

A discusión

OPTIMAL TWO-PART TARIFF LICENSING CONTRACTS WITH DIFFERENTIATED GOODS AND ENDOGENOUS R&D*

Ramón Faulí-Oller and Joel Sandonís**

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Corresponding author: R. Faulí-Oller, Universidad de Alicante, Economics Department. Campus de San Vicente del Raspeig, E-03071 Alicante, Spain. E-mail: fauli@merlin.fae.ua.es.

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** R. Faulí-Oller y J. Sandonis: University of Alicante.

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ABSTRACT

In this paper we get the optimal two-part tariff contract for the licensing of a cost reducing innovation to a differentiated goods industry of a general size. We analyze the cases where the patentee is an independent laboratory or an incumbent firm. We show that, regardless of the number of firms, the degree of product differentiation and the type of patentee, the innovation is licensed to all firms. Moreover, we endogenize R&D investment and get that an internal patentee invests more (less) in R&D when the technological opportunity is low (high).

JEL classification: L11, L13 and L14.

Key words: Patent licensing, two-part tariff contracts, R&D, product differentiation.

1 Introduction

Nowadays, innovation is recognized by economists as a crucial ingredient of growth. This has motivated the development of a huge literature analyzing the determinants of innovation. In particular, one of the main issues is whether the market provides firms with the right incentives to invest in R&D. Since Schumpeter's seminal work, there have been many empirical and theoretical papers addressing the effect of competition on R&D investment. However the results are inconclusive. One common characteristic of many of these papers is that they do not consider the existence of a market for technology. In other words, the return to R&D investment comes from the final good market.

In this paper, we add to this literature by allowing for the existence of a market for technology. We consider the case of a research laboratory that owns a process innovation that allows to reduce the production cost of a given industry. We consider two cases. If the laboratory remains independent as an external patentee, it obtains revenues from the licensing of the patented innovation to the firms producing in this industry. Another possibility is that the laboratory integrates vertically with one of the firms in the industry, becoming an internal patentee. In this case, it may obtain revenues not only from licensing but also from participating in the final goods market.

As the literature on patent licensing has pointed out, the profits of licensing depend on the type of licensing contracts that are available to the patentee. The earlier papers (Kamien and Tauman (1984,1986), Katz and Shapiro (1986) and Kamien et al. (1992)) obtain that licensing through a fixed fee or an auction is more profitable for an independent laboratory than licensing through a royalty. In practice, however, licensing contracts very often include both a fixed fee and a royalty (for example, Rostocker (1984) and Yanagawa and Wada (2000) show that this is the case for approximately half of the licensing contracts). In spite of this empirical fact, only

a few papers have studied these types of contracts (Fauli-Oller and Sandonis (2002, 2006), Sen and Tauman (2007) and Erutku and Richelle (2007)).

In this paper, we obtain the optimal two-part tariff licensing contract for the general case of a n -firms oligopolistic industry producing differentiated goods. We analyze the case where the patentee is an independent laboratory as well as the case where it is an incumbent patentee. We show that, in both cases, and regardless of the size of the innovation, the number of firms in the industry and the degree of product differentiation the innovation is licensed to all firms in the industry.

In the last part of the paper, we endogenize the size of the innovation by allowing the patentee to invest in cost-reducing R&D. This allows us to compare the optimal R&D investment of the laboratory when it is an outsider to the industry and when it is vertically integrated with one of the firms in the industry. We obtain that a vertically integrated laboratory invest more in R&D than an independent laboratory when the R&D investment is costly. This result has a nice empirical implication regarding the internal organization of leading innovative firms. When the technological opportunity of the industry is high, we can expect that innovation is dominated by independent research laboratories. In contrast, when it is low, vertically integrated firms are the ones expected to be the leaders in innovation activities.

There are two papers closely related to ours: Sen and Tauman (2007) and Erutku and Richelle (2007). They analyze the case of homogenous goods. The distinguishing feature of our paper is that we consider differentiated goods. Erutku and Richelle (2007) study the optimal two-part tariff contract to license a cost-reducing innovation for the case of an independent laboratory. They show that, regardless of the number of firms in the industry, the innovation is licensed to all firms. In this paper, we show that their result extends to the case of differentiated goods and also to the case of a vertically integrated laboratory.

On the other hand, Sen and Tauman (2007) study the optimal *auction plus royalty* licensing policy for a general size oligopoly and for the cases of an internal and an external patentee. The difference with respect to a two-part tariff contract is that, in an auction plus royalty contract, the fixed part is determined through an auction. They show that the innovation is licensed to all firms (except perhaps one). They also study the incentives to innovate. They show that the difference between post-innovation and pre-innovation profits is always higher for an external patentee. The intuition is that whereas the independent laboratory earns no profit in the absence of the innovation, an incumbent patentee earns the market profits. As they consider that the R&D investment increases the probability of obtaining a given cost reducing innovation, they obtain that the external patentee invests more in R&D.

In this paper, we adopt a different modelling strategy because R&D, in our model, determines the cost reduction. Therefore, in our setting, what matters is not the difference between post-innovation and pre-innovation profits but the marginal profitability of R&D investment. We do not claim that our approach is more realistic than Sen and Tauman's. We only want to stress that in a framework where R&D allows to reduce production costs in a continuous way, an incumbent patentee may have more incentives to innovate than an independent laboratory. We see our result as complementary to the one in Sen and Tauman (2007).

The rest of the paper is organized as follows: Section 2 presents the model for the cases of an independent laboratory and an incumbent patentee. In Section 3, we compare their incentives to innovate. Finally, we conclude in Section 4. All proofs that are omitted from the text are relegated to an Appendix.

2 Model

We consider n symmetric firms competing in quantities and selling differentiated goods ($i = 1 \dots n$). Firm i sells good i . Inverse demand of good i is given by:

$$\begin{aligned} p_i &= a - q_i - \gamma \sum_{j \neq i} q_j \\ i &= 1 \dots n \end{aligned}$$

where q_i is the quantity sold of good i . All firms produce with marginal cost $c < a$.

We analyze two different settings. In the first model, we assume that there is an independent research laboratory that owns a patented process innovation. In the second model, the laboratory is vertically integrated with one of the competing firms in the industry. The innovation allows the firms to reduce their cost of production to $c - \varepsilon$. We aim to derive the optimal two-part tariff licensing contract (F, r) for both the external and the internal patentee, where F specifies a non-negative fixed fee and r a linear per-unit royalty.

Although most of the papers in the literature impose non-negative royalties (exceptions are Liao and Sen (2005) and Erutku and Richelle (2007)), for simplicity, we solve the model without taking into account this constraint.¹

We start by analyzing the case of an external patentee.

2.1 The case of an external patentee

In this case, the timing of the game is as follows: in the first stage the patentee offers a two-part tariff contract (F, r) to the n competing firms. In the second stage, firms decide whether or not to accept the contract. The ones that accept, pay F to the patentee. Finally, the firms compete à la Cournot with a cost inherited from the licensing stage.

¹Our results would not change if we impose non-negative royalties whenever c is not very low.

Assume that k firms have accepted a licensing contract (F, r) . Firms that have not accepted the contract produce in equilibrium:

$$q_N(k, r) = \begin{cases} \frac{(a-c)(2-\gamma) - \gamma k(\varepsilon - r)}{(2-\gamma)(2+\gamma(n-1))} & \text{if } r > \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma k} \\ 0 & \text{otherwise.} \end{cases}$$

Observe that, if r is very low, the firms that do not accept the contract are driven out of the market. On the other hand, the firms that accept the contract produce in equilibrium:

$$q(k, r) = \begin{cases} \frac{(a-c)(2-\gamma) - (-2 + \gamma(1+k) - \gamma n)(\varepsilon - r)}{(2-\gamma)(2+\gamma(n-1))} & \text{if } r > \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma k} \\ \frac{a-c+\varepsilon-r}{k+1} & \text{otherwise.} \end{cases}$$

Profits of non-accepting and accepting firms are given, respectively, by $\Pi_N(k, r) = (q_N(k, r))^2$ and $\Pi(k, r) = (q(k, r))^2$.

In the second stage, given that $k-1$ firms accept the contract, the k^{th} firm accepts the contract whenever $F \leq \Pi(k, r) - \Pi_N(k-1, r)$. Obviously, as the laboratory maximizes profits, in order for k firms to accept the contract,² it will choose F such that $F = \Pi(k, r) - \Pi_N(k-1, r)$. This implies that the problem of choosing the optimal contract (F, r) is equivalent to that of choosing (k, r) . Then, in the first stage, the external patentee solves the following problem:

$$\underset{k, r}{Max} \ k (\Pi(k, r) - \Pi_N(k-1, r) + r q(k, r)) \quad (1)$$

$$s.t. \quad 1 \leq k \leq n \text{ and } r \leq \varepsilon.$$

We proceed as follows. First of all, we prove that the research laboratory finds profitable to license the innovation to all firms in the industry. Then, we calculate the optimal royalty once we replace k by n in expression (1). As far as the first result is concerned, we know that with a fixed fee contract, the input would be sold to only a subset of firms in order to protect industry profits from competition (Kamien and Tauman (1986)). With a two-part tariff

² As $\frac{\partial(\Pi(k, w) - \Pi_N(k-1, w))}{\partial k} < 0$, this is the only equilibrium in the acceptance stage.

contract, however, the laboratory can always license to one more firm without affecting the level of competition, by choosing an appropriate royalty. Before solving the program, the following lemma shows that it is always profitable for the laboratory to license the innovation to all firms regardless of the total number of them in the industry. Assume that the laboratory licenses the innovation to k firms with a royalty r . The strategy is to show that the laboratory can always increase profits by licensing the innovation to all firms through a higher royalty ($r < r_E < \varepsilon$) such that *total industry output* remains constant. In the particular case of homogeneous goods this is very intuitive because it would imply keeping final price constant³. With differentiated goods, however, it is just a technical condition that helps to get the result.

Lemma 1 *Assume that the laboratory licenses to k firms with a royalty r . It can always increase profits by licensing to all firms with a royalty $r < r_E \leq \varepsilon$ such that $nq(n, r_E) = (n - k)q_N(k, r) + kq(k, r)$.*

Proof. See Appendix. ■

This result is central to the paper and, therefore, it seems interesting to know whether it holds for more general demand functions. In the Appendix we show that, for the case of homogenous goods, it holds for concave demands satisfying a technical restriction concerning the third derivative of the inverse demand. We show that it also holds for the class of demands $P = A - X^b$, where $b \geq 1$.

Using the result in the previous lemma, next proposition derives the optimal two-part tariff contract to license to n firms.

³This argument is used in Sen and Tauman (2007) to prove that with an auction plus royalty contract, the input would be sold to all firms. It is also used in Fauli-Oller and Sandonís (2007) in the context of an input market. In both papers, only the case of homogeneous goods is analyzed.

Proposition 1 *The laboratory optimally licenses the innovation to all firms. The optimal royalty is: $r^*(n) = r_1$ if $\varepsilon < \varepsilon_1$ and $r^*(n) = r_2$ otherwise, where.*

$$\begin{aligned} r_1 &= \frac{\gamma(n-1)((a-c)(-2+\gamma)\gamma + \varepsilon(4 + \gamma(-6 + \gamma + 2n)))}{2(4 + \gamma(4(2-n) + \gamma(6 + \gamma(n-1) + (n-6)n)))}, \\ r_2 &= \frac{(a-c+\varepsilon)\gamma(n-1)}{2 + 2\gamma(n-1)}, \\ \varepsilon_1 &= \frac{(a-c)(4 + \gamma(-6 + \gamma(n-3)(n-1) + 4n))}{\gamma(n-1)(2 + \gamma(n-1))} \end{aligned}$$

Proof. See Appendix ■

Observe that the constraint $r \leq \varepsilon$ is never binding in equilibrium. The reason is that the objective function of the patentee can be expressed as the difference between market profits $n\Pi(n, r) + nrq(n, r)$ and the outside option of the licensees $\Pi_N(n-1, r)$. Market profits are increasing in r up to $r^*(n)$, whereas the outside option is decreasing in r . The balance of the two effects leads to a royalty lower than ε . Notice also that the optimal royalty increases with ε . The reason is that the higher the size of the innovation the lower the outside option of the licensees for a given r . Then, the patentee is less interested in reducing the outside option and increases the royalty to increase market profits.

2.2 The case of an internal patentee

In this subsection, we consider a situation where the innovation is owned by one of the firms in the industry (say firm 1). We have to distinguish two cases: If the innovation is drastic $(-2a + 2c + (a-c+\varepsilon)\gamma > 0)^4$, the patentee gets the monopoly profits in its market by not licensing the innovation. If the innovation is not drastic, monopolization never occurs. The timing of the game is as in the previous subsection. In this situation, in the market stage of the game there are three different cost levels: the owner of the innovation produces at $c - \varepsilon$, the

⁴Observe that this holds when the innovation is important $\varepsilon \geq \frac{(a-c)(2-\gamma)}{\gamma}$.

licensees at cost $c - \varepsilon + r$ and the non-licensees at cost c . As a consequence, equilibrium outputs are given respectively by:

If the innovation is non-drastic:

$$q_P^I(k, r) = \begin{cases} \frac{-2(c - \varepsilon) + a(2 - \gamma) + \gamma(c + \varepsilon(n - k - 2) + kr)}{(2 - \gamma)(2 + \gamma(n - 1))} & \text{if } r > \varepsilon + \frac{-2a + 2c + (a - c + \varepsilon)\gamma}{\gamma k} \\ \frac{(a - c + \varepsilon)(2 - \gamma) + \gamma kr}{(2 - \gamma)(2 + \gamma k)} & \text{otherwise.} \end{cases}$$

$$q^I(k, r) = \begin{cases} \frac{a(2 - \gamma)(c - \varepsilon + r) + \gamma(c + \varepsilon(n - k - 2) - (n - k - 1)r)}{(2 - \gamma)(2 + \gamma(n - 1))} & \text{if } r > \varepsilon + \frac{-2a + 2c + (a - c + \varepsilon)\gamma}{\gamma k} \\ \frac{(a - c - \varepsilon)(2 - \gamma) - 2r}{(2 - \gamma)(2 + \gamma k)} & \text{otherwise.} \end{cases}$$

$$q_N^I(k, r) = \begin{cases} \frac{(a - c)(2 - \gamma) + \gamma(-\varepsilon(k + 1) + kr)}{(2 - \gamma)(2 + \gamma(n - 1))} & \text{if } r > \varepsilon + \frac{-2a + 2c + (a - c + \varepsilon)\gamma}{\gamma k} \\ 0 & \text{otherwise.} \end{cases}$$

If the innovation is drastic, non-licensees do not produce ($q_N^I(k, r) = 0$). In this case, outputs in equilibrium for the patentee and licensees are given respectively by:

$$q_P^I(k, r) = \begin{cases} \frac{(a - c + \varepsilon)}{2} & \text{if } r > \frac{(a - c + \varepsilon)(2 - \gamma)}{2} \\ \frac{(a - c + \varepsilon)(2 - \gamma) + \gamma kr}{(2 - \gamma)(2 + \gamma k)} & \text{otherwise.} \end{cases}$$

$$q^I(k, r) = \begin{cases} 0 & \text{if } r > \frac{(a - c + \varepsilon)(2 - \gamma)}{2} \\ \frac{(a - c - \varepsilon)(2 - \gamma) - 2r}{(2 - \gamma)(2 + \gamma k)} & \text{otherwise.} \end{cases}$$

Observe that subindex P stands for patentee, subindex N for non-licensees and superindex I for the internal case. Market profits of the patentee, the accepting firms and the non-accepting firms are given, respectively, by: $\Pi_P^I(k, r) = (q_P^I(k, r))^2$, $\Pi^I(k, r) = (q^I(k, r))^2$ and $\Pi_N^I(k, r) = (q_N^I(k, r))^2$.

In the second stage, given that $k - 1$ firms accept the contract, the k^{th} firm accepts the contract whenever $F \leq \Pi^I(k, r) - \Pi_N^I(k - 1, r)$. Obviously, as the patentee maximizes licensing

revenues⁵ it will choose F such that $F = \Pi^I(k, r) - \Pi_N^I(k - 1, r)$. This implies that the problem of choosing the optimal contract (F, r) is equivalent to that of choosing (k, r) . Then, in the first stage, the patentee solves the following problem:

$$\begin{aligned} \underset{k, r}{Max} \quad & \Pi_P^I(k, r) + k (\Pi^I(k, r) - \Pi_N^I(k - 1, r) + r q^I(k, r)) \\ \text{s.t.} \quad & 0 \leq k \leq n - 1 \text{ and } r \leq \varepsilon.. \end{aligned} \quad (2)$$

Before solving that program, we are going to show that it is always profitable for the patentee to license the innovation to all firms regardless of the total number of firms in the industry. The strategy of the proof is to show that the patentee can always increase profits by licensing the innovation to one more firm through a higher royalty such that *total industry output* remains constant. In the particular case of homogeneous goods this is very intuitive because it would imply keeping price constant. With differentiated goods, however, it is just a technical condition that helps to get the result.

Lemma 2 *Assume that the patentee licenses to k firms with a royalty r . The patentee can always increase its profits by licensing to all firms with a royalty $r < r_I \leq \varepsilon$ such that*

$$(n - 1)q^I(n - 1, r_I) + q_P^I(n - 1, r_I) = (n - k - 1)q_N^I(k, r) + kq^I(k, r) + q_P^I(k, r)$$

Proof. See Appendix ■

In the Appendix we show that, for the case of homogenous goods, the previous result holds for concave demands satisfying the same technical restrictions as in the case of an external patentee.

Next, we derive the optimal royalty to license the innovation to $n - 1$ firms.

Proposition 2 *The laboratory optimally licenses the innovation to all firms.*

⁵ As $\frac{\partial(\Pi^I(k, r) - \Pi_N^I(k - 1, r))}{\partial k} < 0$, this is the only equilibrium in the acceptance stage.

If $\gamma < \frac{2}{n-1}$, the optimal royalty is: $r^{I*}(n) = \varepsilon$ if $\varepsilon \leq \varepsilon_1^I$, $r^{I*}(n) = r_1^I$ if $\varepsilon_1^I < \varepsilon < \varepsilon_2^I$ and $r^{I*}(n) = r_2^I$ otherwise.

If $\gamma \geq \frac{2}{n-1}$, the optimal royalty is: $r^{I*}(n) = r_I$ if $\varepsilon < \varepsilon_2^I$ and $r^{I*}(n) = r_2^I$ otherwise.

where

$$r_I = \frac{\gamma(a(-2+\gamma)(-2+\gamma(n-1)) + c(-4+\gamma^2 - (-2+\gamma)\gamma n) + \varepsilon(n-1)(4+\gamma(-8+\gamma+2n)))}{2(4+4\gamma(-2+n) + \gamma^2(7+(-7+n)n))}$$

$$r_2^I = \frac{(a-c+\varepsilon)(-2+\gamma)^2\gamma(n-1)}{8-2\gamma(8+3\gamma(n-1)-4n)}$$

$$\varepsilon_1^I = \frac{(a-c)(-2+\gamma)\gamma(-2+\gamma(n-1))}{8-\gamma(-\gamma(6+\gamma)-4(n-3)+\gamma(4+\gamma)n)}$$

$$\varepsilon_2^I = \frac{(a-c)(2-\gamma)(8+8\gamma(n-2)+2\gamma^2(5-6n+n^2)-\gamma^3(2-3n+n^2))}{\gamma(-8+8n-2\gamma^2(n^2-1)+4\gamma(2-3n+n^2)-\gamma^3(2-3n+n^2))}$$

Proof. See Appendix ■

It is interesting to note that $r^{I*}(n) > r^*(n)$. The result is very intuitive because the internal patentee obtains revenues not only from licensing but also by selling the good in the market. Therefore, it is more interested in controlling competition by charging a higher royalty. Observe that for the particular case $n = 2$, the outside option does not depend on the royalty and, therefore, the patentee maximizes industry profits. This implies that $\varepsilon_1^I = \varepsilon_2^I$.

3 Incentives to innovate

In the previous section, we have analyzed licensing contracts assuming that the innovation already existed. The next step in the analysis is to endogenize the level of the innovation through modelling the choice of R&D. Our aim is to compare the R&D investment of both the internal and the external patentee.

We consider that, previous to the licensing stage, the patentee chooses the level of R&D investment ε at the cost $C(\varepsilon) = d\varepsilon^2$, where $d > \frac{n}{4}$. In the licensing stage, the profits of the external patentee ($B(\varepsilon)$), net of the R&D costs, are given by:

$$B(\varepsilon) = \begin{cases} \begin{aligned} & n(\varepsilon^2(-2 + \gamma)^2(2 + \gamma(n-1))^2 + a^2\gamma^4(n-1)^2 + c^2\gamma^4(n-1)^2 - \\ & - 2c\varepsilon(2 + \gamma(n-1))(8 + 8\gamma(n-2) + \gamma^3(n-1) + 2\gamma^2(5 + (n-5)n)) \\ & + \varepsilon(2 + \gamma(n-1))(8 + 8\gamma(n-2) \\ & + 2a(-c\gamma^4(n-1)^2 + \gamma^3(n-1) + 2\gamma^2(5 + (n-5)n))) \end{aligned} & \text{if } \varepsilon < \varepsilon_1 \\ \frac{(a - c + \varepsilon)^2 n}{4 + 4\gamma(n-1)} & \text{otherwise.} \end{cases}$$

For the case of an internal patentee, for simplicity, we focus on the case $\gamma \geq \frac{2}{n-1}$, which avoids corner solutions⁶. In this case, the profits of the internal patentee ($B^I(\varepsilon)$), net of the R&D costs, are given by:

$$\text{If } B^I(\varepsilon) = \begin{cases} \Pi_P^I(k, r_I) + k(\Pi^I(k, r_I) - \Pi_N^I(k-1, r_I) + rq^I(k, r_I)) & \text{if } \varepsilon \leq \varepsilon_2^I \\ \frac{(a - c + \varepsilon)^2 (-\gamma^2 + (2 - \gamma)^2 n)}{4(4 - \gamma(8 + 3\gamma(n-1) - 4n))} & \text{otherwise.} \end{cases}$$

The actual value of $B^I(\varepsilon)$ when $\varepsilon \leq \varepsilon_2^I$ is relegated to the Appendix.

Then, the external patentee will choose:

$$\widehat{\varepsilon} = \arg \max_{\varepsilon} \{B(\varepsilon) - C(\varepsilon)\},$$

and the internal patentee will choose:

$$\widehat{\varepsilon}_I = \arg \max_{\varepsilon} \{B^I(\varepsilon) - C(\varepsilon)\}$$

The specific values of $\widehat{\varepsilon}$ and $\widehat{\varepsilon}_I$ are relegated to the appendix. Condition $d > \frac{n}{4}$ guarantees concavity and interior solutions.

⁶Observe that this implies that $n \geq 3$.

Next we compare both investments. Although interesting, this comparison has not received much attention in the literature. To the best of our knowledge the only exception is Sen and Tauman (2006). They also compare the incentives to innovate of both an external and an internal patentee. They show that the difference between the post-innovation and pre-innovation profits is always higher for an external patentee. The intuition is that whereas the external patentee earns no profit in the absence of the innovation, the internal patentee earns the market profits. We adopt a different modelling strategy because R&D, in our model, is a continuous variable that determines the cost reduction and the post-innovation profits. Therefore, in our setting, what matters is not the incremental profits of the investment but its marginal profitability. In this setting, we are going to show that an internal patentee may have more incentives to innovate. Although we can not explicitly compare the equilibrium investments given their complexity, we next plot in a three dimensional space $\widehat{\varepsilon}_I - \widehat{\varepsilon}$ for different values of γ , n and d . Figure 1 plots the difference for $d = 10$, $\gamma \in [\frac{2}{9}, 1]$ and $n \in [10, 30]$. Ranges for γ and n are chosen such that $\gamma \geq \frac{2}{n-1}$ and $d > \frac{n}{4}$ are satisfied. Figures 2 and 3 plot the difference for values $d = 30$ and $d = 50$ respectively.

Looking at the three figures, it is easy to see that the region where an internal patentee invests more in R&D gets larger as d increases. This has a nice empirical implication regarding the internal organization of leading innovative firms. When the technological opportunity of the industry is high (d is low), we can expect that innovation is dominated by independent research laboratories. In contrast, when it is low, vertically integrated laboratories are the ones expected to be the leaders in innovation activities.

A second finding is that an internal patentee invests more in R&D only when the goods are close enough substitutes. The intuition is that an internal patentee is in a better position to control for the level of competition given that it is an active firm in final good industry.

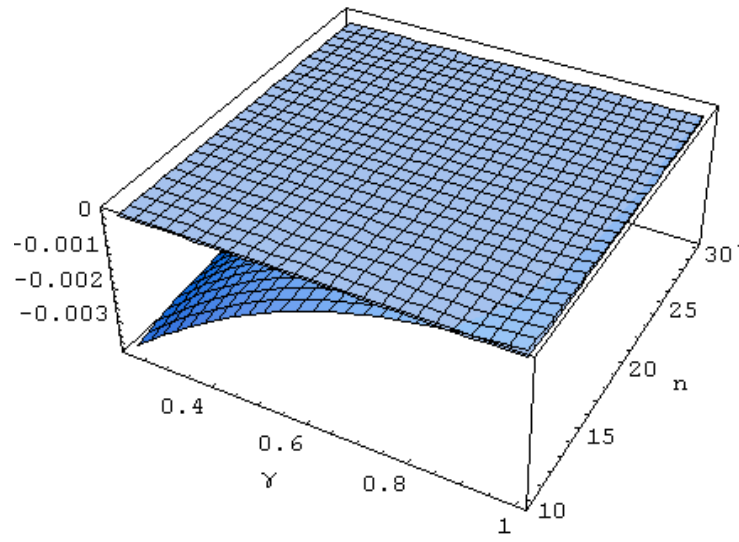


Figure 1: $d = 10$

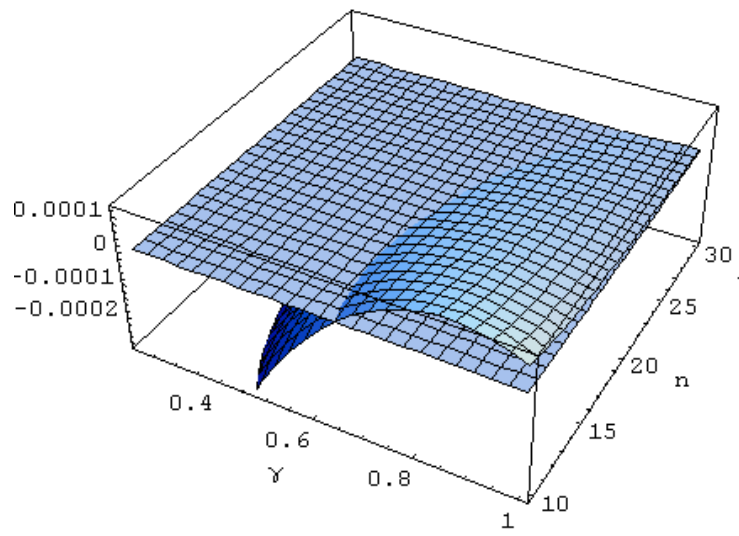


Figure 2: $d = 30$

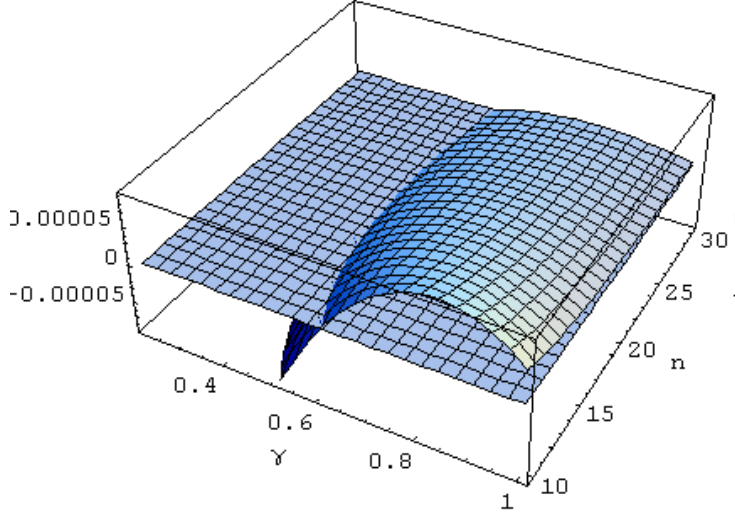


Figure 3: $d = 50$

In order to be able to get explicit results on the R&D comparison, it seems interesting to analyze the particular case where the good is homogenous ($\gamma = 1$). This also facilitates the comparison of our result with the one in Sen and Tauman (2007) that also considers homogenous goods.

3.1 The case of homogeneous goods

In the case $\gamma = 1$, the constraint $d > \frac{1}{2}$ guarantees that

$$\hat{\varepsilon} = \frac{(a - c)n(1 + n(2n - 1))}{-n(1 + n) + 4d(1 + n^3)} \text{ and}$$

$$\hat{\varepsilon}_I = \frac{(a - c)(3 + n(3 + n(-1 + n(-3 + 2n))))}{-3 - n(3 + (-3 + n)n) + 4d(1 + n)^2(3 + (-3 + n)n)}$$

are global maxima. As expected, the higher d , the lower $\hat{\varepsilon}$ and $\hat{\varepsilon}_I$, but we have also that total expenditure in R&D ($d\hat{\varepsilon}^2$ and $d\hat{\varepsilon}_I^2$) is decreasing in d . Therefore $\frac{1}{d}$ measures the degree of technological opportunity of the industry.

The literature has extensively studied the relationship between R&D investment and competition. It is possible to check that $\widehat{e}_I$ is always increasing in n and $\widehat{e}$ increases with n whenever R&D investment is expensive enough (d high enough). When d is low $\widehat{e}$ follows a U-shape with respect to the number of firms.

Next proposition compares both investments.

Proposition 3 *The internal patentee invests more in R&D than the external patentee when $n \geq 3$ and $d > \widehat{d}(n)$, where $\widehat{d}(n) = \frac{n^2(-3 + n(3n - 4))}{2(1 + n)(-3 + (n - 2)n^2)}$.*

This result confirms the intuitions obtained from the figures.

4 Conclusions

In this paper, we obtain the optimal two-part tariff licensing contract for the general case of a n -firms oligopolistic industry producing differentiated goods. We analyze the case where the patentee is an independent laboratory as well as the case where it is an incumbent patentee. We show that, in both cases, and regardless of the size of the innovation, the number of firms in the industry and the degree of product differentiation the innovation is licensed to all firms in the industry.

This result has been previously obtained for the particular case of homogenous goods and an external patentee by Erutku and Richelle (2007). We show that it extends to the case of differentiated goods, even when the patentee is an incumbent firm in the industry. This may seem counterintuitive, because an internal patentee has market profits to protect. However, we have shown that she can always do it not by restricting the number of licensees but by increasing the royalty.

In the second part of the paper, we compare the incentives to innovate of an incumbent patentee with those of an independent laboratory. We obtain that this comparison is ambiguous and depends on the level of technological opportunity of the industry. In particular, we get that when it is low (high), an internal (external) patentee optimally invests more in R&D. Our result is interesting if we compare it with the one in Sen and Tauman (2007). They obtain that an external patentee has always more incentives to innovate. This difference arises because R&D, in our model, determines the cost reduction whereas in Sen and Tauman increases the probability of getting a given innovation. Therefore, in our setting, what matters is not the difference between post-innovation and pre-innovation profits but the marginal profitability of R&D investment.

5 Appendix

Proof of Lemma 1

Let $\pi(k, r)$ represent the laboratory's profit if it licenses to k firms and sets a royalty $r \leq \varepsilon$.

We have that

$$\pi(k, r) = B(k, r) - k(q_N(k-1, r))^2 - (n-k)(q_N(k, r))^2 - \varepsilon(n-k)q_N(k, r).$$

Proof. where

$$B(k, r) = k(q(k, r))^2 + (n-k)(q_N(k, r))^2 + krq(k, r) + \varepsilon(n-k)q_N(k, r)$$

Observe that we have expressed the profits of the incumbent patentee as the difference between total industry profits ($B(k, r)$) and the profits of the remaining firms. The efficiency term appears, because we are computing industry profits as if total output was produced using the new technology.

Let r_E solve $nq(n, r_E) = (n-k)q_N(k, r) + kq(k, r)$.

We next prove that licensing to all firm with royalty $r < r_E \leq \varepsilon$ is more profitable than licensing to k firms with a royalty r . This is equivalent to show that the following expression is positive:

$$\begin{aligned} \pi(n, r_E) - \pi(k, r) &= B(n, r_E) - B(k, r) + k(q_N(k-1, r))^2 + (n-k)(q_N(k, r))^2 + \\ &\quad + \varepsilon(n-k)q_N(k, r) - n(q_N(n-1, r_E))^2. \end{aligned}$$

It is convenient to proceed in two steps. We first prove that $B^I(n-1, r_I) - B^I(n-1, r_I)$ is positive and then prove that the remaining terms are also positive.

If $\varepsilon - \frac{(a-c)(2-\gamma)}{\gamma k} \leq r \leq \varepsilon$, we have that:

$$B(n, r_E) - B(k, r) = \frac{k(1-\gamma)(n-k)(\varepsilon-r)^2}{n(2-\gamma)^2} \geq 0$$

$$k(q_N(k-1, r))^2 + (n-k)(q_N(k, r))^2 - n(q_N(n-1, r_E))^2 = \frac{k\gamma^2(n-k)(\varepsilon-r)^2}{n(2-\gamma)^2(2+\gamma(n-1))^2} \geq 0$$

If $\varepsilon - \frac{(a-c)(2-\gamma)}{\gamma(k-1)} < r < \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma k}$, we have that

$$r_E = \frac{(a-c+\varepsilon)(2-\gamma)(n-k) + k(2+\gamma(n-1))r}{(2+\gamma(k-1))n}$$

where $\varepsilon > r_E > r$ and $q_N(k, r) = 0$.

We have to distinguish two cases:

$$\text{If } \frac{(\varepsilon\gamma k(2+\gamma(n-1))(n-1) + a(-2+\gamma)(2n+\gamma(k+(-2+n)n)) - c(-2+\gamma)(2n+\gamma(k+(-2+n)n)))}{(\gamma k(2+\gamma(n-1))(n-1))} < r \leq \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma k},$$

we have that $q_N(n-1, r_E) > 0$. It is direct to see that:

$$B(n, r_E) - B(k, r) = \frac{k(1-\gamma)(n-k)(a-c+\varepsilon-r)^2}{n(2+\gamma(k-1))^2} \geq 0 \quad (3)$$

Moreover, we have that

$$kq_N(k-1, r) - nq_N(n-1, r_E) = \frac{(n-k)(2+\gamma(n+k-2))((a-c)(-2+\gamma) + \gamma k(\varepsilon-r))}{(2-\gamma)(2+\gamma(k-1))(2+\gamma(n-1))}. \quad (4)$$

It is direct to see that (4) is decreasing in r and it amounts to zero in the upper bound of the region. This implies that

$$k(q_N(k-1, r))^2 - n(q_N(n-1, r_E))^2 > 0$$

$$\text{If } \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma(k-1)} < r \leq \frac{(\varepsilon\gamma k(2+\gamma(n-1))(n-1) + a(-2+\gamma)(2n+\gamma(k+(-2+n)n)) - c(-2+\gamma)(2n+\gamma(k+(-2+n)n)))}{(\gamma k(2+\gamma(n-1))(n-1))},$$

then $q_N(n-1, r_E) = 0$. The previous calculations applied to this case prove the result.

$$\text{If } r \leq \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma(k-1)}, \text{ we have that } q_N(n-1, r_E) = 0 \text{ and } q_N(k-1, r) = 0. \text{ Then}$$

$\pi(n, r_E) - \pi(k, r) = B(n, r_E) - B^I(k, r)$, which is positive by (3). ■

Proof of Proposition 1

$$r_1 = \arg \max_r n(\Pi(n, r) - \Pi_N(n-1, r)) + nrq(n, r)$$

$$r_2 = \arg \max_r n\Pi(n, r) + nrq(n, r)$$

If $\varepsilon = \varepsilon_1$, then $r_1 = r_2$. This implies that the optimal royalty is r_1 when $\varepsilon \leq \varepsilon_1$ and r_2 otherwise.

Proof of Lemma 2

Let $\pi^I(k, r)$ represent the profit of the incumbent patentee if it licenses k firms and sets a royalty r . It is very useful to express this profit as:

$$\pi^I(k, r) = B^I(k, r) - k(q_N^I(k-1, r))^2 - (n-k-1)(q_N^I(k, r))^2 - \varepsilon(n-k-1)q_N^I(k, r),$$

where

$$B^I(k, r) = k(q^I(k, r))^2 + (n-k-1)(q_N^I(k, r))^2 + (q_P^I(k, r))^2 + krq^I(k, r) + \varepsilon(n-k-1)q_N^I(k, r)$$

Proof. Observe that we have expressed the profits of the incumbent patentee as the difference between total industry profits ($B^I(k, r)$) and the profits of the remaining firms. The efficiency term appears, because we are computing industry profits as if total output was produced using the new technology.

$$\text{Let } r_I \text{ solve } (n-1)q^I(n-1, r_I) + q_P^I(n-1, r_I) = (n-k-1)q_N^I(k, r) + kq^I(k, r) + q_P^I(k, r).$$

We next prove that licensing to all firms with royalty $r < r_I \leq \varepsilon$ is more profitable than licensing to k firms with royalty r . This is equivalent to show that next expression is positive:

$$\begin{aligned} \pi^I(n, r_I) - \pi^I(k, r) &= B^I(n-1, r_I) - B^I(k, r) + k (q_N^I(k-1, r))^2 + (n-k-1) (q_N^I(k, r))^2 + \\ &\quad + \varepsilon(n-k-1) q_N^I(k, r) - (n-1) (q_N^I(n-2, r_I))^2. \end{aligned}$$

It is convenient to proceed in two steps. We first prove that $B^I(n-1, r_I) - B^I(k, r)$ is positive and then prove that the remaining terms are also positive. We first analyze the non-drastic case ($-2a + 2c + (a - c + \varepsilon)\gamma < 0$):

If $\varepsilon \geq r > \varepsilon + \frac{-2a + 2c + (a - c + \varepsilon)\gamma}{\gamma k}$, we have that $r \leq r_I = \frac{\varepsilon(n-k-1) + kr}{n-1} \leq \varepsilon$. It is easy to check that:

$$B^I(n-1, r_I) - B^I(k, r) = \frac{k(1-\gamma)(n-k-1)(\varepsilon-r)^2}{(2-\gamma)^2(n-1)} \geq 0.$$

$$\begin{aligned} k (q_N^I(k-1, r))^2 + (n-k-1) (q_N^I(k, r))^2 - (n-1) (q_N^I(n-2, r_I))^2 &= \\ \frac{\gamma^2 k(n-k-1)(\varepsilon-r)^2}{(2-\gamma)^2(n-1)(2+\gamma(n-1))^2} &\geq 0. \end{aligned}$$

If $\frac{(a-c)(-2+\gamma) + \varepsilon\gamma k}{\gamma(k-1)} < r \leq \varepsilon + \frac{-2a + 2c + (a - c + \varepsilon)\gamma}{\gamma k}$, we have that

$$r_I = \frac{(a-c+\varepsilon)(1+k-n)(-2+\gamma) + k(2+\gamma(n-1))r}{(2+\gamma k)(n-1)}$$

, $\varepsilon > r_I > r$ and $q_N^I(k, r) = 0$.

We have to distinguish two cases:

$$\text{If } \frac{(2+\gamma k)(n-1)(\varepsilon + \frac{-2a+2c+(a-c+\varepsilon)\gamma}{\gamma(n-2)} - \frac{(a-c+\varepsilon)(-2+\gamma)(1+k-n)}{(2+\gamma k)(n-1)})}{k(2+\gamma(n-1))} < r \leq \varepsilon + \frac{-2a + 2c + (a - c + \varepsilon)\gamma}{\gamma k},$$

we have that $q_N^I(n-2, r_I) > 0$. It is direct to see that:

$$B^I(n-1, r_I) - B^I(k, r) = \frac{(1-\gamma)k(n-k-1)((a-c+\varepsilon)(-2+\gamma) + 2r)^2}{(2-\gamma)^2(2+\gamma k)^2(n-1)} \geq 0 \quad (5)$$

Moreover, we have that

$$k q_N^I(k-1, r) - (n-1) q_N^I(n-2, r_I) \geq 0,$$

which implies that

$$k \left(q_N^I(k-1, r) \right)^2 - (n-1) \left(q_N^I(n-2, r_I) \right)^2 > 0$$

$$\text{If } \frac{(a-c)(-2+\gamma) + \varepsilon\gamma k}{\gamma(k-1)} < r \leq \frac{(2+\gamma k)(n-1)(\varepsilon + \frac{-2a+2c+(a-c+\varepsilon)\gamma}{\gamma(n-2)} - \frac{(a-c+\varepsilon)(-2+\gamma)(1+k-n)}{(2+\gamma k)(n-1)})}{k(2+\gamma(n-1))},$$

then $q_N^I(n-2, r_I) = 0$. The previous calculations applied to this case prove the result.

$$\text{If } r \leq \frac{(a-c)(-2+\gamma) + \varepsilon\gamma k}{\gamma(k-1)}, \text{ we have that } q_N^I(n-2, r_I) = 0 \text{ and } q_N^I(k-1, r) = 0. \text{ Then}$$

$$\pi^I(n, r_I) - \pi^I(k, r) = B^I(n-1, r_I) - B^I(k, r), \text{ which is positive by (5).}$$

Next we analyze the case of a drastic innovation $(-2a + 2c + (a - c + \varepsilon)\gamma > 0)$:

In this case, non-licensees do not produce and therefore we have that $\pi^I(n, r_I) - \pi^I(k, r) = B^I(n-1, r_I) - B^I(k, r)$.

$$\text{If } \frac{(a-c+\varepsilon)(2-\gamma)}{2} \leq r \leq \varepsilon, \text{ only the incumbent patentee produces and then } r_I = r \text{ and}$$

$$\pi^I(n, r_I) - \pi^I(k, r) = B^I(n-1, r_I) - B^I(k, r) = 0.$$

$$\text{If } r < \frac{(a-c+\varepsilon)(2-\gamma)}{2}, \text{ the licensees and the patentee are active. Then we have that,}$$

$$\pi^I(n-1, r_I) - \pi^I(k, r) = B^I(n-1, r_I) - B^I(k, r) = \frac{(1-\gamma)k(n-k-1)((a-c+\varepsilon)(-2+\gamma) + 2r)^2}{(2-\gamma)^2(2+\gamma k)^2(n-1)} \geq 0.$$

■

Proof of Proposition 2

$$r_1^I = \arg \max_r \Pi_P^I(n-1, r) + (n-1) (\Pi^I(n-1, r) - \Pi_N^I(n-2, r)) + (n-1)r q^I(n-1, r)$$

$$r_2^I = \arg \max_r \Pi_P^I(n-1, r) + (n-1)\Pi^I(n-1, r) + (n-1)r q^I(n-1, r)$$

If $\varepsilon = \varepsilon_2^I$, then $r_1^I = r_2^I$. This implies that the optimal royalty is r_1^I when $\varepsilon \leq \varepsilon_2^I$ and r_2^I otherwise. On the other hand, when $\varepsilon < \varepsilon_1^I$ and $\gamma < \frac{2}{n-1}$, $r_1^I > \varepsilon$ and therefore the participation constraint is binding and the optimal royalty is ε .

The value of $B^I(\varepsilon)$ when $\varepsilon \leq \varepsilon_2^I$

$$\begin{aligned}
& \left(\frac{1}{(-2+\gamma)^2(2+\gamma(n-1))^2} \right) \\
& ((-2+\gamma)\gamma(2+\gamma(n-1))(n-1)(a(-2+\gamma)(-2+\gamma(n-1)) \\
& +c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1)(4+\gamma(-8+\gamma+2n)))((a-c+\varepsilon) \\
& \gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+ \\
& +\varepsilon(n-1)(4+\gamma(-8+\gamma+2n)))) \\
& (-2+\gamma)+\frac{4+\gamma(-8+\gamma+2n))}{4+4\gamma(n-2)+\gamma^2(7+(n-7)n)})) \\
& (2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n)))-(2+\gamma(n-2))(n-1) \\
& \gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1) \\
& (4+\gamma(-8+\gamma+2n))) \\
& (\varepsilon-\frac{2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n))}{(2a(-2+\gamma)-2c(-2+\gamma)+\varepsilon(-2+\gamma n+ \\
& \gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1) \\
& (4+\gamma(-8+\gamma+2n)))} \\
& \frac{4+4\gamma(n-2)+\gamma^2(7+(n-7)n)}{4+4\gamma(n-2)+\gamma^2(7+(n-7)n)}- \\
& \gamma^2(-2+n)\left(\frac{\gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1) \\
& (4+\gamma(-8+\gamma+2n)))}{2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n))}\right)+ \\
& (((a-c+\varepsilon)(-2+\gamma)\left(\frac{\gamma^2(-2+n)\left(\frac{\gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1) \\
& (4+\gamma(-8+\gamma+2n)))}{2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n))}\right)}{\frac{\varepsilon(n-1)(4+\gamma(-8+\gamma+2n))}{2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n))}}\right)\right)^2)
\end{aligned}$$

The values of the optimal R&D investment.

$$\widehat{\varepsilon} = \frac{(a-c)(n(8+8\gamma(n-2)+\gamma^3(n-1)+2\gamma^2(5+(n-5)n)))}{((2+\gamma(n-1))(-(-2+\gamma)^2n+4d(4+\gamma(4(n-2)+\gamma(6+\gamma(n-1)+(n-6)n))))).$$

$$\begin{aligned}
& -(a-c)((-2+\gamma)(\gamma^5(n-1)^3-32n+ \\
& +2\gamma^4(n-1)^2(3+(n-5)n)-16\gamma(1+n(3n-7))- \\
& -4\gamma^3(n-2)(-2+n(10+(n-8)n))-8\gamma^2(-3+n(19+n(3n-16)))))) \\
\widehat{\varepsilon}_I = & \frac{(\gamma^6(n-1)^3+4\gamma^5(n-4)(n-1)^3+64n+64\gamma(n-3)n+16\gamma^2n(16+(n-11)n)- \\
& -16\gamma^3(-2+(n-3)n(3n-5))-4d(-2+\gamma)^2(2+\gamma(n-1))^2 \\
& (4+4\gamma(n-2)+\gamma^2(7+(n-7)n))-4\gamma^4(11+n(-41+n(43+(n-15)n))))}{.}
\end{aligned}$$

Proof of Lemma 1 and 2 for a general demand and homogenous goods

Assume we have n firms and market demand is given by $P(X)$, where $P'(X) < 0$ and $P''(X) \leq 0$. Firms have constant marginal costs. Denote by C the sum of marginal costs. Then in an interior equilibrium we have that:

$$nP(X) - C + P'(X)X = 0 \quad (6)$$

The profits of a firm with cost c is given by:

$$\pi(C) = \frac{(P(X(C)) - c)^2}{-P'(X(c))}$$

where $X(C)$ is implicitly defined in (6). We have that $\pi'(C) > 0$.

We start by Lemma 1. In the case of homogenous goods $B(n, r_E) - B(k, r) = 0$. Then, a sufficient condition for the laboratory to sell to all firms is:

$$k(\pi(C^*) - \pi(\bar{C})) - (n - k)(\pi(\bar{C}) - \pi(\underline{C})) \geq 0 \quad (7)$$

where $C^* = (n - k + 1)c + (k - 1)(c - \varepsilon + r)$, $\bar{C} = (n - 1)(c - \varepsilon + r_E) + c$ and $\underline{C} = (n - k)c + k(c - \varepsilon + r)$.

We have also that $C^* - \bar{C} = \frac{(n - k)(\varepsilon - r)}{n}$ and $\bar{C} - \underline{C} = \frac{k(\varepsilon - r)}{n}$. (7) can be rewritten as:

$$\frac{\pi(C^*) - \pi(\bar{C})}{\pi(\bar{C}) - \pi(\underline{C})} = \frac{\int_{\bar{C}}^{C^*} \pi'(C) dC}{\int_{\underline{C}}^{\bar{C}} \pi'(C) dC} \geq \frac{n - k}{k}$$

A sufficient condition for this to hold is that $\pi''(C) > 0$. Then

$$\frac{\int_{\bar{C}}^{C^*} \pi'(C) dC}{\int_{\underline{C}}^{\bar{C}} \pi'(C) dC} > \frac{(C^* - \bar{C})\pi'(\bar{C})}{(\bar{C} - \underline{C})\pi'(\bar{C})} = \frac{n - k}{k}$$

For the case of an internal patentee (Lemma 2) a similar line of argument applies. For this case we have that $B^I(n - 1, r_I) - B^I(k, r) = 0$. Then, a sufficient condition for the laboratory to sell to all firms is:

$$k (\pi(C^*) - \pi(\overline{C})) - (n - k - 1) (\pi(\overline{C}) - \pi(\underline{C})) \geq 0 \quad (8)$$

where $C^* = c - \varepsilon + (k - 1)(c - \varepsilon + r) + (n - k)c$, $\overline{C} = 2c - \varepsilon + (n - 2)(c - \varepsilon + r_I)$ and $\underline{C} = c - \varepsilon + k(c - \varepsilon + r) + (n - k - 1)c$. We have also that $C^* - \overline{C} = \frac{(n-k-1)(\varepsilon-r)}{n-1}$ and $\overline{C} - \underline{C} = \frac{k(\varepsilon-r)}{n-1}$. (7) can be rewritten as:

$$\frac{\pi(C^*) - \pi(\overline{C})}{\pi(\overline{C}) - \pi(\underline{C})} = \frac{\int_{\overline{C}}^{C^*} \pi'(C) dC}{\int_{\underline{C}}^{\overline{C}} \pi'(C) dC} \geq \frac{n - k - 1}{k}$$

A sufficient condition for this to hold is that $\pi''(C) > 0$. Then

$$\frac{\int_{\overline{C}}^{C^*} \pi'(C) dC}{\int_{\underline{C}}^{\overline{C}} \pi'(C) dC} > \frac{(C^* - \overline{C})\pi'(\overline{C})}{(\overline{C} - \underline{C})\pi'(\overline{C})} = \frac{n - k - 1}{k}$$

It is tedious but direct to show that $\pi''(C) > 0$ if $-P'(X)$ is log-concave and $P'''(X) \leq 0$.

We show that it also holds for the class of demands $P = A - X^b$, where $b \geq 1$.

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