. Carpenter (1998) shows that this type of models performs as well as structural ones and Carr and Linetsky (2000) advocate their use because of their simplicity. Hull and White (2004) illustrate the inclusion of a virtual strike price that triggers early exercise and that can be calibrated so as to match the option expected life.

Our contribution can be placed along the first class of models. We use a simplified version of Ingersoll's (2006) model in which the executive allocates her wealth across the market portfolio and risk free bonds, but the risk factors are reduced to just the market risk. Therefore, the executive is constrained to hold more of the company stock than its corresponding share in the market portfolio.<sup>2</sup> As a result, there are two sources of risk, one coming from the non diversifiable systematic risk factor, underlying the market portfolio, and the other coming from the idiosyncratic component not correlated with market risk. In a well diversified portfolio, the only source of risk would come exclusively from the

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<sup>2</sup>See the introduction to Ingersoll's (2006) article for a list of possible reasons of such undiversified holdings.

market portfolio and any other idiosyncratic component would have vanished. From the maximization of a lifetime utility function, a stochastic discount factor (SDF) is found which, by including those two sources of risk, can be used for pricing ESOs. We extend this model by including a job termination risk along the lines of Jennergren and Näslund (1993). The inclusion of this additional source of risk leads to a valuation model similar to that of Sircar and Xiong (2007). There is nonetheless a basic difference, they solve their model under the risk neutral measure which does not consider the executive's exercise policy. Instead, we use the real measure and obtain a closed form solution for the executive's exercise policy. Under this approach it is possible to provide a closed form expression for both, the subjective and the objective values. The case analyzed by Sircar and Xiong (2007) is only a solution for the ESO market value, which can be obtained in our approach by simply assuming a risk neutral agent (or a well diversified agent).

The merit of this approach lies on its ability to become a reasonable approximation for real situations based on finite maturity ESOs, which are computationally more demanding. We show the bias size for this approximation to be fairly small. We also discuss the impact on the cost of ESOs when the firm is uncertain about how likely the executive will finish the employment relationships. To this end, the firm is endowed with a prior distribution for the job termination risk. Finally, we study the executive's incentives by analyzing the subjective ESO delta and vega.

Section 2 presents the theoretical results for the subjective valuation of ESOs. Section 3 discusses how the subjective valuation is affected by changes in the relevant parameters and shows the size of the bias incurred by the use of the perpetual ESO against the finite maturity case. Section 4 includes the theoretical results about the objective ESO valuation or firm cost, the corresponding goodness of fit in the approximation to the finite maturity case is also discussed and the implications for the accounting standards. This section concludes with a discussion of the impact that uncertainty on the job termination risk has on the cost of ESO. In Section 5, we show how ESO affects executive's performance through his subjective 'greeks'. Section 6 concludes.

## 2 Subjective ESO valuation

Our benchmark model will be a perpetual ESO with a stochastic live arising from exogenous employment shocks which forces the termination of the employment relationship, as in the model of Jennergren and

Näslund (1993). These shocks can arise from either the executive's side, due to voluntary resignation, or from the firm's side, due to the executive dismissal. In any case, the executive is forced to exercise the option if the event occurs after the vesting period, or to forfeit it, if the vesting period has not ended yet. The time at which the employment relationship is terminated is simply modeled as the first event of a Poisson process with hazard rate of  $\lambda$  per unit time. This Poisson process is assumed to be independent of any other stochastic process underlying our menu of assets. The hazard rate leads to jumps in the ESO price, but not in the underlying stock price, as in Jennergren and Näslund (1993). We assume that the job termination risk is not priced, so that it can be diversified away. This assumption is very common in the literature. See, for instance, Jennergren and Näslund (1993), Carpenter (1998), Carr and Linetsky (2000), Hull and White (2004), Sircar and Xiong (2007) and Leung and Sircar (2009).

Those employment shocks force exogenous exercise of ESOs. Endogenous exercise results from the optimizing behavior of the executive which typically is assumed to solve a constrained maximization problem. We shall consider the same menu of assets as in Ingersoll (2006), a risk-free bond, the market portfolio and the company stock. The equations describing the dynamics of the company stock and the market portfolio prices will be given by

$$\begin{aligned}\frac{dS}{S} &= (\mu_S - q_S)dt + \sigma_S dZ_S , \\ \frac{dM}{M} &= (\mu_M - q_M)dt + \sigma_M dZ_M ,\end{aligned}$$

where  $\mu_S$  and  $\mu_M$  denote the growth rate of the stock and the market portfolio respectively, while  $q_S$  and  $q_M$  stand for the corresponding continuous dividend yields. The firm's stock and the market portfolio are assumed to be imperfectly correlated. Formally, the Wiener processes satisfy the following relationship:

$$\sigma_S dZ_S = \beta \sigma_M dZ_M + \sigma_I dZ_I , \tag{1}$$

where the parameter  $\beta$  is the conventional market beta, and  $dZ_M$  and  $dZ_I$  are independent standard Wiener processes. Notice that  $\sigma_I$  is not an independent parameter, since it must satisfy the restriction  $\sigma_I^2 = \sigma_S^2 - \beta^2 \sigma_M^2$  as equation (1) makes clear. In short, we can write the equation for the stock price dynamics as

$$\frac{dS}{S} = (\mu_S - q_S)dt + \beta \sigma_M dZ_M + \sigma_I dZ_I .$$

As Ingersoll (2006), we assume that the executive is infinitely lived and maximizes an expected

lifetime utility of the constant relative risk aversion class. To capture the degree of undiversification, we define the parameter  $\theta$  as the excess of company's stock holding over the optimal level already incorporated in the market portfolio.<sup>3</sup> Therefore, the executive's problem is

$$\max_{C, \omega} \mathbb{E}_0 \left\{ \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right\} \quad (2)$$

where  $\mathbb{E}_0[\cdot]$  denotes the conditional expectation and it is subject to the following dynamic budget constraint:

$$dW = \{[r + \omega(\mu_M - r) + \theta(\mu_S - r)]W - C\} dt + \omega\sigma_M W dZ_M + \theta\sigma_S W dZ_S, \quad (3)$$

with initial condition  $W(0) = W_0$ . For simplicity, no wage income is assumed. Assuming that CAPM holds, i.e.  $\mu_S = r + \beta(\mu_M - r)$ , and using the orthogonal decomposition described in equation (1), we can rewrite equation (3) as

$$dW = \{(r + (\omega + \theta\beta)(\mu_M - r))W - C\} dt + (\omega + \theta\beta)\sigma_M W dZ_M + \theta\sigma_I W dZ_I. \quad (4)$$

Given the above conditions, we can obtain the SDF in the following lemma.

**Lemma 1** *The SDF,  $\Theta$ , that prices the derivative will obey the following ordinary differential equation (ODE):*

$$\frac{d\Theta}{\Theta} = -\hat{r}dt - \left( \frac{\mu_M - r}{\sigma_M} \right) dZ_M - \gamma\theta\sigma_I dZ_I \quad (5)$$

where  $\hat{r} = r - \gamma\theta^2\sigma_I^2$ .

This proof is straightforward by following Ingersoll (2006). Notice that, when the executive is either risk neutral,  $\gamma = 0$ , or has a well diversified portfolio,  $\theta = 0$ , the SDF does not include any term reflecting the (diversifiable) idiosyncratic risk of the stock. Hence, the resulting value coincides with the risk neutral price for marketable options.

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<sup>3</sup>Let  $\underline{\theta}$  denote the minimum amount of the company stock that the executive is constrained to hold. If  $\xi^*$  denotes the optimal share of the company stock in the market portfolio and  $\omega^*$  the optimal share of the market portfolio in the executive's total portfolio, then  $\theta$  would satisfy the following condition  $\theta = \underline{\theta} - \omega^*\xi^* \geq 0$ .



## 2.1 Pricing without vesting period

For pricing ESOs, we use the no arbitrage condition  $\mathbb{E}_0[d(\Theta V)] = 0$  where  $V$  denotes the perpetual ESO value. Thus,

$$\begin{aligned} 0 = \mathbb{E}_0 [d(\Theta V)] &= \mathbb{E}_0 [d(\Theta V) | \text{no employment shock}] (1 - \lambda dt) \\ &+ \mathbb{E}_0 [d(\Theta V) | \text{employment shock}] \lambda dt . \end{aligned} \quad (6)$$

Given equation (6), we get the following result:

**Lemma 2** *Under the no arbitrage condition in equation (6) and assuming that CAPM holds, we get the following fundamental ODE for the subjective perpetual ESO price:*

$$\left(\frac{\sigma_s^2}{2}\right) V_{SS} S^2 + (\hat{r} - \hat{q}_s) V_s S - (\hat{r} + \lambda) V + \lambda \Psi(S) = 0 , \quad (7)$$

where  $\hat{r} = r - \gamma\theta^2\sigma_I^2$ ,  $\hat{q}_s = q_s + \gamma\theta(1 - \theta)\sigma_I^2$  and  $\Psi(S)$  denotes the payoff of the ESO holder if there is an employment shock defined as  $\Psi(S) = (S - K)\mathbb{1}_{\{S > K\}}$ , where  $\mathbb{1}_{\{A\}}$  is the indicator function such that  $\mathbb{1}_{\{A\}} = 1$  if  $A$  is true and  $\mathbb{1}_{\{A\}} = 0$ , otherwise.

*Proof.*- See Appendix A.

Equation (7) is the ODE defining the executive's value of the ESO in the continuation or waiting region. The structure of the problem implies that there is a threshold price,  $S^*$ , such that the optimal policy is waiting while  $S < S^*$  and exercising as soon as  $S \geq S^*$ . Hence, at the boundary with the exercise region, the following conditions must hold:

$$V(S^*) = S^* - K, \quad (8)$$

$$V'(S^*) = 1, \quad (9)$$

where the threshold price  $S^*$  is determined endogenously as part of the complete solution. Considering equations (7), (8) and (9), we obtain the following proposition:

**Proposition 3** *Assume that there is no vesting period and the ESO is a perpetual American call option, then the solution to the ODE defined in (7) subject to the following boundary conditions,  $V(0) = 0$  and*

equations (8) and (9), provides the value for the ESO holder, whose explicit solution is given by

$$V(S) = \begin{cases} \hat{A}_1 K^{1-\hat{\alpha}_1} S^{\hat{\alpha}_1} & \text{if } S \leq K \\ \hat{B}_1 K^{1-\hat{\alpha}_1} S^{\hat{\alpha}_1} + \hat{B}_2 K^{1-\hat{\alpha}_2} S^{\hat{\alpha}_2} + \lambda \left( \frac{S}{\lambda + \hat{q}_s} - \frac{K}{\lambda + \hat{r}} \right) & \text{if } K < S \leq \hat{S}^* \\ S - K & \text{if } S > \hat{S}^* \end{cases} \quad (10)$$

where  $\hat{\alpha}_1$  and  $\hat{\alpha}_2$  are the solutions to the quadratic equation  $(\sigma_s^2/2)\hat{\alpha}^2 + (\hat{r} - \hat{q}_s)\hat{\alpha} - (\hat{r} + \lambda)$  and the values of the constants  $\hat{A}_1$ ,  $\hat{B}_1$  and  $\hat{B}_2$  are defined in Appendix B by equations (27), (26) and (22) respectively. Finally, the threshold price  $\hat{S}^*$  is uniquely defined by solving the next equation:

$$\lambda \left( \frac{\hat{S}^*}{K} \right)^{\hat{\alpha}_2} = -(1 - \hat{\alpha}_2)\hat{r} - \hat{\alpha}_2\hat{q}_s \left( \frac{\hat{S}^*}{K} \right). \quad (11)$$

*Proof.*- See Appendix B.

$V(S)$  is homogeneous of degree one in both  $S$  and  $K$ . The first two rows of equation (10) show the subjective ESO value when the price is below the optimal subjective threshold. Both belong to a situation in which the executive is better-off waiting rather than exercising the option. Consider the following intuitive explanation of these gains for the second waiting region. Its first component comes from the possible increase in the future price of the underlying stock. The second one concerns the possibility of exercising the ESO if an employment shock occurs at any future time with a probability of  $\lambda$ . Hence, the term  $\lambda(S/(\lambda + \hat{q}_s) - K/(\lambda + \hat{r}))$  denotes the expected ESO present value when it is in-the-money.<sup>4</sup> Of course, in the first waiting region this term does not appear since the option is out-of-the-money.

Some remarks about equation (11) are in order. By contrast with the paper of Sircar and Xiong (2007), we have found a closed form expression for the executive's exercise policy which is also valid for non-risk neutral executives. The exercise policy is homogeneous of degree one in  $K$ , that is, any change in the strike price implies a change in the same proportion in the threshold price. Finally, although it is not easy to find general comparative static results, all numerical simulations have provided results

<sup>4</sup>This expected present value is obtained by solving the following integral:

$$\int_0^\infty \lambda e^{-\lambda t} \left[ e^{-rt} \left( S e^{(r-q_s)t} - K \right) \right] dt$$

where the time for the first employment shock follows an exponential distribution with mean  $1/\lambda$ .

that conform with intuition. For instance, a higher value of the employment shock probability tends to reduce the threshold price.

Figure 1 shows the typical shape of  $V(S)$  derived in Proposition 3. This figure displays several values for the stock excess holdings. Note that the situation of  $\theta = 0$  is equivalent to the ESO risk neutral valuation. This value acts as an upper boundary for other situations in which the executive is less diversified.

## 2.2 Pricing with vesting period

In this subsection, we extend the results of Proposition 3 to the case of a positive vesting period,  $\nu$ . The subjective ESO value at the granting date,  $t = 0$ , is obtained in the following proposition.

**Proposition 4** *The subjective ESO value at the granting date,  $t = 0$ , with a vesting period of length  $\nu$  is given by*

$$V_0^{SUB} = e^{-\lambda\nu} \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} V(S_\nu) \right] \quad (12)$$

where the exponential term is the probability that the executive will remain employed till the end of the vesting period and

$$\begin{aligned} \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} V(S_\nu) \right] &= e^{-\hat{r}\nu} \mathbb{E}_0 \left[ (S_\nu - K) \mathbf{1}_{\{S_\nu > \hat{S}^*\}} \right] + \\ &+ e^{-\hat{r}\nu} \mathbb{E}_0 \left[ \left( \hat{B}_1 K^{1-\hat{\alpha}_1} S_\nu^{\hat{\alpha}_1} + \hat{B}_2 K^{1-\hat{\alpha}_2} S_\nu^{\hat{\alpha}_2} + \lambda \left( \frac{S_\nu}{\lambda + \hat{q}_s} - \frac{K}{\lambda + \hat{r}} \right) \right) \mathbf{1}_{\{K < S_\nu \leq \hat{S}^*\}} \right] \\ &+ e^{-\hat{r}\nu} \mathbb{E}_0 \left[ (\hat{A}_1 K^{1-\hat{\alpha}_1} S_\nu^{\hat{\alpha}_1}) \mathbf{1}_{\{S_\nu \leq K\}} \right] \end{aligned} \quad (13)$$

for any real numbers  $a$ ,  $b$  and  $c$  verifying

$$\mathbb{E}_0 [S_\nu^c \mathbf{1}_{\{a \leq S_\nu \leq b\}}] = \exp \left\{ c\mu + c^2 \frac{\sigma^2}{2} \right\} \times \left\{ \Phi \left( \frac{\ln b - \mu - c\sigma^2}{\sigma} \right) - \Phi \left( \frac{\ln a - \mu - c\sigma^2}{\sigma} \right) \right\},$$

where  $\mu = \ln S_0 - (\hat{r} - \hat{q}_s - \sigma_s^2/2)\nu$ ,  $\sigma = \sigma_s \sqrt{\nu}$  and  $\Phi(\cdot)$  represents the cumulative standard normal distribution.

*Proof.*- See Appendix C.

### 3 Discussion

In this section, we begin studying the impact on the ESO valuation due to changes in several parameters of interest. These results can be found in the subsection of sensitivity analysis. We turn next to examine the robustness of our perpetual ESO valuation by comparing with American-style ESOs with finite maturities.

For all situations, we assume the risk free interest rate  $r = 6\%$ , the continuous dividend yield  $q_s = 1.5\%$  and the market volatility  $\sigma_M = 20\%$ . All these parameter values are taken on a yearly basis. We assume ESOs are granted at the money. The strike price,  $K$ , and the stock price at the granting date,  $S_0$ , are equal to \$30.

#### 3.1 Sensitivity analysis

We illustrate the effect of varying each of the following parameters: (i) the market beta,  $\beta$ , which can be either 0 or 1; (ii) the total yearly volatility of the stock return,  $\sigma_s$ , which can be 30%, 40% or 60%; (iii) the vesting period,  $\nu$ , which can be either 0 or 3 years; (iv) the employment shock captured through the yearly Poisson intensity parameter,  $\lambda$ , with values of either 10% or 20%; (v) the excess stock holding,  $\theta$ , ranging from 0% to 40%; and finally, (vi) the risk aversion parameter,  $\gamma$ , with value of 2 or 4. The results of this analysis are displayed in Table 1. This table is divided in four panels labelled from A to D according to different values of  $\lambda$  and  $\nu$ . Namely, Panel A ( $\lambda = 10\%$ ,  $\nu = 0$ ), Panel B ( $\lambda = 10\%$ ,  $\nu = 3$ ), Panel C ( $\lambda = 20\%$ ,  $\nu = 0$ ) and Panel D ( $\lambda = 20\%$ ,  $\nu = 3$ ). Next, we comment on several features of interest.

First, the ESO market price,  $V^{RN}$ , is obtained under the restriction of  $\theta = 0\%$ . It is shown that  $V^{RN}$  increases with the total volatility of the stock return. This feature is independent of the risk decomposition into common risk or beta and specific or idiosyncratic risk,  $\sigma_I$ . The reason is that a well diversified agent does not worry about the size of  $\sigma_I$ . Note that, as expected,  $V^{RN}$  acts as an upper boundary for  $V^{SUB}$ .

Second, the higher the value of  $\beta$  the higher  $V^{SUB}$ . This effect has already been addressed by Tian (2004). A value of  $\beta = 0$  suggests that the market portfolio is useless for hedging the risk of large holdings of the company stock. Observe that under  $\beta = 0$ , it holds that  $\sigma_s = \sigma_I$ . Thus, for any level of

both the excess stock holding and risk aversion, an increase in the size of beta such that  $\sigma_s$  is keeping constant (i.e, a displacement across any row of Table 1), implies a decrease in the specific risk which improves the executive's utility.

Third, the higher the risk aversion or the excess stock holding, the lower  $V^{SUB}$ . Fourth, the higher the employment shock intensity the lower  $V^{SUB}$ . Compare panels A and C when there is no vesting period, while panels B and D for the positive one. Fifth, the higher the vesting period the lower  $V^{SUB}$ . Compare panels A and B for  $\lambda = 10\%$ , or panels C and D for  $\lambda = 20\%$ .

Last but not least, increases in  $\sigma_s$  have ambiguous effects on  $V^{SUB}$ . This is due to the fact that, as  $\beta$  is kept constant, changes in  $\sigma_I^2$  are one-to-one changes in  $\sigma_s^2$ . The conventional analysis of American call options shows that a higher variance tends to increase  $V^{SUB}$ . If this increase is due to a higher idiosyncratic volatility, the adjusted interest rate falls and the adjusted dividend yield increases and, pursuing further the analogy with American calls options, these changes tend to decrease  $V^{SUB}$ . In Table 1 we see that  $V^{SUB}$  tends to fall as  $\sigma_I$  increases when either  $\gamma$  or  $\theta$  are large. For instance,  $V^{SUB}$  increases with  $\sigma_I$  in the column of Panel A for the values of  $\beta = 1$ ,  $\gamma = 2$  and  $\theta = 10\%$ . By contrast,  $V^{SUB}$  decreases in the column of Panel A for the values of  $\beta = 1$ ,  $\gamma = 2$  and  $\theta = 30\%$ . There are also some transitional cases in which  $V^{SUB}$  exhibits an U-shaped behavior or its inverse. For instance, the column of Panel A for the values of  $\beta = 1$ ,  $\gamma = 2$  and  $\theta = 20\%$  illustrates the U-shaped case, and the column of Panel B for the values of  $\beta = 0$ ,  $\gamma = 2$  and  $\theta = 20\%$ , the inverted U-shaped case.

Notice, however, that a higher value of  $\lambda$  tends to reverse the former relationship, making  $V^{SUB}$  increasing in  $\sigma_I$ . However, this effect only appears for short vesting periods, compare the values in Panel A with those in Panel C.<sup>5</sup> Therefore, for the typical length of vesting periods observed in practice, this effect does not seem to modify substantially the behavior of  $V^{SUB}$  with respect to changes in  $\sigma_I$ .

### 3.2 Perpetual versus finite maturities

It becomes interesting to analyze if ESOs having finite maturities are adequately approximated by perpetual ones. We calculate ESO prices with finite maturities using the least-squares Monte Carlo algorithm (LSMC henceforth) of Longstaff and Schwartz (2001). We simulate the risk neutral price process but replacing  $r$  and  $q_s$  respectively by  $\hat{r}$  and  $\hat{q}_s$ , as defined in Lemma 2.

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<sup>5</sup>In additional results, not reported here, for a vesting period of one year, an increase in  $\lambda$  from 10% to 20% enlarges the set of cases for which  $V^{SUB}$  increases when  $\sigma_I$  increases.

The LSMC uses a backward induction procedure that works as follows. At maturity, the ESO is exercised if it is in-the-money, then the subjective value of the ESO is  $V_T^{SUB} = (S_T - K)^+$ .<sup>6</sup> One period before, at  $T - \delta t$  where  $\delta t$  is the length of one time step, on one hand there is a probability equal to  $1 - e^{-\lambda \delta t}$  to abandon the firm and the payoff of the ESO would be  $(S_{T-\delta t} - K)^+$ . On the other hand, with probability  $e^{-\lambda \delta t}$  the executive remains in the firm and thus, he must decide either to hold or to exercise voluntarily the ESO. The executive will exercise the ESO if  $S_{T-\delta t} - K > e^{-\hat{r} \delta t} \mathbb{E}_{T-\delta t} [V_T^{SUB}]$ , in this case the ESO value will be  $S_{T-\delta t} - K$ . Otherwise, the payoff will be the discounted expected one period ahead ESO value. Thus, the ESO value at any time  $t$ , such that  $T > t > \nu$ , is computed as

$$V_t^{SUB} = e^{-\lambda \delta t} \left[ X_t \mathbb{1}_{\{S_t - K < X_t\}} + (S_t - K)^+ \mathbb{1}_{\{S_t - K \geq X_t\}} \right] + (1 - e^{-\lambda \delta t}) (S_t - K)^+$$

where  $X_t = e^{-\hat{r} \delta t} \mathbb{E}_t [V_{t+1}^{SUB}]$  is the discounted expectation of the ESO value.<sup>7</sup> The conditional expected ESO value is computed by least-squares such that for those paths in-the-money, the one period ahead ESO value is regressed over some basis functions of the current stock price. We work backwards until the vesting or grant date with this scheme. We use 20,000 paths simulated with monthly frequency<sup>8</sup> and we take the average of 50 previous estimations using 25 different seeds plus the corresponding 25 antithetics.

Figure 2 exhibits alternative subjective ESO values without vesting period for different values of  $\beta$  and  $\lambda$  from Table 1. The values of the risk aversion and the total volatility are, respectively,  $\gamma = 2$  and  $\sigma_s = 30\%$ . The remaining parameters are the same as in Table 1. The selected time to maturities,  $T$ , go from 5 years until 25 years, denoted as  $V_T^{SUB}$ . It is observed that ESO prices do not change from about 15 years until the end and considering alternative values of  $\theta$ , see the graphics on the left. The graphics of relative biases,  $(V^{SUB} - V_T^{SUB})/V_T^{SUB}$ , where  $V^{SUB}$  are the perpetual ESO values from Table 1 displayed on the right lead to a decreasing pattern as  $T$  increases. Indeed, this positive bias tends to be lower than 10% for the benchmark American-style ESO with a maturity of ten years. In short, the benchmark ESO value is well approximated through the perpetual one.<sup>9</sup> For the shortest maturity,  $T = 5$ , the largest bias is around 25% for  $\lambda = 10\%$ , while it decreases significantly to

<sup>6</sup>Note that  $(S_T - K)^+$  is the same as  $(S_T - K) \mathbb{1}_{\{S_T > K\}}$ .

<sup>7</sup>Hull and White (2004) and Ammann and Seiz (2004) also introduce in the same way the exit rate for the backwards induction in their binomial tree models.

<sup>8</sup>Stentoft (2004) obtains that the LSMC method with 10 exercise points per year produce very accurate prices compared with the ones obtained using a binomial model with 50,000 time steps. He argues that more accurate prices are obtained when increasing the simulated paths or the number of basis functions used as regressors.

<sup>9</sup>It is available upon request, though not reported here, a table similar to Table 1 for the case of an ESO with  $T = 10$  years containing the ESO values underlying the four graphics on the left-hand side of Figure 2.

approximately 10% for  $\lambda = 20\%$ . Therefore, the larger the probability of employment shock, the better the approximation through the perpetual ESO.

## 4 ESO cost for firms

ESOs are extensively used as a payment scheme that may help firms in retaining and motivating its executives. As a result, an ever increasing part of the executive's compensation packages have taken the form of ESOs and this has motivated a discussion about the precise way in which this cost should be evaluated. There is not yet a consensus on this issue because of the lack of a general agreement about what should be the right model. Despite of this, there is a general agreement of the role played by the subjective value in the computation of the ESO cost for firms. This cost depends on the executive's exercise policy which is a function of his subjective ESO valuation. Since the firm is not restricted to hedge the risk associated with the stock options, the objective valuation will be a straightforward modification of the risk neutral one. That is, the ESO cost for the firm is the amount that would receive if the ESO were sold to a well diversified investor with the risk-averse executive's exercise policy, to which we should also add, facing exogenous employment shocks that may end the employment relationship.

### 4.1 Objective ESO valuation

We shall denote by  $V^{OBJ}$  the objective ESO value. It is immediate that the former definition implies that  $V^{OBJ}$  is the solution to the ODE described in equation (7) for the case of risk neutral agents (or, equivalently, well diversified agents). Recall that this case follows from setting the parameter  $\gamma$ , or either the parameter  $\theta$ , to zero. In this case the adjusted risk free rate,  $\hat{r}$ , and the adjusted dividend yield,  $\hat{q}_S$ , become respectively  $r$  and  $q_S$ . For ease of comparison, the ODE corresponding to the risk neutral valuation is:

$$\left(\frac{\sigma_S^2}{2}\right) V_{SS} S^2 + (r - q_S) V_S S - (r + \lambda) V + \lambda \Psi(S) = 0, \quad (14)$$

where the boundary conditions are now given by

$$V(0) = 0 \tag{15}$$

$$V(\hat{S}^*) = \hat{S}^* - K, \tag{16}$$

such that  $\hat{S}^*$  is the executive's threshold price obtained in Proposition 3.<sup>10</sup>

We start getting the explicit expression for  $V^{OBJ}$  when there is no vesting. This turns out to be a slight modification of Proposition 3. The precise result is stated in the following corollary.

**Corollary 5** *Assuming that there is no vesting period and the ESO is a perpetual American call option, the objective ESO value is given by the solution to the ODE defined in equation (14) subject to the boundary conditions (15) and (16):*

$$V(S) = \begin{cases} \tilde{A}_1 K^{1-\alpha_1} S^{\alpha_1} & \text{if } S \leq K \\ \tilde{B}_1 K^{1-\alpha_1} S^{\alpha_1} + \tilde{B}_2 K^{1-\alpha_2} S^{\alpha_2} + \lambda \left( \frac{S}{\lambda + q_S} - \frac{K}{\lambda + r} \right) & \text{if } K < S \leq \hat{S}^* \\ S - K & \text{if } S > \hat{S}^* \end{cases} \tag{17}$$

where  $\hat{S}^*$  is the executive's threshold price obtained in Proposition 3,  $\alpha_1$  and  $\alpha_2$  are the roots corresponding to the risk neutral case and the values of  $\tilde{A}_1$ ,  $\tilde{B}_1$  and  $\tilde{B}_2$  are defined in Appendix D by equations (34), (33) and (32), respectively.

*Proof.*- See Appendix D.<sup>11</sup>

This result can also be extended to the case of a positive vesting period. Specifically, by plugging both the risk neutral SDF and the executive's threshold price,  $\hat{S}^*$ , into Proposition 4 we get the next corollary.

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<sup>10</sup>Notice that  $V^{OBJ}$  can also be obtained as the solution to a perpetual up-and-in barrier option with  $\hat{S}^*$  as the upper barrier. See Ingersoll (2006) for the finite maturity case.

<sup>11</sup>As in the subjective valuation case,  $V(S)$  is linearly homogeneous in  $S$  and  $K$ .



**Corollary 6** *The objective ESO cost is given by*

$$\begin{aligned}
V_0^{OBJ} &= e^{-\lambda\nu} \left\{ e^{-r\nu} \mathbb{E}_0 \left[ (S_\nu - K) \mathbb{1}_{\{S_\nu > \hat{S}^*\}} \right] + \right. \\
&+ e^{-r\nu} \mathbb{E}_0 \left[ \left( \tilde{B}_1 K^{1-\alpha_1} S^{\alpha_1} + \tilde{B}_2 K^{1-\alpha_2} S^{\alpha_2} + \lambda \left( \frac{S_\nu}{\lambda + q_S} - \frac{K}{\lambda + r} \right) \right) \mathbb{1}_{\{K < S_\nu \leq \hat{S}^*\}} \right] \\
&\left. + e^{-r\nu} \mathbb{E}_0 \left[ (\tilde{A}_1 K^{1-\alpha_1} S_\nu^{\alpha_1}) \mathbb{1}_{\{S_\nu \leq K\}} \right] \right\} \quad (18)
\end{aligned}$$

where the constants  $\tilde{A}_1$ ,  $\tilde{B}_1$  and  $\tilde{B}_2$  are those obtained in Corollary 5.

*Proof.*- The above equation follows easily from equation (13) of Proposition 4 by setting  $\gamma = 0$ , or  $\theta = 0$ , and taking  $\hat{S}^*$  in Proposition 3 as the threshold price.

As in Section 3.1 we can show, with the help of the former results, the effects on  $V^{OBJ}$  of changes in several parameters of interest. Figure 3 summarizes the main findings concerning the impact on  $V^{OBJ}$  of different values for  $\gamma$ ,  $\theta$  and  $\beta$ . We take the values of  $\sigma_S = 30\%$ ,  $\sigma_M = 20\%$ ,  $\nu = 3$  and  $\lambda = 20\%$  as our benchmark values. The straight line at the top of the figure represents  $V^{RN} = 6.240$  which corresponds to the second row of Panel D in Table 1. The picture of the left hand side shows that  $V^{OBJ}$  decreases with the degree of risk aversion,  $\gamma$ , and with the undiversification parameter,  $\theta$ . Conversely, a higher value of  $\beta$  implies, for given total volatility of the company stock and the market portfolio, a lower idiosyncratic volatility and hence, a higher value of  $V^{OBJ}$ . The intuition is that all those changes reduce the threshold price that triggers the ESO exercise. The picture on the right hand side displays the ratio between the subjective and the objective value. We can see that, although the former changes affect  $V^{SUB}$  and  $V^{OBJ}$  in the same way, the ESO ratio decreases significantly with the degree of risk aversion,  $\gamma$ , the undiversification parameter,  $\theta$  and the size of the idiosyncratic risk,  $\sigma_I$  (displayed in the figure as a lower value for  $\beta$ ). This is in line with the results of Tian (2004), Ingersoll (2006) and Chung, Hu and Fu (2008) for the case of European ESOs.

This fall in the ratio is explained by the fact that, although all these changes affect both, the impact on  $V^{OBJ}$  is only through a change in the threshold whereas the impact on  $V^{SUB}$  also includes the changes in  $\hat{r}$  and  $\hat{q}_S$ , which are absent in the former because it is computed under the risk-neutral measure (see either Corollary 5 or 6).

Finally, we have also explored the effect of different values of the intensity rate. As expected,  $V^{OBJ}$  decreases with higher values of  $\lambda$  and this effect turns out to be larger, the larger the ESO

vesting period. The ratio  $V^{SUB}/V^{OBJ}$  moves up only slightly with higher values of  $\lambda$  and they are not reported here to save space (but see Table 2 for an illustration of the impact on  $V^{OBJ}$  of higher values of  $\lambda$ ).

## 4.2 Perpetual versus finite maturity

In this section, we perform the same exercise made in subsection 3.2 and compare the objective perpetual ESO valuation,  $V^{OBJ}$ , with that corresponding to the finite maturity case,  $V_T^{OBJ}$ . We consider a maturity of ten years, which is frequently used in practice. Additionally, in subsection 3.2, the subjective perpetual ESO value turns out to perform quite well for this maturity. Thus, it seems natural to use this maturity to check how well our valuation method approximates the finite maturity case. This comparison is exhibited in Table 2. Both perpetual and finite maturity ESOs are displayed on the left and right hand-sides respectively for alternative parameter values of  $\lambda$ ,  $\beta$ ,  $\gamma$  and  $\theta$  and different vesting periods (one, three and five years in panels A to C respectively). In this table, we have only considered the case of  $\sigma_s = 40\%$ . Other values also yield similar results.

Inspection of Table 2 shows that the difference between both ESO values is not large and decreases for larger values of  $\gamma$ ,  $\theta$  and  $\lambda$ . For instance, it is shown that relative biases, measured as  $(V^{OBJ} - V_T^{OBJ})/V_T^{OBJ}$ , go from about 10% to 0% when  $\theta$  increases for the case of  $\lambda = 10\%$  and  $\gamma = 2$ . This bias can be negative with respect to the former situation for large values of  $\gamma$ , though the size is nearly the same as above. The case of  $\lambda = 20\%$  and  $\gamma = 4$  also exhibits positive relative biases ranging from 5% to 0% as  $\theta$  increases. Again, this bias becomes negative in some situations when  $\theta$  is large. In short, we can conclude that the perpetual valuation is a good approximation due to the small bias size and its lower computational intensity.

## 4.3 Accounting implications

As a result of the increasing relevance of ESOs in the managers compensation packages and the need to converge with other international standards, the FASB has revised its statement N° 123. The new FAS 123R requires firms to disclose the method used for estimating the grant-date fair value of their ESO compensation packages. Among the valuation techniques that the FAS 123R consider acceptable are both lattice and closed form models, such as the binomial model and the Black-Scholes-Merton

formula, respectively. Since ESOs are typically exercised before maturity, the FAS 123R (paragraph A26) explicitly requires that this fair value be based on its expected term, or expected life, rather than its maturity term. Furthermore, this expected life must be disclosed by firms as part of the shareholders' available information (FAS 123R, paragraph A240). In general, this expected life must be estimated. Kulatilaka and Marcus (1994), among others, have warned against the use of historical data with the purpose of estimating the option expected term. This is so because the ESO expected life is linked to the stock market performance during the relevant period and this might lead to poor predictions. As an example of an acceptable method for estimating this parameter, the FAS 123R suggests using the estimated ESO fair value, as obtained from a lattice model, as an input of a closed form model, such as an augmented Black-Scholes-Merton model, from which the implied expected term can be calculated.

Now, we illustrate the usefulness of the perpetual option approximation to the real finite maturity option to estimate the ESO expected term. Specifically, we take the expected life of the objective finite maturity ESO which is implicit in the corresponding value of Table 2, and compare it with the implied expected life that one would obtain by using the objective perpetual ESO price as the value of the FAS 123 adjusted European call price. This value is given by,

$$V^{FAS} = \exp(-\lambda\nu) \times BS(L) \quad (19)$$

where  $BS(L)$  denotes the Black-Scholes (1973) formula with a time to maturity equal to  $L$  and a vesting period of length  $\nu$ .

Figure 4 displays the results of this comparison through four pictures for two different values of the risk of the employment shock,  $\lambda = 10\%$  and  $\lambda = 20\%$ , and two values of the degree of risk aversion,  $\gamma = 2$  and  $\gamma = 4$ . In all cases, the objective finite ESO has a term to maturity of ten years. The remaining parameters are  $r = 6\%$ ,  $q_s = 1.5\%$ ,  $\beta = 1$  and  $\sigma_s = 40\%$  which are used for the evaluation of both, the objective perpetual and finite ESO. As it can be seen, the objective perpetual approximation is good whenever either the risk of the employment shock or the degree of relative risk aversion is high, or whenever the degree of portfolio diversification is low.

## 4.4 Uncertainty in employment shocks

We study the effects of uncertainty in the likelihood of an employment shock,  $\lambda$ , that is one of the main parameters in this work. We implement a simple exercise just to motivate this situation. Notice that the employment relationship may terminate by the side of either the firm or the executive because of some exogenous event, for instance, the executive finds out a better available job. Each part is more uncertain about the likelihood the other part attaches to this event. Assume that this uncertainty arises only from the executive's side. Further, we also assume the firm's manager ignores the precise value of  $\lambda$ , although he has some a priori probability distribution for it. Of course, this will also generate a probability distribution for the objective value.

For simplicity, we assume a Triangular distribution<sup>12</sup> for the values of  $\lambda$ . We consider three prior distributions for  $\lambda$ , that might be representative of the executive's outside opportunities. For instance, in a highly concentrated industry (few firms), the executive's employment alternatives would tend to be scarce, so that the probability of leaving his actual job would be lower than in less concentrated industries. These prior distributions are classified according to the skewness behavior capturing alternative industry concentration levels. An asymmetric distribution with positive (negative) skewness represents a higher probability mass for small (large) values of  $\lambda$  because the industry is highly (scarcely) concentrated. And finally, we also consider a symmetric distribution for an intermediate situation.

For each assumed distribution of  $\lambda$  a total of 10,000 values have been simulated. For the cases of positive, zero and negative skewness, the values of  $\lambda$  going from the first to the third quartiles are, respectively, [7.5%, 21%], [14.1%, 25.6%] and [18.6%, 32.2%].

Figure 5, containing four pictures denoted from I to IV, displays the box-plots for the objective values,  $V^{OBJ}$ , implied by the above three distributions under several combinations of values for the diversification restriction,  $\theta$ , and the total volatility,  $\sigma_s$ . The pairs represented in figures I to IV are respectively:  $\{\sigma_s = 30\%, \theta = 10\%\}$ ,  $\{\sigma_s = 30\%, \theta = 40\%\}$ ,  $\{\sigma_s = 60\%, \theta = 10\%\}$  and  $\{\sigma_s = 60\%, \theta = 40\%\}$ . We select a value for  $\beta$  of one and a level of relative risk-aversion of four. Clearly, the implied distributions of  $V^{OBJ}$  coming from the distributions of  $\lambda$  with positive skewness (boxplots labeled as A) exhibit a higher median value than those distributions of  $\lambda$  with negative skewness (boxplots C). This is a clear effect of the negative correlation between the size of  $\lambda$  and  $V^{OBJ}$ . For instance, these

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<sup>12</sup>See chapter 40 in Evans et al. (2000).

differences go from a median value of 8 (boxplot A) to 5 (boxplot C) in picture I. Note also that a higher volatility leads to a higher median for the objective value. This value is higher the lower is the restriction on the portfolio diversification.

## 5 Incentive effects

The literature has paid great attention to the optimal design of the executive compensation package as a way of affecting their incentives. In general, these compensation packages consist of a fixed cash component, a certain amount of restricted stock and ESOs. Typically, the role of ESOs on executives' incentives has been approached by examining the sign and size of the ESO greeks. See, for instance, Ingersoll (2006), Tian (2004) and Chang et al. (2008) among others. Of course, it is understood that the relevant ESO greeks are those coming from the subjective valuation, those related with executive's perception of incentives. In this regard, there are two ways in which ESOs may affect executives' incentives. First, as part of an agency problem, they align executives and shareholders interests in raising stock price. The perceived reward from acting this way is measured by the subjective delta greek, that is, the partial derivative of  $V^{SUB}$  with respect to the initial price  $S_0$ . And second, when a new investment project is taken, the distribution of firm's total risk between the systematic component and the specific one will generally change. Given that they affect  $V^{SUB}$  very differently, there might be a moral hazard problem because of executives' risk taking behavior. However, the results of subsection 3.1 show that executives have strong incentives to reduce the firm's specific component and to increase the systematic component of total volatility. Although both results might seem interesting from the shareholders' point of view, they are not quite so. For the first implication, notice that shareholders do not face any restrictions to diversify their portfolios. The second one confirms the well known result in the literature that ESOs generate incentives to increase the systematic component of total risk since it raises the firm's expected return. See for instance, Tian (2004).<sup>13</sup>

Despite all this, there are at least two reasons to use option greeks in the discussion of how ESOs affect executives' incentives. In the first place, there is a gap between subjective and objective ESO valuation and, hence, there is a distinction between the incentives perceived by executives and the cost

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<sup>13</sup>In all numerical simulations performed, we have found that the subjective vega for firm's specific risk is strongly negative when total risk is held constant, and that the subjective vega for systematic risk is positive. Although these results are obtained for perpetual American ESOs, they have not been reported in the main text, since they are analogous to those obtained for the finite European ESOs which are typically considered in the literature.

to the firm of providing those incentives. In particular, Chang et al. (2008) have considered the ratio between the subjective and the objective greek for finite European ESOs as a measure of the size of the perceived incentives per unit cost of the firm. Since our perpetual American ESOs approximates reasonably the finite case as shown in subsection 3.2, we extend their analysis to the case of perpetual American ESOs. Secondly, it turns out that the ratio between subjective and objective greeks might be of help in the design of the composition of the executive compensation package, provided the parameters are conveniently interpreted. We consider next each issue in turn.

Let us consider first the incentives to raise shareholders wealth as measured by the subjective delta,  $\Delta^{SUB}$ . The firm's cost of providing these incentives are measured by the objective delta,  $\Delta^{OBJ}$ . The ratio  $\Delta^{SUB}/\Delta^{OBJ}$  is then the reward perceived by the executive per unit cost of the firm. A ratio below one suggests that the executive's incentives are lower than those intended by the firm. As Chang et al. (2008) have observed a ratio of 0.5, it means that the executive only perceives half of the incentive effects intended by shareholders. Hence, to provide the same level of incentives, he must be offered at least twice as many ESOs incurring in a higher cost.<sup>14</sup> We show that  $\Delta^{SUB}/\Delta^{OBJ}$  is typically below one when plotted as a function of  $S_0$ . With regard to the impact of unemployment shocks, a higher value of  $\lambda$  reduces somewhat the size of this ratio.<sup>15</sup>

This is depicted in picture I of Figure 6. The ratio  $\Delta^{SUB}/\Delta^{OBJ}$  has been computed numerically by evaluating both partial derivatives using  $S_0 = K$  as a midpoint, e.g. for at-the-money ESOs. It turns out that this ratio is monotonically decreasing with  $\theta$  and below one, except for the case  $\theta = 0$  where subjective and objective valuations coincide. This result is not modified by the consideration of different values of  $\lambda$ . Hence, the maximum perceived incentives to increase firm's market value per unit cost are achieved when the executive does not hold any restricted stock, for at-the-money ESOs.

The former analysis can also be extended to examine executive's incentives for investments in risky projects. This is measured by the subjective systematic risk vega, denoted by  $\Lambda_{\beta}^{SUB}$ , that shows the change in  $V^{SUB}$  resulting from a unit change in market beta. The firm's cost of providing those incentives is measured analogously by  $\Lambda_{\beta}^{OBJ}$ . As before, the ratio  $\Lambda_{\beta}^{SUB}/\Lambda_{\beta}^{OBJ}$ , is represented in picture II of Figure 6 against  $\theta$  for the case of at-the-money ESOs. The corresponding numerical partial derivatives have been evaluated using  $\beta = 1$  as the midpoint. Now, the shape of the curve suggests the

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<sup>14</sup>With decreasing marginal benefits, the amount of ESOs offered to executives should be more than twice the initial level.

<sup>15</sup>These results are available upon request.

existence of a maximum value of the ratio for a certain value of  $\theta$  denoted as  $\theta^*$ . Furthermore, this maximum value turns out to be greater than one.

This issue has been further explored in Figure 7, in which we plot  $\Lambda_{\beta}^{SUB}/\Lambda_{\beta}^{OBJ}$  as a function of  $\theta$  for several values of the idiosyncratic volatility and of the employment shock likelihood. As it is shown, the higher the idiosyncratic component,  $\sigma_I$ , the lower the required amount of restricted stock,  $\theta^*$ . A higher value of the likelihood of employment shocks,  $\lambda$ , leads to a rise in  $\theta^*$ . By the other hand, the maximum value of the ratio is clearly decreasing.

## 6 Conclusions

In this paper we have examined how the valuation of perpetual ESOs can be achieved by using a stochastic discount factor derived from the constrained intertemporal optimization problem faced by the executive. Following Ingersoll (2006), this constraint comes from the obligation to hold a proportion of the company stock higher than the optimal one. We obtain a stochastic discount factor that, by pricing the risk free asset, the market portfolio and the firm's stock, can also price the option. In a first step, we obtain a closed form expression for the executive's exercise policy. From this, a closed formula for the subjective value of ESOs with and without a vesting period is obtained. Furthermore, shareholders' cost follows easily from the subjective threshold price that characterizes the executive's exercise policy.

Despite considering perpetual ESOs, our valuation method approximates reasonably well the more realistic finite maturity case. This is true for both, the subjective and the objective ESO price. Since the latter is the ESO cost for firms and this must be calculated as part of the firm financial statements, our approach can be useful for this purpose. Furthermore, in Section 4.3, we have checked that our approach can also approximate reasonably well the expected term of the ESO, which is also part of the information the firm must disclosure to their shareholders.

Finally, we have also examined the impact of ESOs on executives incentives, highlighting the role of the employment shock likelihood. In particular, we have found that a higher job turnover rate implies lower incentives for both raising the company market value and taking on projects with high correlation with market expected return. In this latter case, there appears to be a critical value of  $\theta$  for

which the executive's perceived incentive per unit cost of the firm achieves a maximum. This maximum is decreasing with respect to the size of the idiosyncratic volatility.



## Appendix A

Using Ito's lemma, the money value of the ESO,  $V$ , obeys the following stochastic differential equation:

$$dV = \left[ (\mu_S - q_S)V_S S + \frac{\sigma_S^2}{2} V_{SS} S^2 \right] dt + \beta \sigma_M V_S S dZ_M + \sigma_I V_S S dZ_I.$$

This equation together with equation (5) will lead to our fundamental equation (7). Indeed, by omitting terms of order higher than  $dt$ , we obtain:

$$\begin{aligned} \mathbb{E}_0[V d\Theta] &= -(r - \gamma\theta^2\sigma_I^2)V\Theta dt \\ \mathbb{E}_0[\Theta dV] &= (\mu_S - q_S)V_S S \Theta dt + \frac{\sigma_S^2}{2} V_{SS} S^2 \Theta dt \\ \mathbb{E}_0[d\Theta dV] &= -(\mu_S - r)V_S S \Theta dt - \gamma\theta\sigma_I^2 V_S S \Theta dt \end{aligned}$$

where  $\mathbb{E}_0$  denotes the conditional expectation operator under the real measure and the CAPM condition,  $\mu_S = r + \beta(\mu_M - r)$ , has been used to obtain the third equation. Now, by straightforward substitution, we get:

$$0 = \mathbb{E}_0[V d\Theta + \Theta dV + d\Theta dV] (1 - \lambda dt) + \mathbb{E}_0 \left[ V d\Theta + \Theta (\Psi(S) - V) + d\Theta (\Psi(S) - V) \right] \lambda dt.$$

Hence,

$$-(r - \gamma\theta^2\sigma_I^2)V + (r - q_S - \gamma\theta\sigma_I^2)V_S S + \frac{\sigma_S^2}{2} V_{SS} S^2 + \lambda [\Psi(S) - V] = 0.$$

Finally, by defining  $\hat{r} = r - \gamma\theta^2\sigma_I^2$  and  $\hat{q}_S = q_S + \gamma\theta(1 - \theta)\sigma_I^2$  we obtain equation (7).

## Appendix B

The solution to equation (7) can be easily shown to be

$$V(S) = \begin{cases} \hat{a}_1 S^{\hat{\alpha}_1} & \text{if } S < K \\ \hat{b}_1 S^{\hat{\alpha}_1} + \hat{b}_2 S^{\hat{\alpha}_2} + \left( \frac{\lambda S}{\lambda + \hat{q}_S} - \frac{\lambda K}{\lambda + \hat{r}} \right) & \text{if } S \geq K \end{cases} \quad (20)$$

where  $\hat{\alpha}_1$  and  $\hat{\alpha}_2$  are, respectively, the positive and negative root of the quadratic equation  $\hat{\alpha}^2 + (\hat{b} - 1)\hat{\alpha} - \hat{c}$ , for

$$\hat{c} \equiv \frac{2}{\sigma_S^2}(\hat{r} + \lambda) = -\hat{\alpha}_1 \hat{\alpha}_2 \quad \text{and} \quad \hat{b} \equiv \frac{2}{\sigma_S^2}(\hat{r} - \hat{q}_S) = 1 - \hat{\alpha}_1 - \hat{\alpha}_2. \quad (21)$$

In equation (7) the negative root has been eliminated in the region  $S < K$  by imposing the boundary condition  $V(0) = 0$ . The constants  $\hat{a}_1$  and  $\hat{b}_2$  can be solved in terms of  $\hat{b}_1$  and  $K$  by using the usual conditions of value

matching and smooth pasting for  $S = K$ :

$$\begin{aligned}\hat{a}_1 K^{\alpha_1} &= \hat{b}_1 K^{\alpha_1} + \hat{b}_2 K^{\alpha_2} + \left( \frac{\lambda}{\lambda + \hat{q}_S} \right) \left( \frac{\hat{r} - \hat{q}_S}{\lambda + \hat{r}} \right) K \\ \alpha_1 \hat{a}_1 K^{\alpha_1} &= \alpha_1 \hat{b}_1 K^{\alpha_1} + \alpha_2 \hat{b}_2 K^{\alpha_2} + \left( \frac{\lambda}{\lambda + \hat{q}_S} \right) K\end{aligned}$$

The solution for  $\hat{b}_2$  is:

$$\hat{b}_2 = \frac{2\lambda}{\sigma_S^2} \left( \frac{1}{\hat{\alpha}_2(\hat{\alpha}_2 - 1)(\hat{\alpha}_1 - \hat{\alpha}_2)} \right) K^{1-\alpha_2} \equiv \hat{B}_2 K^{1-\alpha_2} \quad (22)$$

And for  $\hat{a}_1$ :

$$\hat{a}_1 = \hat{b}_1 + \frac{2\lambda}{\sigma_S^2} \left( \frac{1}{\hat{\alpha}_1(\hat{\alpha}_1 - 1)} \right) \frac{K^{1-\alpha_1}}{(\hat{\alpha}_1 - \hat{\alpha}_2)} \quad (23)$$

To determine the remaining constant,  $\hat{b}_1$ , and the threshold price,  $S^*$ , we use equations (8) and (9) to get:

$$\hat{b}_1 \hat{S}^{*\alpha_1} + \hat{b}_2 \hat{S}^{*\alpha_2} + \left( \frac{\lambda \hat{S}^*}{\lambda + \hat{q}_S} - \frac{\lambda K}{\lambda + \hat{r}} \right) = \hat{S}^* - K, \quad (24)$$

$$\alpha_1 \hat{b}_1 \hat{S}^{*\alpha_1} + \alpha_2 \hat{b}_2 \hat{S}^{*\alpha_2} + \frac{\lambda \hat{S}^*}{\lambda + \hat{q}_S} = \hat{S}^*. \quad (25)$$

Solving first for  $\hat{b}_1$  in equation (25) one gets:

$$\hat{b}_1 = \frac{-\hat{\alpha}_2}{\hat{\alpha}_1} \hat{b}_2 (\hat{S}^*)^{\alpha_2 - \alpha_1} + \frac{1}{\hat{\alpha}_1} \left( 1 - \frac{\lambda}{\lambda + \hat{q}_S} \right) (\hat{S}^*)^{1-\alpha_1}.$$

Or, after substituting for  $\hat{b}_2$  and simplifying

$$\hat{b}_1 = \frac{1}{\hat{\alpha}_1} \left\{ \left( \frac{2\lambda}{\sigma_S^2} \right) \frac{1}{(1 - \hat{\alpha}_2)(\hat{\alpha}_1 - \hat{\alpha}_2)} \left( \frac{\hat{S}^*}{K} \right)^{\alpha_2 - \alpha_1} + \left( 1 - \frac{\lambda}{\lambda + \hat{q}_S} \right) \left( \frac{\hat{S}^*}{K} \right)^{1-\alpha_1} \right\} K^{1-\hat{\alpha}_1} \equiv \hat{B}_1 K^{1-\hat{\alpha}_1}. \quad (26)$$

Hence, we can write  $\hat{a}_1$  as:

$$\hat{a}_1 = \hat{B}_1 K^{1-\hat{\alpha}_1} + \frac{2\lambda}{\sigma_S^2} \left( \frac{1}{\hat{\alpha}_1(\hat{\alpha}_1 - 1)} \right) \frac{K^{1-\alpha_1}}{(\hat{\alpha}_1 - \hat{\alpha}_2)} \equiv \hat{A}_1 K^{1-\hat{\alpha}_1} \quad (27)$$

so that, equation (10) in the main text is obtained.

Finally, by combining equations (24) and (25) we get the implicit equation for solving for  $\hat{S}^*$  which, by using the relations given in equation (21), can be written as equation (11) in the main text.

## Appendix C

We want to solve the following conditional expectation:

$$\begin{aligned} \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} V(S_\nu) \right] &= \hat{A}_1 K^{1-\hat{\alpha}_1} \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} S_\nu^{\hat{\alpha}_1} \mathbb{1}_{\{S_\nu \leq K\}} \right] + \hat{B}_1 K^{1-\hat{\alpha}_1} \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} S_\nu^{\hat{\alpha}_1} \mathbb{1}_{\{K \leq S_\nu \leq \hat{s}^*\}} \right] + \\ &+ \hat{B}_2 K^{1-\hat{\alpha}_2} \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} S_\nu^{\hat{\alpha}_2} \mathbb{1}_{\{K \leq S_\nu \leq \hat{s}^*\}} \right] + \frac{\lambda}{\lambda + \hat{q}_S} \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} S_\nu \mathbb{1}_{\{K \leq S_\nu \leq \hat{s}^*\}} \right] - \frac{\lambda}{\lambda + \hat{r}} K \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} \mathbb{1}_{\{K \leq S_\nu \leq \hat{s}^*\}} \right] + \\ &+ \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} S_\nu \mathbb{1}_{\{S_\nu \geq \hat{s}^*\}} \right] - K \mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} \mathbb{1}_{\{S_\nu \geq \hat{s}^*\}} \right]. \end{aligned}$$

Thus, all expectations take the general form  $\mathbb{E}_0 \left[ \frac{\Theta_\nu}{\Theta_0} S_\nu^c \mathbb{1}_{\{a \leq S_\nu \leq b\}} \right]$  for  $c$  any given real number. Given the stochastic dynamics driving  $S_\nu^c$  and  $(\Theta_\nu/\Theta_0)$  we have explicit expressions for each one of them, namely:

$$\begin{aligned} S_\nu^c &= S_0^c \cdot \exp \left\{ c \left( \mu - q_S - \frac{\sigma_S^2}{2} \right) \nu + c \sigma_S \sqrt{\nu} \varepsilon \right\} \\ \frac{\Theta_\nu}{\Theta_0} &= \exp \left\{ - \left( \hat{r} + \frac{1}{2} \left( \frac{\mu_M - r}{\sigma_M} \right)^2 + \frac{\gamma^2 \theta^2 \sigma_I^2}{2} \right) \nu - \left( \frac{\mu_M - r}{\sigma_M} \right) \sqrt{\nu} \varepsilon_M - \gamma \theta \sigma_I \sqrt{\nu} \varepsilon_I \right\} \end{aligned}$$

where  $\varepsilon$ ,  $\varepsilon_M$  and  $\varepsilon_I$  are *independent standard normal variables* satisfying  $\sigma_S \varepsilon = \beta \sigma_M \varepsilon_M + \sigma_I \varepsilon_I$ . Then, the expectation we seek to solve is given by a double integral of the form:

$$\begin{aligned} &\int_{\varepsilon_M} \int_{\varepsilon_I} S_0^c \exp \left\{ c \left( \mu_S - q_S - \frac{\sigma_S^2}{2} \right) \nu + c (\beta \sigma_M \varepsilon_M + \sigma_I \varepsilon_I) \sqrt{\nu} \right\} \times \\ &\exp \left\{ - \left( \hat{r} + \frac{1}{2} \left( \frac{\mu_M - r}{\sigma_M} \right)^2 + \frac{\gamma^2 \theta^2 \sigma_I^2}{2} \right) \nu - \left( \frac{\mu_M - r}{\sigma_M} \right) \sqrt{\nu} \varepsilon_M - \gamma \theta \sigma_I \sqrt{\nu} \varepsilon_I \right\} \times \\ &\phi(\varepsilon_M) \phi(\varepsilon_I) d\varepsilon_M d\varepsilon_I \end{aligned} \quad (28)$$

where  $\phi(\cdot)$  denotes the density function of a standard normal variable. Notice that the range of integration for  $\varepsilon_M$  and  $\varepsilon_I$  must be such that  $a \leq S_\nu \leq b$ .

Following Cochrane and Saa-Requejo (1999), we define the new variables:

$$\delta_1 = \frac{\beta \sigma_M \varepsilon_M + \sigma_I \varepsilon_I}{\sigma_S} \quad ; \quad \delta_2 = \frac{\sigma_I \varepsilon_M - \beta \sigma_M \varepsilon_I}{\sigma_S}.$$

Notice that  $\delta_1$  and  $\delta_2$  are too *independent standard normal variables*. By reversing the change we have the following expression in terms of  $\varepsilon_M$  and  $\varepsilon_I$ :

$$\varepsilon_M = \frac{\beta \sigma_M \delta_1 + \sigma_I \delta_2}{\sigma_S} \quad ; \quad \varepsilon_I = \frac{\sigma_I \delta_1 - \beta \sigma_M \delta_2}{\sigma_S}.$$

After substitution into equation (28), we get the following expression:

$$\begin{aligned}
& S_0^c \exp \left\{ c \left( \mu_S - q_S - \frac{\sigma_S^2}{2} \right) \nu - \left( \hat{r} + \frac{1}{2} \left( \frac{\mu_M - r}{\sigma_M} \right)^2 + \frac{\gamma^2 \theta^2 \sigma_I^2}{2} \right) \nu \right\} \times \\
& \int_{\delta_1} \int_{\delta_2} \exp \left\{ \left[ c \sigma_S - \left( \frac{\beta(\mu_M - r) + \gamma \theta \sigma_I^2}{\sigma_S} \right) \right] \sqrt{\nu} \delta_1 \right\} \times \\
& \exp \left\{ - \left[ \left( \frac{\mu_M - r}{\sigma_M} \right) - \beta \sigma_M \gamma \theta \right] \frac{\sigma_I}{\sigma_S} \sqrt{\nu} \delta_2 \right\} \phi(\delta_1) \phi(\delta_2) d\delta_1 d\delta_2 .
\end{aligned} \tag{29}$$

We turn next to the specification of the integration region for each of the new variables. Clearly, for  $a \leq S_\nu \leq b$  we have the following boundaries in terms of the  $\delta_1$  variable:

$$\frac{\ln(a/S_0) - (\mu_S - q_S - \sigma_S^2/2)\nu}{\sigma_S \sqrt{\nu}} \leq \delta_1 \leq \frac{\ln(b/S_0) - (\mu_S - q_S - \sigma_S^2/2)\nu}{\sigma_S \sqrt{\nu}}$$

or more compactly  $A \leq \delta_1 \leq B$ . By the other hand the range of integration for  $\delta_2$  is unrestricted. Hence, by omitting the exponential term that appears outside the double integral (29), we are left with

$$\begin{aligned}
& \frac{1}{\sqrt{2\pi}} \int_{\delta_2} \exp \left\{ - \left[ \left( \frac{\mu_M - r}{\sigma_S} \right) - \beta \sigma_M \gamma \theta \right] \left( \frac{\sigma_I}{\sigma_S} \right) \sqrt{\nu} \delta_2 - \frac{1}{2} \delta_2^2 \right\} d\delta_2 \times \\
& \frac{1}{\sqrt{2\pi}} \int_A^B \exp \left\{ \left[ c \sigma_S - \left( \frac{\beta(\mu_M - r) + \gamma \theta \sigma_I^2}{\sigma_S} \right) \right] \sqrt{\nu} \delta_1 - \frac{1}{2} \delta_1^2 \right\} d\delta_1 .
\end{aligned}$$

Each integral is solved by completing the square. Thus, for the first integral, we get

$$\exp \left\{ \frac{1}{2} \left[ \left( \frac{\mu_M - r}{\sigma_S} \right) - \beta \sigma_M \gamma \theta \right]^2 \left( \frac{\sigma_I}{\sigma_S} \right)^2 \nu \right\},$$

whereas for the second integral, we obtain:

$$\exp \left\{ \frac{1}{2} \left[ c \sigma_S - \left( \frac{\beta(\mu_M - r) + \gamma \theta \sigma_I^2}{\sigma_S} \right) \right]^2 \nu \right\} \times \left\{ \Phi \left[ \frac{\ln(b) - \mu - c\sigma^2}{\sigma} \right] - \Phi \left[ \frac{\ln(a) - \mu - c\sigma^2}{\sigma} \right] \right\},$$

for  $\mu = \ln(S_0) + (\hat{r} - \hat{q}_S - (\sigma_S^2/2))\nu$  and  $\sigma = \sigma_S \sqrt{\nu}$ . In the computation of this integral we have made use of the relationship  $\hat{r} - \hat{q}_S = r - q_S - \gamma \theta \sigma_I^2$  and the CAPM condition  $(\mu_S - r) = \beta(\mu_M - r)$ .

Finally and after some algebra, the product of the three remaining exponentials can be greatly simplified to

$$\exp \{ -\hat{r}\nu \} \exp \left\{ c\mu + c^2 \frac{\sigma^2}{2} \right\} .$$

Summing up, we obtain:

$$\mathbb{E}_0 \left[ \left( \frac{\Theta_\nu}{\Theta_0} S_\nu^c \right) \mathbb{1}_{\{a \leq S_\nu \leq b\}} \right] = \exp \{ -\hat{r}\nu \} \exp \left\{ c\mu + c^2 \frac{\sigma^2}{2} \right\} \times \left\{ \Phi \left( \frac{\ln b - \mu - c\sigma^2}{\sigma} \right) - \Phi \left( \frac{\ln a - \mu - c\sigma^2}{\sigma} \right) \right\} .$$

## Appendix D

Following the same steps as in Appendix C, we find that the solution to equation (14) is given by

$$V(S) = \begin{cases} \tilde{a}_1 S^{\alpha_1} & \text{if } S < K \\ \tilde{b}_1 S^{\alpha_1} + \tilde{b}_2 S^{\alpha_2} + \left( \frac{\lambda S}{\lambda + q_S} - \frac{\lambda K}{\lambda + r} \right) & \text{if } K \leq S \leq \hat{S}^* \\ S - K & \text{if } S \geq \hat{S}^* \end{cases} \quad (30)$$

where  $\alpha_1$  and  $\alpha_2$  are, respectively, the positive and negative root of the quadratic equation  $\alpha^2 + (b-1)\alpha - c$ , for

$$c \equiv \frac{2}{\sigma_S^2}(r + \lambda) = -\alpha_1 \alpha_2 \quad \text{and} \quad b \equiv \frac{2}{\sigma_S^2}(r - q_S) = 1 - \alpha_1 - \alpha_2 . \quad (31)$$

Again, a similar procedure leads to the following values for the constants  $\tilde{a}_1$ ,  $\tilde{b}_2$  and  $\tilde{b}_1$ :

$$\tilde{b}_2 = \frac{2\lambda}{\sigma_S^2} \left( \frac{1}{\alpha_2(\alpha_2 - 1)(\alpha_1 - \alpha_2)} \right) K^{1-\alpha_2} := \tilde{B}_2 K^{1-\alpha_2} \quad (32)$$

$$\begin{aligned} \tilde{b}_1 &= \left( 1 - \frac{(2\lambda/\sigma_S^2)}{(\alpha_1 - 1)(1 - \alpha_2)} \right) (\hat{S}^*)^{1-\alpha_1} - \left( 1 + \frac{(2\lambda/\sigma_S^2)}{\alpha_1 \alpha_2} \right) K (\hat{S}^*)^{\alpha_1} - \tilde{b}_2 (\hat{S}^*)^{\alpha_2 - \alpha_1} = \\ &= \left\{ \left( 1 - \frac{(2\lambda/\sigma_S^2)}{(\alpha_1 - 1)(1 - \alpha_2)} \right) \left( \frac{\hat{S}^*}{K} \right)^{1-\alpha_1} - \left( 1 + \frac{(2\lambda/\sigma_S^2)}{\alpha_1 \alpha_2} \right) \left( \frac{\hat{S}^*}{K} \right)^{\alpha_1} - \tilde{B}_2 \left( \frac{\hat{S}^*}{K} \right)^{\alpha_2 - \alpha_1} \right\} K^{1-\alpha_1} \\ &\equiv \tilde{B}_1 K^{1-\alpha_1} \end{aligned} \quad (33)$$

$$\tilde{a}_1 = \tilde{b}_1 + \frac{2\lambda}{\sigma_S^2} \left( \frac{1}{\alpha_1(\alpha_1 - 1)} \right) \frac{K^{1-\alpha_1}}{(\alpha_1 - \alpha_2)} \equiv \tilde{A}_1 K^{1-\alpha_1} . \quad (34)$$

Table 1: Subjective perpetual ESO valuation

		$\beta = 0$					$\beta = 1$				
		0.00	0.10	0.20	0.30	0.40	0.00	0.10	0.20	0.30	0.40
$\gamma$	$\sigma_S$	Panel A: $\lambda = 0.1, \nu = 0$									
2	0.30	11.000	8.802	7.404	6.420	5.683	11.000	9.673	8.714	7.987	7.416
	0.40	12.859	9.617	7.813	6.624	5.768	12.859	10.266	8.694	7.611	6.810
	0.60	16.248	10.866	8.410	6.923	5.911	16.248	11.273	8.899	7.432	6.419
4	0.30	11.000	7.273	5.415	4.281	3.516	11.000	8.605	7.127	6.105	5.348
	0.40	12.859	7.644	5.456	4.215	3.416	12.859	8.527	6.453	5.190	4.332
	0.60	16.248	8.174	5.538	4.185	3.345	16.248	8.656	6.007	4.596	3.719
Panel B: $\lambda = 0.1, \nu = 3$											
2	0.30	9.778	7.692	6.365	5.415	4.689	9.778	8.516	7.613	6.928	6.392
	0.40	11.324	8.232	6.450	5.221	4.310	11.324	8.858	7.337	6.259	5.446
	0.60	14.095	8.756	6.054	4.356	3.216	14.095	9.187	6.616	4.961	3.823
4	0.30	9.778	6.233	4.382	3.169	2.311	9.778	7.505	6.098	5.101	4.343
	0.40	11.324	6.259	3.892	2.466	1.560	11.324	7.159	5.012	3.617	2.645
	0.60	14.095	5.733	2.636	1.205	0.533	14.095	6.296	3.193	1.635	0.828
Panel C: $\lambda = 0.2, \nu = 0$											
2	0.30	8.296	6.930	5.994	5.306	4.774	8.296	7.485	6.863	6.370	5.969
	0.40	9.951	7.863	6.597	5.720	5.066	9.951	8.297	7.216	6.434	5.837
	0.60	13.080	9.380	7.507	6.311	5.468	13.080	9.677	7.884	6.718	5.885
4	0.30	8.296	5.920	4.612	3.766	3.170	8.296	6.803	5.807	5.086	4.536
	0.40	9.951	6.492	4.856	3.865	3.197	9.951	7.116	5.610	4.639	3.952
	0.60	13.080	7.340	5.179	3.990	3.238	13.080	7.716	5.573	4.360	3.574
Panel D: $\lambda = 0.2, \nu = 3$											
2	0.30	6.240	5.062	4.263	3.668	3.199	6.240	5.538	5.009	4.593	4.258
	0.40	7.365	5.560	4.433	3.625	3.011	7.365	5.939	4.996	4.300	3.761
	0.60	9.450	6.120	4.290	3.108	2.304	9.450	6.404	4.674	3.528	2.729
4	0.30	6.240	4.192	3.019	2.215	1.630	6.240	4.954	4.100	3.470	2.978
	0.40	7.365	4.320	2.743	1.758	1.120	7.365	4.894	3.495	2.550	1.878
	0.60	9.450	4.077	1.901	0.874	0.389	9.450	4.464	2.296	1.184	0.602

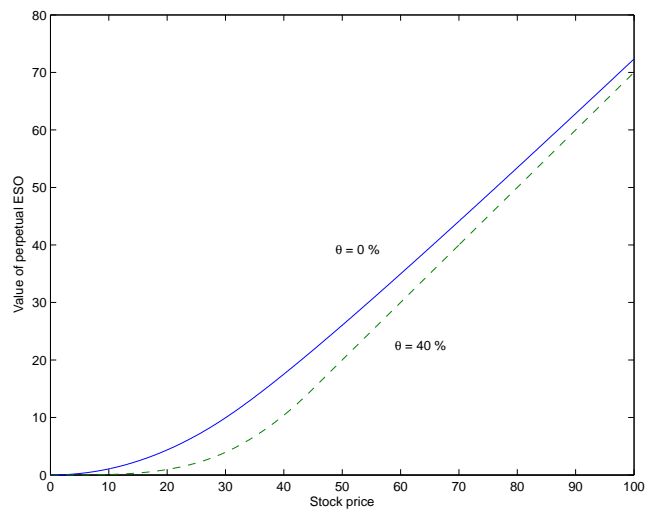
This table shows the subjective ESO value obtained by using either equation (10) or (13). The first column,  $\gamma$ , contains different risk-aversion coefficients. The second column,  $\sigma_S$ , contains the different levels of firm's stock volatility. The next five columns are obtained with  $\beta = 0$  and the remaining ones with  $\beta = 1$ . We also consider different values of  $\theta$  ranging from 0% to 40%. The table is divided in four panels for different combinations of the employment shock intensity,  $\lambda$ , and the vesting period,  $\nu$ . Other parameters for ESO valuation are:  $S_0 = K = \$30$ , corresponding to the initial firm's stock price and the option strike price respectively, the yearly risk-free rate,  $r = 6\%$ , the yearly stock continuously compounded dividend rate,  $q_S = 1.5\%$  and the yearly market portfolio volatility,  $\sigma_M = 20\%$ .

Table 2: Objective ESO valuation: perpetual vs. finite maturity (10 years)

$\lambda$	$\beta$	$\gamma$	Perpetual ESO price					Finite maturity ESO price				
			$\theta$					$\theta$				
			0.00	0.10	0.20	0.30	0.40	0.00	0.10	0.20	0.30	0.40
Panel A: $\nu = 1$												
0.1	0	2	12.534	11.907	11.007	10.217	9.572	11.424	10.932	10.377	9.892	9.479
		4	12.534	10.893	9.310	8.280	7.595	11.424	10.328	9.332	8.568	7.960
	1	2	12.534	12.136	11.500	10.892	10.361	11.424	11.082	10.680	10.293	9.960
		4	12.534	11.404	10.084	9.099	8.383	11.424	10.620	9.825	9.175	8.627
0.2	0	2	9.339	9.002	8.481	8.011	7.622	8.429	8.609	8.249	7.938	7.659
		4	9.339	8.420	7.479	6.840	6.398	8.429	8.207	7.573	7.070	6.672
	1	2	9.339	9.130	8.766	8.407	8.087	8.429	8.702	8.446	8.190	7.977
		4	9.339	8.716	7.940	7.345	6.900	8.429	8.407	7.899	7.475	7.116
Panel B: $\nu = 3$												
0.1	0	2	11.324	10.795	10.175	9.701	9.342	10.217	9.919	9.664	9.452	9.253
		4	11.324	10.103	9.201	8.665	8.318	10.217	9.645	9.197	8.794	8.460
	1	2	11.324	10.975	10.500	10.102	9.784	10.217	9.981	9.780	9.623	9.478
		4	11.324	10.435	9.625	9.089	8.718	10.217	9.778	9.432	9.110	8.821
0.2	0	2	7.365	7.120	6.830	6.606	6.433	6.986	6.821	6.690	6.572	6.454
		4	7.365	6.800	6.372	6.108	5.929	6.986	6.681	6.421	6.199	6.010
	1	2	7.365	7.204	6.981	6.793	6.640	6.986	6.854	6.757	6.664	6.584
		4	7.365	6.953	6.573	6.316	6.132	6.986	6.745	6.557	6.379	6.219
Panel C: $\nu = 5$												
0.1	0	2	9.905	9.520	9.120	8.827	8.610	8.826	8.708	8.587	8.470	8.368
		4	9.905	9.075	8.526	8.205	7.998	8.826	8.577	8.341	8.118	7.933
	1	2	9.905	9.644	9.326	9.074	8.878	8.826	8.737	8.654	8.565	8.482
		4	9.905	9.284	8.781	8.459	8.237	8.826	8.642	8.463	8.290	8.129
0.2	0	2	5.465	5.319	5.168	5.056	4.972	5.125	5.070	5.011	4.954	4.901
		4	5.465	5.152	4.942	4.814	4.728	5.125	5.006	4.889	4.785	4.698
	1	2	5.465	5.366	5.245	5.149	5.073	5.125	5.081	5.042	4.999	4.959
		4	5.465	5.231	5.040	4.915	4.826	5.125	5.036	4.949	4.868	4.790

This table shows the objective value for the perpetual ESO (left side) and for a finite maturity ESO of ten years (right side) denoted as  $V^{SUB}$  and  $V_T^{SUB}$ , respectively. The first column,  $\lambda$ , contains different values of the employment shock intensity. The second column,  $\beta$ , contains different levels of the market beta. The third column,  $\gamma$ , represents different levels of executive's risk aversion.  $V^{SUB}$  and  $V_T^{SUB}$  are obtained under five different values of  $\theta$  ranging from 0% to 40%. We also consider three different vesting period lengths in Panel A ( $\nu = 1$  years), Panel B ( $\nu = 3$  years) and Panel C ( $\nu = 5$  years). The perpetual values have been obtained using Corollary 6. For the finite maturity cases, the procedure to obtain these prices are described in subsection 3.2. The values for the remaining parameters are:  $S_0 = K = \$30$ ,  $r = 6\%$ ,  $q_S = 1.5\%$ ,  $\sigma_S = 40\%$  and  $\sigma_M = 20\%$ .

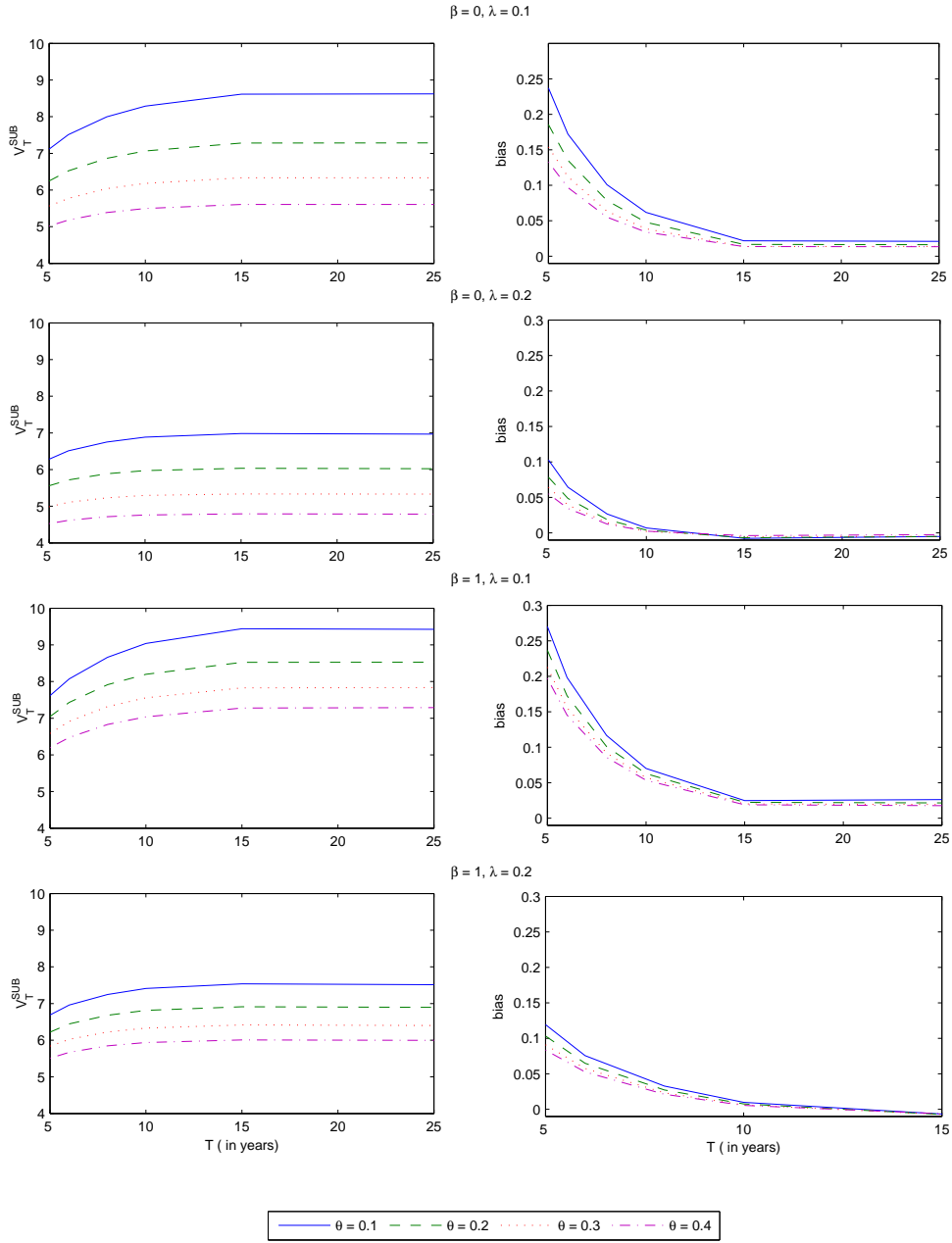
Figure 1: Subjective ESO price and diversification level



The figure plots the subjective ESO price,  $V^{SUB}$ , as a function of the stock price for different values of the portfolio undiversification parameter,  $\theta$ . Note that the continuous line for  $\theta = 0$  represents the market ESO price, which acts as an upper bound for  $V^{SUB}$ . The values of the remaining parameters are:  $r = 6\%$ ,  $q_S = 1.5\%$ ,  $\gamma = 4$ ,  $\lambda = 20\%$ ,  $\sigma_M = 20\%$ ,  $\sigma_S = 40\%$ ,  $K = \$30$  and  $\nu = 0$ .

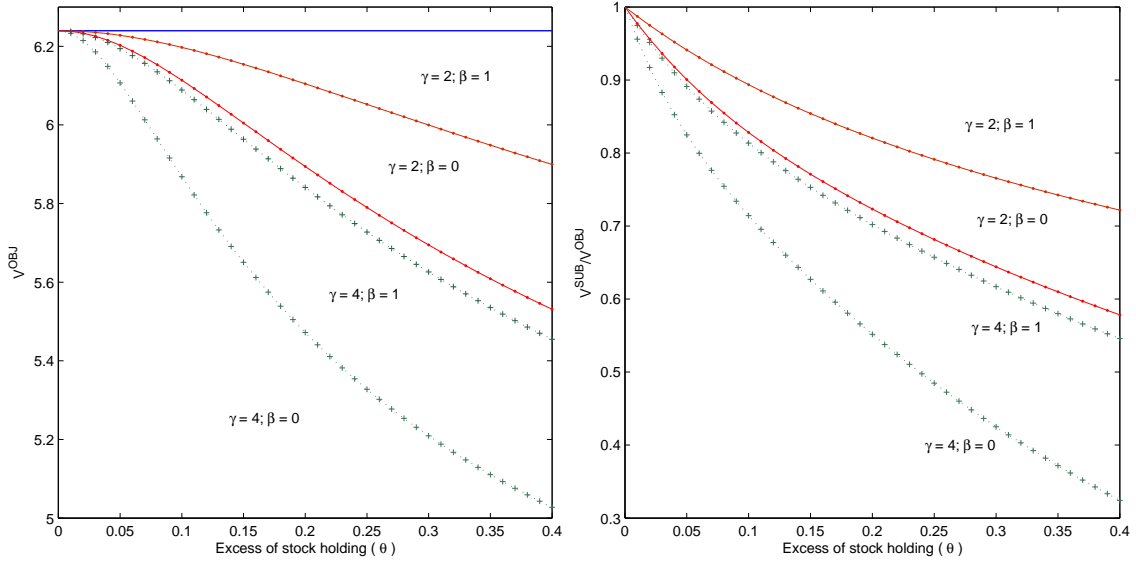


Figure 2: Subjective ESO price and time to maturity



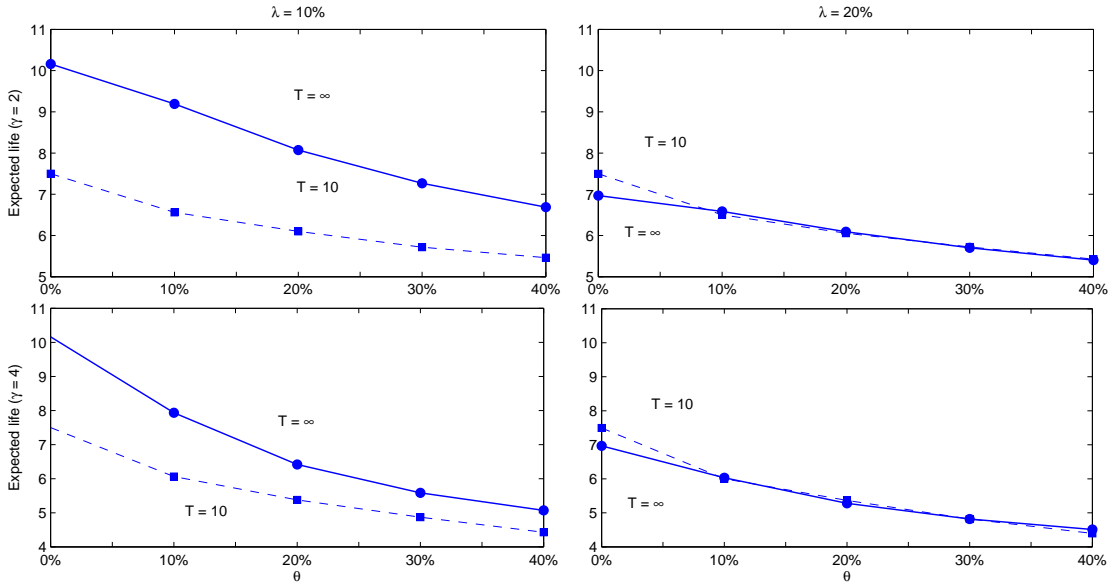
This figure shows the subjective ESO value (left-hand graphics) for different finite maturities (in x-axis) ranging from  $T = 5$  to  $T = 25$  years, denoted as  $V_T^{SUB}$ . Each graphic exhibits a different pair of values for  $(\beta, \lambda)$  and each line, inside each graphic, corresponds to a different value of  $\theta$ . Any right-hand graphic displays the relative bias of the perpetual ESO computed as  $(V^{SUB} - V_T^{SUB})/V_T^{SUB}$ , where  $V^{SUB}$  is the perpetual ESO price from Table 1. The procedure to obtain  $V_T^{SUB}$  is described in Subsection 3.2. The values for the remaining parameters are:  $S_0 = K = \$30$ ,  $r = 6\%$ ,  $q_S = 1.5\%$ ,  $\lambda = 20\%$ ,  $\sigma_S = 40\%$ ,  $\sigma_M = 20\%$  and  $\nu = 3$  years.

Figure 3: Firm cost of perpetual ESO



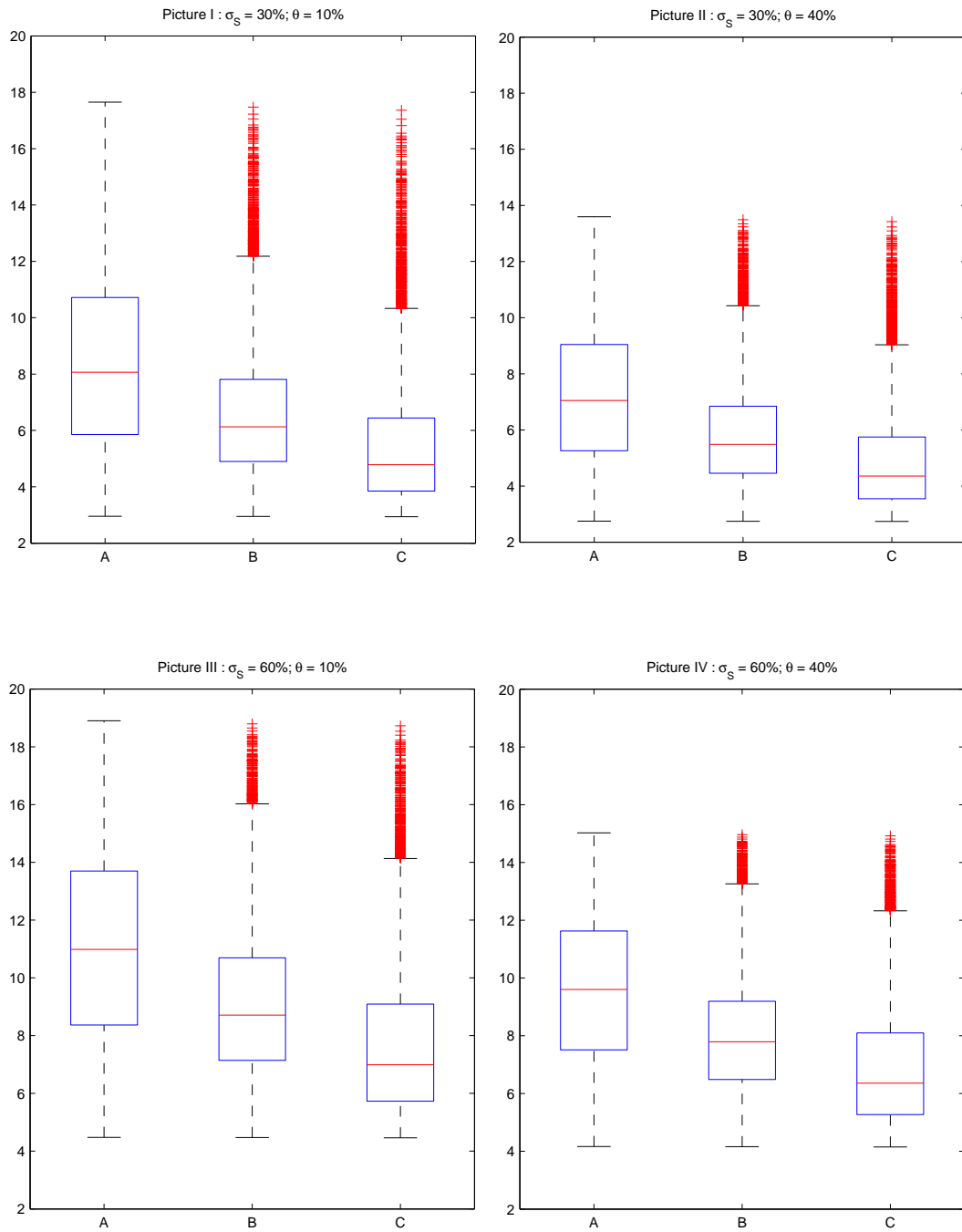
The left hand graphic represents the firm cost,  $V^{OBJ}$ , of a perpetual ESO for several values of  $\gamma$  and  $\beta$ . In this figure, the continuous line represents the risk neutral value,  $V^{RN}$ , of the perpetual ESO. The right hand graphic displays the ratio between the subjective and the objective values. The values of the other parameters are:  $r = 6\%$ ,  $q_S = 1.5\%$ ,  $\sigma_S = 30\%$ ,  $\sigma_M = 20\%$ ,  $\lambda = 20\%$ ,  $S_0 = K = \$30$  and  $\nu = 3$ .

Figure 4: Expected term and FAS 123R



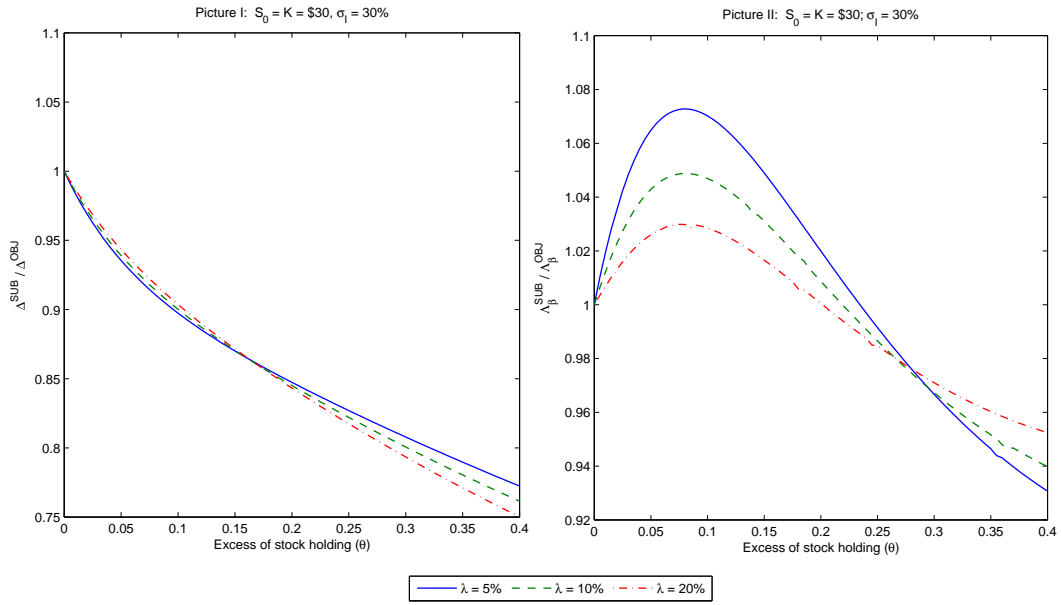
This figure compares the expected term from a finite maturity ESO of 10 years and a vesting period of 3 years (discontinuous line) with that obtained by calibrating equation (19) using a price equal to the perpetual ESO value (solid line). The remaining values of the parameters are:  $r = 6\%$ ,  $q_S = 1.5\%$ ,  $\sigma_S = 40\%$  and  $S_0 = K = \$30$ .

Figure 5: Box-plot for objective ESO prices



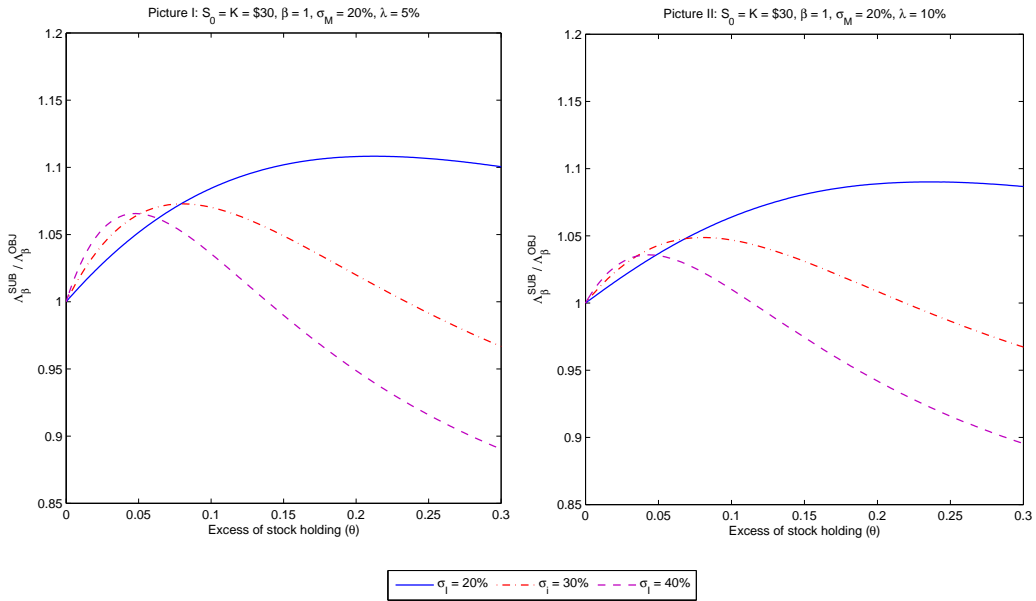
This figure shows the distribution for objective ESO values implied by the corresponding random samples drawn from alternative Triangular distributions of  $\lambda$ , denoted as A, B and C which have been described in subsection 4.4. The values for the remaining parameters are:  $r = 6\%$ ,  $q_S = 1.5\%$ ,  $\gamma = 4$ ,  $\beta = 1$ ,  $\sigma_S = 40\%$ ,  $\beta = 1$ ,  $S_0 = \$30$  and  $K = \$30$ .

Figure 6: Greek ratios and employment shock



Picture I displays the delta ratio,  $\Delta^{SUB}/\Delta^{OBJ}$ , as a function of  $\theta$  taking  $S_0 = K$  as a midpoint in the computation of the numerical partial derivatives. Picture II displays the systematic risk ratio,  $\Lambda_{\beta}^{SUB}/\Lambda_{\beta}^{OBJ}$ , as a function of  $\theta$  using  $\beta = 1$  as a midpoint. Each picture depicts several plots for different values of the likelihood of employment shocks,  $\lambda$ . The values for the remaining parameters are:  $r = 6\%$ ,  $q_S = 1.5\%$ ,  $\gamma = 2$ ,  $\beta = 1$ ,  $\sigma_M = 20\%$  and  $\nu = 3$ .

Figure 7: Greek ratios and idiosyncratic risk



Pictures I and II display the systematic vega ratio,  $\Lambda_{\beta}^{SUB}/\Lambda_{\beta}^{OBJ}$ , as a function of  $\theta$  for  $\lambda = 5\%$  and  $\lambda = 10\%$ , respectively. In both cases, the ratio is computed for at-the-money ESOs and  $\gamma = 2$ . Each picture depicts several plots for different idiosyncratic risk values,  $\sigma_I$ . The values for the remaining parameters are:  $r = 6\%$ ,  $q_S = 1.5\%$ ,  $\beta = 1$ ,  $\sigma_M = 20\%$  and  $\nu = 3$ .

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