

# ***A discusión***

## **STRATEGIC EFFECTS OF AIRLINE ALLIANCES\***

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# STRATEGIC EFFECTS OF AIRLINE ALLIANCES

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## ABSTRACT

This paper looks at the endogenous formation of airline alliances by means of a two-stage game where first airlines decide whether to form an alliance and then fares are determined. We analyze the profitability and the strategic effects of airline alliances when two complementary alliances, following different paths, may be formed to serve a certain city-pair market. The formation of a complementary alliance is shown to hurt outsiders and that fares decrease in the interline market. Contrary to what might be expected, we find that complementary alliances are not always profitable, even in the presence of economies of traffic density. The interplay between market size, the degree of product differentiation and the intensity of economies of traffic density determines whether the market equilibrium entails no alliances, a single alliance or a double alliance.

JEL classification: L13, L2, L93.

Keywords: complementary airline alliances, economies of traffic density, product differentiation.

# 1 Introduction

The air transportation sector has witnessed a number of changes since the deregulation processes of the US industry (in the 1980s) and of the European industry (in the 1990s). These changes include the substantial decline in the number of major carriers, the intensified reorganization of routes into hub-and-spoke networks and, still taking place, the formation of strategic alliances among international carriers.<sup>1</sup> In particular, a formal explanation to the rise of international airline alliances seems to be lacking. The present paper identifies conditions under which airline alliances are strategically advantageous, examines the effects on carriers outside the alliance and studies how travel volumes and fares are affected.

Airline alliances are designed to offer passengers a seamless service in order to minimize some of the inconveniences of interline multi-carrier trips. They allow the carriers to rely on a partner to provide flight to destinations where they lack route authority. Cooperation adopts several forms - which in many instances come close to effective merger - and includes codesharing agreements, the coordination of flight schedules and the joint use of frequent flyer programs. Collaboration between airlines can be traced as far back as the 1940s when, for instance, Air France was involved in setting up the operations of many African airlines, such as Air Afrique, Royal Air Maroc and Tunisair. The first major multi-partner alliance was that between Delta Air Lines, Singapore Airlines and Swissair in the early 1990s. At present, almost every major airline belongs to a big international alliance: *Oneworld*, *Star Alliance*, and *The Sky Team*.<sup>2</sup> The structure of the industry is constantly changing as with the recent merger (September 2004) between Air France and KLM, and the announced agreement to merge between America West Airlines and US Airways (May 2005).

Since the major alliances enjoy antitrust immunity, another advantage of an alliance is related with cooperative pricing in interline trips. It seems that IATA's

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<sup>1</sup>See Morrison and Winston (1995) for an overview of developments in the industry.

<sup>2</sup>The *Oneworld* Alliance includes British Airways, American Airlines, Iberia, Lan Chile, Aer Lingus, Qantas, Cathay Pacific and Finnair among others. The most important airlines in the *Star Alliance* are Lufthansa, United Airlines, US Airways, Air Canada, All Nippon Airways, Thai Airways, Singapore Airlines, Air New Zealand, Varig Brasil, SAS, the Austrian Group and British Midland. Finally, *The Sky Team* is mainly composed of Air France, Delta Air Lines, KLM, Northwest, Continental Airlines, Alitalia, Korean Air, CSA and Malaysian Air System.

(International Air Transport Association) influence over negotiations on interline fares has declined and there is evidence that, after the deregulation processes both in the US and in Europe, airline cooperation has pushed fares down whereas travel volumes have increased.<sup>3</sup> In a sense, this is what theory would predict since joint pricing of complementary flights through a hub airport internalizes the negative effect of separate pricing. An airline alliance is an agreement within a network which involves multiproduct competition and, since the airline industry shows evidence of increasing returns to traffic density, this means that competition transmits across routes through costs, a feature that becomes particularly relevant in an environment with strategic interaction. It therefore suggests that antitrust treatment on this issue should be carefully looked at. Although alliances can be beneficial to firms and/or consumers (for instance by enhancing efficiency and service quality), they can as well significantly reduce or eliminate competition on routes where the allied companies were former competitors. In Europe, competition rules (i.e. Articles 81 and 82 of the EU Treaty) are fully applicable to air transport. Indeed the parties, in their seeking for authorization, try and offer arguments so that the Commission gives an individual exemption based on Article 81(3).<sup>4</sup> The Commission clears or prohibits airline mergers on the basis of Merger Regulation 4064/89, amended by Regulation 1310/97. In the United States, the Department of Transportation (DOT) had the authority over airline mergers until 1989. Since then, the Antitrust Division of the Department of Justice reviews airline mergers and acquisitions, although the DOT retains authority over some matters. Thus, the Antitrust Division has challenged agreements in violation of section 2 of the Sherman Act and/or section 7 of the Clayton Act. Overall, there are no clear guidelines but many airline alliances receive antitrust immunity subject to conditions, which range from the surrender of take-off and landing slots and the guarantee that partners do not increase frequencies to obstruct entry, to the limitation or the extension in the use of frequent

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<sup>3</sup>Brueckner (2003), using data from the U.S. Department of Transportation's Passenger Origin-Destination Survey, concludes that airline alliances lead to lower interline fares. His analysis shows that, when combined codesharing and antitrust immunity, airline cooperation leads to a reduction in interline fares between 17% and 30%.

<sup>4</sup>Article 81(1) prohibits agreements between firms that encompass fixing prices, sharing markets and so on. However, paragraph 3 allows for an exemption on the application of paragraph 1 if the proposed agreements satisfy certain requirements such as the promotion of technical progress to the consumers benefit.

flyer programs. One way or the other, the partners receive a lenient treatment by antitrust authorities since they can determine fares for interline trips in a way that would not otherwise be possible.<sup>5</sup> By now, close cooperation between European and US authorities has been intensified regarding transatlantic alliance cases; with regard to "open skies" agreements, the EU and the US are relaunching negotiations to create a common air space.

To illustrate our analysis let us consider the following simple network structure. Suppose that a passenger wishes to travel from Madrid to Washington. She can fly via Chicago O'Hare International or via Amsterdam Schiphol. In the former case, Madrid - Chicago is provided by Iberia (e.g. IB6275) and Chicago - Washington R. Reagan National is operated by American Airlines (e.g. AA1730). In the latter, the passenger can fly with KLM/AF from Madrid to Amsterdam (e.g. KL1708) and then make the trip between Amsterdam and Washington Dulles International with Northwest (e.g. NW8651). As it turns out, Iberia and American Airlines belong to the *Oneworld* alliance. On the other hand, KLM/AF and Northwest are partners in *The Sky Team*. Alternatively, a passenger travelling from Frankfurt to Minneapolis may fly either with Lufthansa (e.g. LH4670) and Northwest (e.g. NW0041) via Amsterdam Schiphol; or with Delta Airlines (e.g. DL0027) and Airtran Airways (e.g. FL857) via Atlanta Hartsfield-Jackson.<sup>6</sup> As it turns out, these four firms are independent carriers. We will provide a theoretical explanation to this type of observed phenomena as a result of strategic behavior.

Travellers perceive a composite trip using two or more airlines as a differentiated product from other substitute composite trips. That differentiation can be explained by a number of reasons such as brand loyalty, frequent flyer programs, frequency of services, quality considerations and so on. In addition, there is empirical evidence

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<sup>5</sup>The reader might like to look at "Airline Alliances and Mergers - The Emerging Commission Policy" (2001) by J. Stragier; the statement by R. Hewitt Pate from the Antitrust Division concerning "International Aviation Alliances: Market Turmoil and the Future of Airline Competition" (2001); and the report "Entry and Competition in the US Airline Industry: Issues and Opportunities" (1999), by the Transportation Research Board.

<sup>6</sup>The reader can access [www.airwise.com](http://www.airwise.com) and find plenty of examples where a passenger must change planes on their way to final destination where carriers belong to the same alliance or not; trips can be made through different hubs.

that the airline industry, after deregulation, exhibits increasing returns to traffic density (see Brueckner and Spiller, 1994, and Creel and Farrell, 2001). The model that we propose will consider a network structure to capture competition between routes through different hubs. It is our purpose to evaluate the role played by market size, the degree of product differentiation and the intensity of economies of traffic density as factors that cause strategic airline alliances to show up. We will look at the endogenous formation of airline alliances by means of a two-stage game where first airlines simultaneous and independently decide whether to form an alliance and then, given the inherited outcome of the first stage, fares are simultaneous and independently determined. Specifically, the aim of our paper is to analyze the profitability and the strategic effects of two complementary alliances, following different paths, that may be formed to serve a certain city-pair market. We wish to answer the following questions: when are alliances profitable for the potential partners?, what are their effects on outsiders?, what is the outsiders' optimal response? and how are fares and travel volumes affected?

Our main findings can be summarized as follows. Firstly, the formation of a complementary alliance is shown to hurt outsiders and that fares will decrease in the interline market. Secondly, in contrast to what might be expected, the formation of two alliances (*double alliance*) is not always the equilibrium outcome. This is a particularly remarkable result because integration of complementary goods together with the presence of economies of traffic density are elements that favor the profitability of alliance agreements. Thus, a sufficiently high degree of product differentiation is enough to ensure the *double alliance* equilibrium, regardless of market size and the size of the economies of traffic density, since the intensity of competition is weak. At the other extreme, if product differentiation is low enough then *no alliance* will occur in equilibrium. This may also occur for small economies of traffic density when a low enough degree of product differentiation is combined with high enough market size. Interestingly enough, asymmetric equilibrium with a *single alliance* may arise as the degree of product differentiation increases. Broadly speaking, the *single alliance* equilibrium shows up for intermediate values of market size together with economies of traffic density that are not too significant. This result provides an explanation about why some carriers decide to remain non-allied

(e.g. Japan Airlines) and in several routes only one alliance offers interline tickets.<sup>7</sup> Furthermore, the market equilibrium can be a *double alliance* and yet the carriers be better off in a *no alliance* situation for a sufficiently low degree of product differentiation.

The existing literature on airline alliances is sparse. A number of papers have focused on providing reasons and conditions under which hub-and-spoke networks are equilibrium structures.<sup>8</sup> There only exist a few theoretical contributions on airline alliances, first initiated by Brueckner and Spiller (1991) who developed a hub-and-spoke model where an airline is considered as a multiproduct firm with cost complementarities. These theoretical analyses include Park (1997), Brueckner (2001), Brueckner and Whalen (2000), Hassin and Shy (2004) and Bilotkach (2005). The latter three references assume product differentiation although only Brueckner and Whalen (2000) considers economies of traffic density. It is Park (1997) who examined the consequences of parallel and complementary alliances on output levels, profits and welfare. Complementary alliances are analyzed by Brueckner and Whalen (2000) and Brueckner (2001) under different network structures. We will also address this type of alliances where emphasis is put on the strategic rationale for alliance formation. Brueckner (2001) considers two airlines to study the effect of alliances on traffic levels and fares both in the inter-hub and the domestic markets. Hassin and Shy (2004) also examine codesharing agreements among airlines competing on international routes and show that codesharing including all carriers is welfare improving. Brueckner and Whalen (2000) contemplate that an international passenger may normally opt between several carrier pairs when making an interline trip. They develop a particular example where aggregate demand does not vary either with the degree of substitutability or the number of products. This is a satisfactory property when dealing with more than two firms but there may be some routes that are better accounted for when total travel volumes are not fixed. Brueckner and Whalen (2000) consider the *no alliance*, the symmetric and the asymmetric alliance situations where domestic markets are operated by two competing carriers; their focus is on the theoretical and empirical price effects of

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<sup>7</sup>For instance, most of the one-stop routes between Italy and France are only served by *The Sky Team* (e.g. the route Rome Fiumicino - Nantes Atlantique).

<sup>8</sup>See Oum *et al.* (1995), Berechman *et al.* (1998), Hendricks *et al.* (1999), Barla and Constantatos (2000) and Brueckner (2004).

international airline alliances.<sup>9</sup> Our paper is also related with Bilotkach (2005), who develops a model of price competition among four carriers where two alliances are possible to examine their effects on fares and travel volumes with assuming constant returns to scale; unlike Brueckner and Whalen (2000), this author includes routes between hub airports. Our analysis complements these earlier contributions and takes a game theoretical approach to examine strategic alliance formation. A complete characterization of the different equilibria is provided in terms of market size, the degree of product differentiation and the intensity of economies of traffic density.

The paper is structured as follows. Section 2 introduces the model presenting the *pre-alliance* equilibrium and the equilibria arising from the situations with a *single* and a *double alliance*, respectively. The effects on fares and travel volumes are analyzed in Section 3. A simultaneous two-stage game of airline alliances is then presented in Section 4. A brief concluding section closes the paper.

## 2 The model

### *Basic Assumptions*

The model's network structure is shown in Figure 1.

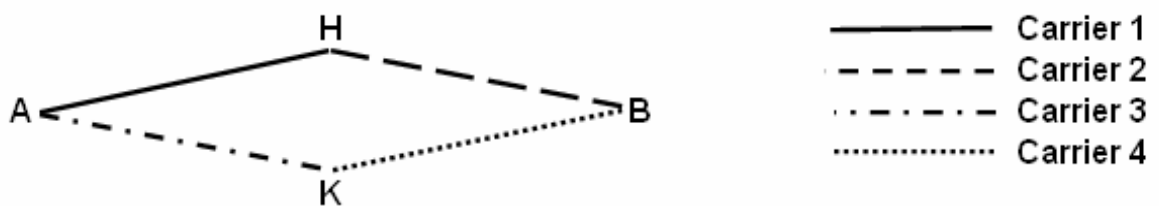


Figure 1: The Network

Airline 1 operates route  $AH$ , airline 2 serves route  $HB$ , airline 3 provides the flight  $AK$  and airline 4 operates route  $KB$  and all of them enjoy monopoly power

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<sup>9</sup>The effect of airline alliances has also been empirically investigated by Oum *et al.* (1996), Park and Zhang (1998) and Brueckner (2003), among many others. These studies provide evidence that international alliances lead to lower fares, increases in the number of passengers on the relevant routes and that airline cooperation generates important benefits for interline passengers.



in their respective routes. They face standard linear demand functions  $q_i = \alpha - p_i$ , where  $q_i$  represents the travel volume,  $p_i$  denotes the fare charged by airline  $i$  for  $i = 1, 2, 3, 4$  and  $\alpha$  is a positive parameter that measures market size.<sup>10</sup> In addition, there are travellers wishing to fly from city  $A$  to city  $B$  (interline market) either through airport  $H$  or through airport  $K$ . We assume there are no travellers willing to fly between  $H$  and  $K$ .<sup>11</sup> In this case, travellers must fly by either combining airlines 1 and 2 or by combining airlines 3 and 4 so that routes  $AH$  and  $HB$  are regarded as complementary products (just as routes  $AK$  and  $KB$  together). However, the trips through airport  $H$  and airport  $K$  are viewed by travellers as substitute trips. There is no direct flight connecting cities  $A$  and  $B$  so that passengers have to interline at the hubs  $H$  and  $K$ . The proposed network structure is the simplest possible configuration including rivalry between two composite one-stop trips. It aims at capturing the particular aspect of alliances that enjoy antitrust immunity, where two different carriers may cooperate to offer interline trips in which passengers need to travel with both of them. We consider that cooperation is full, which implies that the alliance will behave as a single carrier in the market for which it is formed at the eyes of the passengers.

We will assume that demand functions for air travel between cities  $A$  and  $B$  are linear as follows,

$$\begin{aligned} Q_{12} &= \alpha - (p_1 + p_2) + d(p_3 + p_4) \\ Q_{34} &= \alpha - (p_3 + p_4) + d(p_1 + p_2), \end{aligned} \tag{1}$$

where  $Q_{12}$  and  $Q_{34}$  represent the travel volumes on the two interline flights in the market and  $d$ , that ranges between 0 and 1, captures the degree of product differentiation taking value 0 when products are independent and 1 when they are perfectly homogeneous. This demand system for differentiated products follows from solving

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<sup>10</sup>Note that, for low and medium density routes, the proportion of monopolist city-pair markets is very important. For the case of the low density routes, this proportion in Europe and the US accounts for 90% and 77% on short routes and for 79% and 94% on medium-haul routes, respectively. For the case of the medium density routes, this proportion in Europe and the US accounts for 87% and 86% on short routes and for 35% and 71% on medium-haul routes, respectively. See the IATA report "Air Transport Markets in Europe and the US, a comparison" (2001).

<sup>11</sup>In considering an inter-hub demand each carrier would operate in three markets. Then, the effects of alliances would be more complex because a complementary alliance in an interline market would have effects in all the three markets for each carrier.

the optimization problem of a representative passenger with a quasi-linear utility function ala Dixit (1979)<sup>12</sup> and it reflects that composite products are substitutes for one another up to some extent.<sup>13</sup>

Brueckner and Whalen (2000) provide an interesting interpretation of this kind of demand functions in terms of brand loyalty, since an increase in price of one composite product permits the competitor to "steal" a certain amount of traffic volume (total travel volume is constant) but both competitors remain in the market.

$A$  and  $B$  are assumed equidistant to  $H$  and  $K$  and a common cost function  $C(q_i + Q_{xy})$  applies to each of the four links in the network where  $q_i + Q_{xy}$  accounts for the total traffic volume using a particular link with  $xy = 12$  for  $i = 1, 2$  and  $xy = 34$  for  $i = 3, 4$ . As in Brueckner and Spiller (1991), we assume linear marginal cost functions of the form:

$$C'(q_i + Q_{xy}) = 1 - \theta(q_i + Q_{xy}), \quad (2)$$

that reflect increasing returns to traffic density,<sup>14</sup> which is a required assumption in airline markets.<sup>15</sup> The intensity of the economics of traffic density is measured by

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<sup>12</sup>A representative passenger maximizes  $U(Q_{12}, Q_{34}) = (\frac{\alpha}{1-d})(Q_{12} + Q_{34}) - \frac{Q_{12}^2 + 2dQ_{12}Q_{34} + Q_{34}^2}{2(1-d^2)}$ . This type of utility function has been used by Singh and Vives (1984) and by Economides and Salop (1992). We are therefore assuming an equal size of the market ( $\alpha$ ) both for the interline and for the short markets. A natural extension of the model would be to introduce asymmetric market sizes by supposing larger short markets. This extension complicates the presentation without offering any additional insights. Results are qualitatively similar as long as the difference between the two market sizes remains sufficiently small.

<sup>13</sup>As in other papers in the literature, we assume that the interline fare is the sum of the fares for markets  $AH$  and  $BH$  (or  $AK$  and  $BK$ ). Such an assumption implies that a carrier gives equal treatment to all passengers on its flight. In contrast, it seems closer to reality to consider that a carrier sets a fare for passengers stopping at the hub and a "subfare" for those doing the interline trip, as done by Brueckner and Whalen (2000) and Brueckner (2001). This practice, which would lead to different constraints on the parameters, entails some degree of coordination between carriers even under the *pre-alliance* situation. We keep the former pricing behavior for analytical reasons and to better capture the move from a purely non-cooperative setting to others involving cooperation.

<sup>14</sup>For instance, the cost function corresponding to link  $AH$  is:  $C(q_1 + Q_{12}) = (q_1 + Q_{12})(1 - \frac{\theta(q_1 + Q_{12})}{2})$ .

<sup>15</sup>Brueckner *et al.* (1992) and Park (1997) also use this marginal cost function, suggested by Brueckner and Spiller (1991), to model economics of traffic density that can be considered as a stylized fact of the airline industry as showed in the empirical literature on airlines. See for instance Brueckner and Spiller (1994) or Creel and Farrell (2001).

$\theta \geq 0$  where constant returns correspond to  $\theta = 0$ .

### *The pre-alliance equilibrium*

We begin by characterizing the *pre-alliance* solution. Airlines choose non-cooperatively their respective profit-maximizing fares. In this situation the fare to travel from  $A$  to  $B$  (either through  $H$  or through  $K$ ) is just the sum of the fares of the two short markets (either  $p_1 + p_2$  or  $p_3 + p_4$ ). The profit functions are  $\pi_i = p_i(q_i + Q_{xy}) - C(q_i + Q_{xy})$  again with  $xy = 12$  for  $i = 1, 2$  and  $xy = 34$  for  $i = 3, 4$ . The system of first order conditions is given by  $\partial\pi_1/\partial p_1 = 0$ ,  $\partial\pi_2/\partial p_2 = 0$ ,  $\partial\pi_3/\partial p_3 = 0$  and  $\partial\pi_4/\partial p_4 = 0$ . Fulfillment of the second order conditions require that  $\theta < 1$ . The equilibrium price, which is symmetric across markets (hence we omit subscripts  $i$  and  $xy$ ), is given by,

$$p^{na} = \frac{2(\alpha(1 - 2\theta) + 1)}{5 - 2d - 6\theta + 4d\theta},$$

where superscript  $na$  denotes the no-alliances scenario. Equilibrium travel volumes and profits are the following:

$$q^{na} = \frac{\alpha(3 - 2d - 2\theta) - 2}{5 - 2d - 6\theta + 4d\theta}, \quad Q^{na} = \frac{\alpha(1 + 2d - 4\theta d + 2\theta) - 4(1 - d)}{5 - 2d - 6\theta + 4d\theta},$$

$$\pi^{na} = (p^{na} - 1 + \frac{\theta(q^{na} + Q^{na})}{2})(q^{na} + Q^{na}). \quad (3)$$

The equilibrium profits consist of the margin (price minus average cost) times the travel volume in both the short and the interline markets. Clearly, there exist some conditions regarding market size,  $\alpha$ , the degree of product differentiation,  $d$ , and the size of economies of traffic density,  $\theta$ , for which positive prices, positive travel volumes, positive margins and positive marginal costs are obtained.

### *The single alliance equilibrium*

With a *single alliance*, either airlines 1 and 2 set the fare for flight from  $A$  to  $B$  through hub  $H$  cooperatively while competition with flight through hub  $K$  remains, or it is airlines 3 and 4 that set cooperatively the fare through hub  $K$  while competition with flight through hub  $H$  remains. Denote by  $P_p$  the fare of the interline flight established jointly by the partners in the alliance. The alternative interline flight is priced separately,  $p_{o1} + p_{o2}$ , where subscripts  $o1$  and  $o2$  stand for

the two outsiders. Both alliance partners are symmetric and both outsiders too. The demand functions for the short markets do not change but, in market  $AB$  they now take the form:  $Q_p = \alpha - P_p + d(p_{o1} + p_{o2})$  for the alliance partners and  $Q_o = \alpha - (p_{o1} + p_{o2}) + dP_p$  for outsiders. Thus, alliance partners choose two fares since they price separately the short and the interline market in which they operate. The joint profit function for the partners,  $p1$  and  $p2$ , becomes  $\pi_p = p_{p1}q_{p1} + p_{p2}q_{p2} + P_pQ_p - C(q_{p1} + Q_p) - C(q_{p2} + Q_p)$  and for the outsiders are the same as before. Solving the system formed by  $\partial\pi_p/\partial p_{p1} = 0$ ,  $\partial\pi_p/\partial p_{p2} = 0$ ,  $\partial\pi_o/\partial p_{o1} = 0$ ,  $\partial\pi_o/\partial p_{o2} = 0$  and  $\partial\pi_p/\partial P_p = 0$  yields, given symmetry, the following equilibrium prices,

$$p_p^a = \frac{4d\theta(1 + \alpha - 2\alpha\theta) + 2d^2(2\theta - 1)(\alpha(3\theta - 1) - 1) - (6\theta - 5)(\alpha(5\theta - 2) - 2)}{54\theta - 20 - 36\theta^2 + 2d^2(2 + \theta(10\theta - 9))},$$

$$p_o^a = \frac{4(2 - 3\theta)(\alpha(2\theta - 1) - 1) + d(1 - 2\theta)(\alpha(7\theta - 2) - 4)}{54\theta - 20 - 36\theta^2 + 2d^2(2 + \theta(10\theta - 9))},$$

$$P_p^a = \frac{4d(2 - 5\theta)(\alpha(2\theta - 1) - 1) + (5 - 6\theta)(\alpha(7\theta - 2) - 4)}{54\theta - 20 - 36\theta^2 + 2d^2(2 + \theta(10\theta - 9))},$$

where superscript  $a$  identifies the *single alliance* scenario.<sup>16</sup> The second order conditions for a maximum now require that  $\theta < \frac{2}{3}$ . This condition follows from the negativity of the Hessian matrix corresponding to the optimization problem of the allied carriers. Equilibrium travel volumes are,

$$Q_p^a = \frac{4d^2(1 - 2\theta)(\alpha\theta - 2) + (5 - 6\theta)(4 - \alpha(2 + \theta)) - 4d(2 - \theta)(1 - \alpha(2\theta - 1))}{54\theta - 20 - 36\theta^2 + 2d^2(2 + \theta(10\theta - 9))},$$

$$Q_o^a = \frac{2(3\theta - 2)(\alpha + 2\alpha\theta - 4) - 2d^2(5\theta - 2)(\alpha(2\theta - 1) - 2) - d(2\theta - 3)(\alpha(7\theta - 2) - 4)}{54\theta - 20 - 36\theta^2 + 2d^2(2 + \theta(10\theta - 9))},$$

$$q_p^a = \frac{4d\theta(\alpha(2\theta - 1) - 1) - (2 + \alpha(\theta - 2))(6\theta - 5) + 2d^2(2\theta - 1)(1 + \alpha(2\theta - 1))}{54\theta - 20 - 36\theta^2 + 2d^2(2 + \theta(10\theta - 9))},$$

$$q_o^a = \frac{d(2\theta - 1)(\alpha(7\theta - 2) - 4) - 2(3\theta - 2)(2 + \alpha(2\theta - 3)) + 2d^2\alpha(2 + \theta(10\theta - 9))}{54\theta - 20 - 36\theta^2 + 2d^2(2 + \theta(10\theta - 9))}.$$

The equilibrium profits for each of the two outsiders are given by:

$$\pi_o^a = (p_o^a - 1 + \frac{\theta(q_o^a + Q_o^a)}{2})(q_o^a + Q_o^a). \quad (4)$$

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<sup>16</sup>It is important to know that, when the long market is priced jointly, the stability of the network requires non-arbitrage conditions to apply. These conditions are of two types. The first type prevents passengers willing to do a short market trip ( $AH$  or  $BH$ ) from buying an interline ticket ( $AB$  ticket) and then get off at the hub airport ( $H$  airport). The second type of conditions ensure that nobody would buy an interline ticket if breaking down the trip into two parts were cheaper.

However, the joint equilibrium profits for the allied carriers are now:

$$\pi_p^a = (p_p^a - 1 + \frac{\theta(q_p^a + Q_p^a)}{2})2q_p^a + (P_p^a - 2 + \theta(q_p^a + Q_p^a))Q_p^a. \quad (5)$$

The alliance "partially unbundles" the interline from the short markets since the interline market is priced separately. Nevertheless, partners are not able to extract monopoly profits because markets remain connected through the link-dependent cost function. One can observe that (3) and (4) have the same structure whereas (5) is different. In (5) there are two margins: one corresponding to the short market (price minus the average cost) and another one corresponding to the interline market (price minus twice the average cost) since the interline market needs to use two links of the network while the short market just needs to use one link.

#### *The double alliance equilibrium*

We now characterize the situation when two interline alliances are formed, that is, when airlines 1 and 2 behave cooperatively in setting fares and so do airlines 3 and 4. Denote by  $P_{12}$  and  $P_{34}$  the fare of the interline flight through hub  $H$  and hub  $K$ , respectively. The interline market demands are now given by  $Q_{12} = \alpha - P_{12} + dP_{34}$  and  $Q_{34} = \alpha - P_{34} + dP_{12}$ . Joint profit maximization results in the following symmetric equilibrium prices (hence we omit subscripts):

$$P^{aa} = \frac{4 + 2\alpha - 7\alpha\theta}{4 - 2d - 6\theta + 5d\theta}, \quad p^{aa} = \frac{(3d - 5)\alpha\theta + (1 + \alpha)(2 - d)}{4 - 2d - 6\theta + 5d\theta}$$

and we employ superscripts  $aa$  for the case where both alliances occur. As in the *single alliance* equilibrium, the second order conditions for a maximum impose that  $\theta < \frac{2}{3}$ . The corresponding travel volumes and profits are given by,

$$Q^{aa} = \frac{(2 + \theta)\alpha - 4 + d(4 - 2\alpha\theta)}{4 - 2d - 6\theta + 5d\theta}, \quad q^{aa} = \frac{(2d - 1)\alpha\theta - (\alpha - 1)(d - 2)}{4 - 2d - 6\theta + 5d\theta}$$

and the equilibrium profits can be written as follows:

$$\pi^{aa} = (p^{aa} - 1 + \frac{\theta(q^{aa} + Q^{aa})}{2})2q^{aa} + (P^{aa} - 2 + \theta(q^{aa} + Q^{aa}))Q^{aa}. \quad (6)$$

Before characterizing the effects of alliances and the Nash equilibrium in alliance formation, there are a number of restrictions that must be borne in mind. This amounts to comparing a number of bounds on market size  $\alpha$ .

**Assumption 1** A market in our model is defined by a triple  $\{d, \theta, \alpha\}$ . We restrict attention to markets  $\{d, \theta, \alpha\} \in R$  where  $R$  is the relevant region in our analysis, i.e., the region ensuring positive prices, quantities, marginal costs, margins and the compliance with non-arbitrage conditions are guaranteed in the three scenarios under consideration (*pre-alliance*, *single alliance* and *double alliance*). See Appendix 1 for the details.

### 3 Effects on Fares and Travel Volumes

We will begin by comparing the changes in firms' fares and travel volumes in a move from the *pre-alliance* situation to the *single alliance* situation. Then, we will establish the corresponding variations when two alliances are formed rather than a single one.

**Proposition 1** *For any market  $\{d, \theta, \alpha\} \in R$ , the formation of a complementary alliance has the following effects:*

- i) For the partners, the fare  $P_p^a$  is lower than the pre-alliance fare  $p^{na} + p^{na}$  and travel volume increases, i.e.  $Q_p^a > Q^{na}$  (direct effect).*
- ii) Outsiders' fares and travel volumes are lower, i.e.  $p_o^a < p^{na}$  and  $Q_o^a < Q^{na}$  (outsider effect).*
- iii) As for the short markets, since  $p_o^a < p^{na}$ , we observe that  $q_o^a > q^{na}$ . For the allied carriers, the effects on  $p_p$  and  $q_p$  are unclear.*

**Proof.** See Appendix 2. ■

The above results partially confirm Cournot's (1838) model of complementary duopoly. Cournot considered the merger of two monopolists that produce complementary goods (zinc and copper) into a single (fused) monopolist that produces the combination of them (brass). The price of the composite good is lower than under independent ownership (*direct effect*). The alliance between airlines that offer complementary services internalizes the externality that arises when they set fares independently thus ignoring the effects on their individual markups. This result, part *i)* above, has already been suggested by earlier theoretical work by Brueckner (2001) and Brueckner and Whalen (2000) although under somewhat different modeling and network structures. Indeed empirical evidence in Brueckner and Whalen

(2000) and Brueckner (2003) find that alliance partners charge interline fares that are below those charged when they were non-allied.

To see the intuition, note that the first order conditions in the *single alliance* situation implicitly define the corresponding reaction functions in prices.<sup>17</sup> Whenever  $\theta < \frac{1}{2}$ , which is almost always the case, the effects implied by the reaction functions are  $P_p = f(p_p, p_o)$ ,  $p_p = f(P_p, p_o)$  and  $p_o = f(P_p)$ . Therefore, the decrease in the interline fare induced by the alliance leads to a reduction in the fares set by outsiders since  $P_p$  and  $p_o$  are strategic complements. Rather obviously a lower interline fare permits partners to capture a higher travel volume and this occurs at the expense of a lower interline traffic by the rivals. On the other hand, since the outsiders' fares are lower than under the *pre-alliance* situation it follows that there is a gain in travel volumes for short trips.

Besides, the decrease in the interline fare induces a reduction in the fare  $p_p$  but this fare increases due to the indirect effect via the decrease in the fare of outsiders. The final effect on partners' fare and travel volumes for their short trips depend on market size, the degree of product differentiation and the intensity of economies of traffic density. Although allied carriers are monopolists in their short markets, they cannot extract monopoly profits because the equilibrium prices and quantities are linked through the cost function because costs are link-dependent and not market dependent. Therefore, there is an output reallocation effect affecting the partners in their short market equilibrium values causing a non-monotonic relationship of prices and quantities when an alliance to serve the interline market is formed.

An interesting by-product of the previous analysis is that, for outsiders, the revenue in the interline market decreases since both the fare and the travel volume are lower in a move to a *single alliance* setting (*outsider effect*). Further, revenue in the short market is unclear because the variation in fare goes in the opposite direction to the one in travel volume. Finally, as total travel volume by the outsiders decreases this means that average costs are larger; the output reallocation makes carriers to take less profit from the presence of economies of traffic density. Consequently, outsiders will very likely be harmed by the formation of an alliance. Boyer (1992)

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<sup>17</sup>These are the three reaction functions for the *single alliance* case:

$P_p = \frac{2+\alpha(1-4\theta)+2(1-2\theta)dp_o+2\theta p_p}{2(1-\theta)}$ ,  $p_p = \frac{1+\alpha(1-2\theta)+\theta P_p-2\theta dp_o}{2-\theta}$  and  $p_o = \frac{2+2\alpha(1-2\theta)+(1-2\theta)dP_p}{5-6\theta}$ . These functions are not always upward or downward sloping since they depend on whether  $\theta$  is higher or lower than  $\frac{1}{2}$ .

suggests that non-participating firms in a merger may be harmed by a merger. He argues that this is more likely to occur if there are inter-market connections. This is the case in the current setting where markets are linked through the cost function.

The next proposition summarizes the comparison of firms' fares and travel volumes involved in the *single* and *double alliance* equilibria. Notice that the term outsiders will now allude to the airlines that had previously formed an alliance.

**Proposition 2** *For any market  $\{d, \theta, \alpha\} \in R$ , the formation of two complementary alliances as opposed to a single alliance has the following effects:*

- i) For the new partners, the fare  $P^{aa}$  is lower than their single alliance fare  $2p_o^a$  and travel volume increases, i.e.  $Q^{aa} > Q_o^a$  (direct effect).*
- ii) Outsiders' fares and travel volumes are lower, i.e.  $P^{aa} < P_p^a$  and  $Q^{aa} < Q_p^a$  (outsider effect).*
- iii) We observe that  $p^{aa} > p_p^a$  (and  $q^{aa} < q_p^a$ ), which is the opposite to the move from the no alliance to the single alliance situation, in the outsiders's short markets. For the new allied carriers, the effects on  $p_o$  and  $q_o$  are unclear.*

**Proof.** See Appendix 2. ■

Again, the alliance of the new partners results in lower fares for their interline market. Given that prices set by the new partners and by outsiders in the interline market are strategic complements (upward sloping reaction functions), it follows that the interline fare of the already allied carriers (now outsiders) goes down. Thus, the *direct* and the *outsider effects* work in the same way as in Proposition 1. However, the effect on the outsiders' short markets is now different. In the case of Proposition 1 (*iii*) above, the decrease in the price in the short market was a consequence of the outsider effect since the price on the interline market was the sum of the price of two short markets. Now this is not the case anymore and the outsiders react to the alliance by increasing their short market fares.

As in the move from *no alliance* to *single alliance*, there is an output reallocation effect affecting the new allied carriers in their short market equilibrium values causing a non-monotonic relationship of prices and quantities when moving from a *single* to a *double alliance* scenario.

To sum up, concerning outsiders' interline market, alliance formation is disadvantageous in terms of revenue no matter they set fares cooperatively (Proposition 2) or non-cooperatively (Proposition 1) in the interline market. Hence, a setting



with interline alliances leads to lower fares, which is consistent with some observed facts in the airline industry, as found by Brueckner (2003).

## 4 A simultaneous game of airline alliances

The foregoing analysis suggests that airline alliances are profitable only under some circumstances. The formation of airline alliances is endogenously obtained as a result of the following two-stage game. In the first stage airlines 1 and 2 and airlines 3 and 4 decide simultaneously and independently whether to form an alliance. In stage two, given the inherited outcome from the first stage, airlines set fares.

Given the symmetry in the model it suffices to study the best-response of the potential partners (either airlines 1 and 2, or alternatively 3 and 4) *i*) when the rivals decide not to form a complementary alliance, and *ii*) when the rivals decide to form a complementary alliance. Let us define

$$\Psi^a(d, \theta, \alpha) = \frac{\pi_p^a}{2} - \pi^{na}. \quad (7)$$

Therefore  $\Psi^a(d, \theta, \alpha) > 0$  defines when airlines 1 and 2 (alternatively 3 and 4) will form an alliance given that the rivals do not.<sup>18</sup> The following lemma results from the analysis of this unilateral incentive to form an alliance. Sufficient conditions are given in parenthesis.

**Lemma 1** *For any market  $\{d, \theta, \alpha\} \in R$ , whenever the rivals decide not to form an alliance, two potential partners:*

- i) will form an alliance either for a sufficient degree of product differentiation ( $d \leq \underline{d}^a \equiv 0.802$ ) or for a sufficiently high intensity of economies of traffic density ( $\theta > \bar{\theta}^a \equiv 0.08$ );*
- ii) will not form an alliance whenever the two routes are close enough substitutes ( $d > \bar{d}^a \equiv 0.856$ );*
- iii) will either form or not form an alliance for values of  $d \in (\underline{d}^a, \bar{d}^a]$ . In this case, an alliance will not be formed for high values of  $\alpha$  combined with low values of  $\theta$ .*

**Proof.** See Appendix 2. ■

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<sup>18</sup>It is assumed that profits are equally shared by partners once they decide to ally and set up a complementary alliance.

This lemma states that a sufficiently high degree of product differentiation guarantees at least one alliance in equilibrium. On the other hand, when products are close substitutes, which supposes that competition intensity is strong, it is preferable to remain non-allied given that the rivals do not form an alliance. In this case, it is strategically profitable for carriers not to cooperate. For values between the specified thresholds, either strategy might be the best-response (*BR*) depending on the specific value of the parameters. Figure 2 below constitutes a representative example for  $d \in (\underline{d}^a, \bar{d}^a]$ ,<sup>19</sup> where  $\alpha(\theta)$  is obtained from solving  $\Psi^a(d = 0.835, \theta, \alpha) = 0$ .

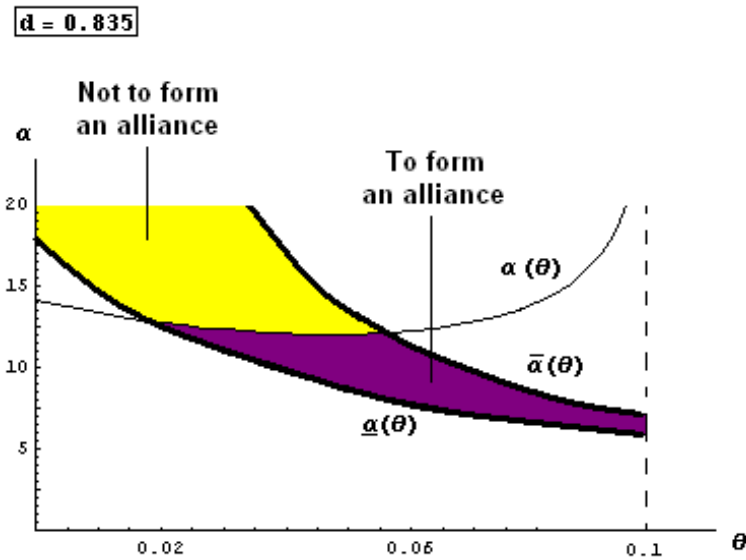


Figure 2: *BR* when rivals do not form an alliance

Values below (above) the function  $\alpha(\theta)$  identify those cases where, as long as both the lower and the upper bounds on  $\alpha$  are respected, the best-response is (not) to form an alliance. One can observe that larger values of  $\theta$  create incentives for alliance formation since they foster efficiency gains. In fact, as stated in the lemma, a sufficiently high  $\theta$  ensures a *single alliance* for any market  $\{d, \theta, \alpha\} \in R$ . On the other hand, it seems that carriers operating in large markets ( $\alpha$ ) are less willing to form alliances when the rivals do not do it. Furthermore, as the degree of product differentiation decreases (higher values of  $d$ ), the function  $\alpha(\theta)$  shifts downwards, thereby enlarging the region for *no alliance* to be the best-response.

<sup>19</sup>As stated in Assumption 1, markets  $\{d, \theta, \alpha\} \in R$  require to respect some bounds on  $\theta$  and  $\alpha$ . Following the notation in Appendix 1,  $\bar{\theta}(d) = L2$ ,  $\bar{\alpha}(d, \theta) = B2$  and  $\underline{\alpha}(d, \theta) = B3$  when  $d = 0.835$ .

To see the intuition note that the change in a carrier's profits when moving to a *single alliance* setting can be decomposed in three terms. The first one is related with the variation in revenue from the interline market ( $\Delta R_I$ ); the second one comes from the difference in revenue in the short market ( $\Delta R_S$ ); and the third one has to do with the change in costs due to the reallocation in travel volumes. Only the latter variation has got an unambiguous sign. It happens that total travel volume by the allied carriers increases ( $Q_p^a + q_p^a > Q^{na} + q^{na}$ ). Since we have already argued that the joint cost function exhibits increasing returns to traffic density in the supply of interline and short trips, higher travel volumes imply a better exploitation of these economies of traffic density. Concerning the first term, it has been shown above that the interline fare goes down while travel volume goes up (*direct effect*). It can be proven that  $\Delta R_I = f(\underline{d}, \underline{\theta}, \underline{\alpha})$ , i.e. the fare effect is more likely to dominate the travel volume effect, other things equal, the higher the market size; the higher the degree of product differentiation; or the smaller the intensity of economies of traffic density. As for the second term, the variation associated with the revenue from the short market cannot be neatly established since it involves non-monotonic effects, but numerical examples indicate that  $\Delta R_S$  is more likely to increase when  $d$  does not take intermediate values.

To sum up, there is a positive effect coming from the efficiency gains due to cost savings, that may be offset by possible revenue losses (either in the short or in the interline market). Lemma 1 states that typically, but not always, the positive effects outweigh negative ones, this meaning that there is a unilateral incentive to form a complementary alliance given that the rivals do not form an alliance.

The interesting conclusion from our analysis is that, contrary to what one might expect for an alliance with complementary trips in presence of economies of traffic density, it is not necessarily optimal for carriers to create an alliance when the other potential partners remain non-allied. If there were no competition from a substitute flight, then the alliance would always turn out profitable - as in Cournot's example. However, the presence of other airlines serving the interline market unveils that the alliance will be profitable only under certain circumstances. Therefore, when the two possible one-stop interline trips are "sufficiently substitutes" at the eyes of the traveller, the best-response for two potential partners is not to form an alliance when the other carriers remain non-allied (e.g. Japan Airlines).

Next we study the best-response of the potential partners when the rivals form a

complementary alliance. Thus, let us define

$$\Psi^{aa}(d, \theta, \alpha) = \frac{\pi^{aa}}{2} - \pi_o^a. \quad (8)$$

Hence,  $\Psi^{aa}(d, \theta, \alpha) > 0$  defines when airlines 1 and 2 (alternatively 3 and 4) will form an alliance given that the rivals do. The analysis of  $\Psi^{aa}(d, \theta, \alpha)$  leads to the following result.

**Lemma 2** *For any market  $\{d, \theta, \alpha\} \in R$ , whenever the rivals decide to form an alliance, two potential partners:*

*i) will also form an alliance either for a sufficient degree of product differentiation ( $d \leq \underline{d}^{aa} \equiv 0.707$ ) or for a sufficiently high intensity of economies of traffic density ( $\theta > \bar{\theta}^{aa} \equiv 0.195$ );*

*ii) will not form an alliance whenever the two routes are rather close substitutes ( $d > \bar{d}^{aa} \equiv 0.828$ );*

*iii) will either form or not form an alliance for values of  $d \in (\underline{d}^{aa}, \bar{d}^{aa}]$ . In this case, an alliance will not be formed for intermediate values of  $\alpha$  combined with low values of  $\theta$ .*

**Proof.** See Appendix 2. ■

Again, a sufficiently high degree of product differentiation makes alliances strategically profitable. On the other hand, when products are loose substitutes, *double alliance* is not an equilibrium. For values between the specified bounds, either strategy might be the best-response. Figure 3 below displays a representative example for  $d \in (\underline{d}^{aa}, \bar{d}^{aa}]$ ,<sup>20</sup>

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<sup>20</sup>As stated in Assumption 1, markets  $\{d, \theta, \alpha\} \in R$  require to respect some bounds on  $\theta$  and  $\alpha$ . Following the notation in Appendix 1,  $\bar{\theta}(d) = L2$ ,  $\bar{\alpha}(d, \theta) = B2$  and  $\underline{\alpha}(d, \theta) = B3$  when  $d = 0.750$ .

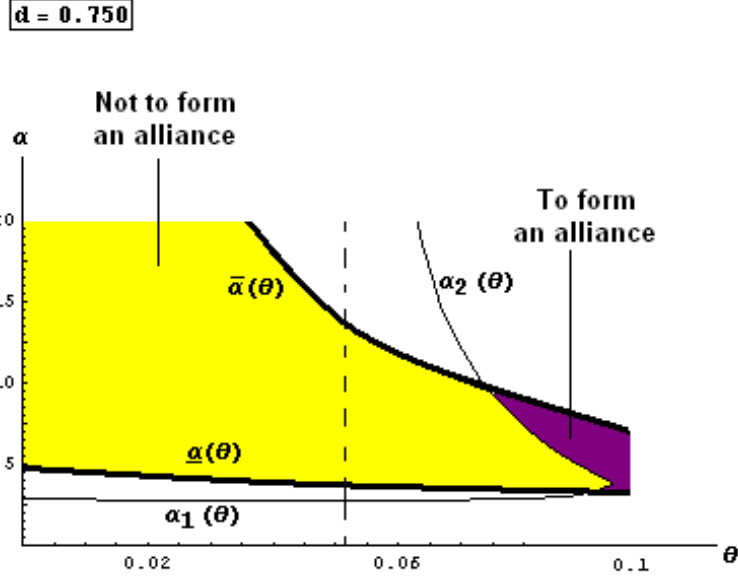


Figure 3: BR when rivals form an alliance

where we observe, as in Lemma 1, that larger values of  $\theta$  create incentives for alliance formation. In this case, a larger market size seems to foster alliance formation. The functions  $\alpha_1(\theta)$  and  $\alpha_2(\theta)$  are the ones that solve  $\Psi^{aa}(d = 0.750, \theta, \alpha) = 0$ . Thus, values between these two functions indicate that a *single alliance* is the best-response. As the degree of product differentiation decreases (higher values of  $d$ ), the functions  $\alpha_1(\theta)$  and  $\alpha_2(\theta)$  move far apart thereby enlarging the region where *single alliance* is the best-response.

Although the interpretation of Lemma 2 is as before, it is important to notice that the sufficient conditions to ensure a *double alliance* are now more stringent, i.e. either a higher degree of product differentiation ( $\underline{d}^{aa} < \underline{d}^a$ ) or a higher intensity of economies of traffic density ( $\bar{\theta}^{aa} > \bar{\theta}^a$ ) is required. Therefore, we can anticipate that there will be cases in which the best-response can be to form an alliance if the rivals do not; but not to form an alliance if the rivals decide to do so, i.e.  $\Psi^a(d, \theta, \alpha) > 0$  and  $\Psi^{aa}(d, \theta, \alpha) < 0$ . The fact that a setting with two alliances may not always be privately profitable, certainly opens the door to *asymmetric equilibria* which is sometimes a real issue in the airline industry; in fact, there is an important number of routes where only one international alliance provides interline services (e.g. most of the one-stop routes between Italy and France are only served by *The Sky Team*). Nevertheless, it must be noted that  $\Psi^a(d, \theta, \alpha) < 0$  is not compatible with  $\Psi^{aa}(d, \theta, \alpha) > 0$  so that *multiple equilibria* cannot arise.

In the *double alliance* setting, total travel volume for the new partners increases, that is,  $Q^{aa} + q^{aa} > Q_o^a + q_o^a$ . This means that the alliance produces an output reallocation effect which allows the new partners to take advantage of the economies of traffic density. Besides, a similar analysis to the previous lemma unveils that revenues in the interline market are higher for a sufficient degree of product differentiation or for small enough economies of traffic density, other things being equal. On the other hand, the variation in revenues from the short market is negative for intermediate values of product differentiation or for large enough economies of traffic density, other things equal. Thus, the above result states that the gains stemming from travel volume restructuring and interline earnings more than compensate the losses originated from lower revenues in the short market as long as competition intensity be sufficiently weak, no matter the size of the economies of traffic density. Nevertheless, the losses may be more important than the gains as either competition intensity becomes stronger, or if economies of traffic density are not too significant or market size is large enough.

In the light of the best-responses studied above, we can easily identify the sub-game perfect equilibrium of the game. The next proposition combines the results stated in the previous two lemmas and singles out sufficient conditions under which the three possible scenarios are an equilibrium of the game in alliance formation. In particular, it addresses the corresponding equilibrium for the whole range of the values of the degree of product differentiation. It should nonetheless be kept in mind that  $\theta > \bar{\theta}^a$  ensures at least one alliance and that  $\theta > \bar{\theta}^{aa}$  ensures two alliances.<sup>21</sup> Since we have  $\underline{d}^{aa}$ ,  $\underline{d}^a$ ,  $\bar{d}^{aa}$  and  $\bar{d}^a$ , there are five regions to be considered.

**Proposition 3** *For any market  $\{d, \theta, \alpha\} \in R$ , the equilibrium in alliance formation is:*

- i) no alliance for  $d > \bar{d}^a$ ;*
- ii) either no alliance or single alliance for  $d \in (\bar{d}^{aa}, \bar{d}^a]$ . No alliance will be the equilibrium for high values of  $\alpha$  combined with low values of  $\theta$ ;*
- iii) no alliance, single alliance or double alliance may be the equilibrium for  $d \in (\underline{d}^a, \bar{d}^{aa}]$  depending on the joint effect of  $\alpha$  and  $\theta$ ;*
- iv) either single alliance or double alliance for  $d \in (\underline{d}^{aa}, \underline{d}^a]$ . Single alliance will be for intermediate values of  $\alpha$  combined with low values of  $\theta$ ;*

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<sup>21</sup>This means that the *no alliance* equilibrium in the next proposition, can only arise for markets  $\{d, \theta, \alpha\} \in R$  with  $\theta < \bar{\theta}^a$ .

v) *double alliance* for  $d \leq \underline{d}^{aa}$ .

The proof is straightforward. We conclude that a setting with complementary alliances arises when the two possible interline trips are strongly differentiated, whereas alliances will not take place in equilibrium when trips are viewed by travellers as close substitutes. It is worth mentioning that, for intermediate values of the degree of product differentiation, an *asymmetric equilibrium* in which only one alliance occurs is possible in the current setting. Moreover, the transition from the region where the equilibrium is *no alliance* to the one where the equilibrium is *double alliance* is parsimonious since in part *ii)* above the *single alliance* equilibrium is a possibility, in part *iii)* anything may happen, and in part *iv)* *no alliance* is not a possibility anymore.

The following contention may be useful to illustrate the above proposition. Let us fix values for market size and economies of traffic density and let the degree of product differentiation open. Now a pair of carriers have to decide whether to form an alliance. As argued before, total travel volume by the potential partners increases when they form an alliance, allowing for a better exploitation of economies of traffic density. However the effects coming from the interline and short market revenues are not straightforward. Suppose that  $d$  is close to one so that competition intensity is strong. In this case, the internalization of competition that occurs under cooperation when setting fares results in greater revenues in the short market since the increase in short trip traffic is larger than the decrease in fares; the opposite happens to revenues from the interline market. In this framework, the latter negative effect outweighs the other two positive effects and the carriers are better off not forming an alliance.

However, as the degree of product differentiation increases and competition intensity softens, carriers also obtain gains in the interline market thus making the alliance privately profitable.<sup>22</sup> Concerning the rivals, their equilibrium travel volumes go down and the corresponding cost inefficiency incurred suffices to offset any likely revenue gains so that alliance formation hurts outsiders. This occurs both in a move to the *single alliance* equilibrium and from this one to the *double alliance* equilibrium. Finally, a sufficiently large degree of product differentiation ensures an

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<sup>22</sup>Although there might be losses in the short market for intermediate values of  $d$ , these are offset by the efficiency gains due to traffic reallocation and the revenue gains from the interline market.

equilibrium with airline alliances.<sup>23</sup>

A deeper analysis of carriers' profits in the *no alliance* and the *double alliance* settings reveals that the agents may well get engaged in a prisoner's dilemma situation. Specifically, let us define

$$\Psi^{pd}(d, \theta, \alpha) = \pi^{na} - \frac{\pi^{aa}}{2}, \quad (9)$$

where superscript *pd* stands for prisoner's dilemma. Hence,  $\Psi^{pd}(d, \theta, \alpha) < 0$  concludes that the *double alliance* is profitable per se; and  $\Psi^{pd}(d, \theta, \alpha) > 0$  means that the *double alliance* leads to a prisoner's dilemma situation as long as forming an alliance is a dominant strategy. The fact that outsiders end up worse off when the rivals form an alliance is behind this possibility. Therefore, the *double alliance* can be the equilibrium outcome and yet the four airlines might indeed find themselves earning lower profits had neither alliance occurred. This can in some sense be considered as an outcome stemming from a "war of alliances".<sup>24</sup> The next corollary summarizes this discussion.

**Corollary 1** *For any market  $\{d, \theta, \alpha\} \in R$ , whenever double alliance is the equilibrium in alliance formation, airlines get engaged in a prisoner's dilemma situation for a sufficiently low degree of product differentiation ( $d > d^{pd} \equiv 0.355$ ).*

**Proof.** See Appendix 2. ■

We will finish this section with presenting the case of constant returns to traffic density, i.e. when  $\theta = 0$ . The previous analysis discloses the difficulties in the characterization of the equilibrium due to the interplay between market size, economies of traffic density and the degree of product differentiation. The particular solution for  $\theta = 0$  allows us to illustrate the results when the travel volume reallocation effect is not present.

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<sup>23</sup>A similar reasoning can be made by letting  $\theta$  or  $\alpha$  vary while keeping the other two variables fixed.

<sup>24</sup>A recent reference by Fridolfsson and Stennek (2005) distinguishes mergers that are profitable per se from those which these authors call "defensive" mergers. The latter would correspond with a prisoner's dilemma situation in the current setting.



When  $\theta = 0$ , a market is characterized by  $\{d, \theta = 0, \alpha\}$  and region  $R$  is defined accordingly. There is no upper bound on market size  $\alpha$  but there remains a lower bound  $\underline{\alpha}(d)$ .<sup>25</sup> Figure 4 below depicts region  $R$ .

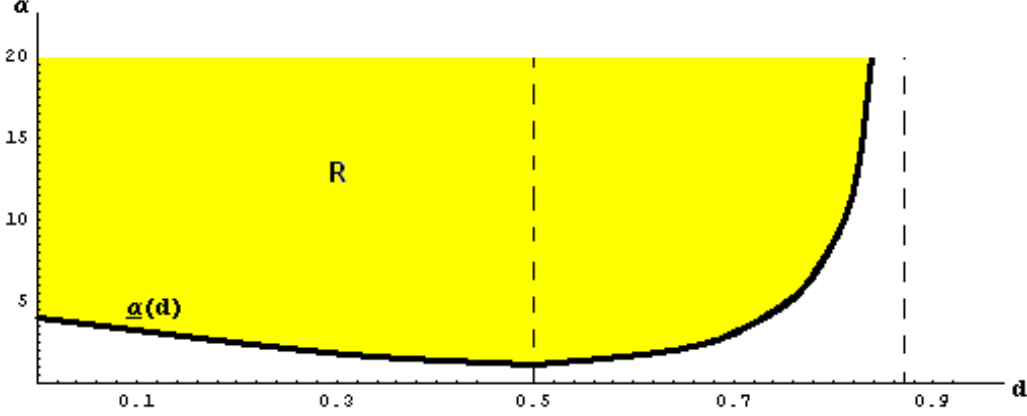


Figure 4: Region  $R$  when  $\theta = 0$

The next proposition illustrates the complete characterization of the equilibrium in the formation of airline alliances as a function of the degree of product differentiation and market size. The functional forms of the expressions  $\alpha_1(d)$ ,  $\alpha_2(d)$ ,  $\alpha_3(d)$  and  $\alpha_4(d)$  are conveniently specified in Appendix 2. Subscript 0 identifies values corresponding to the case of constant returns to traffic density.

**Proposition 4** *Suppose  $\theta = 0$ . Then, for any market  $\{d, \theta = 0, \alpha\} \in R$ , the equilibrium in alliance formation is:*

- i) no alliance for  $d > \bar{d}_0^a$ ;*
- ii) either no alliance or single alliance for  $d \in (\underline{d}_0^a, \bar{d}_0^a]$ . No alliance will be the equilibrium for  $\alpha \geq \alpha_1(d)$ ;*
- iii) single alliance for  $d \in (\bar{d}_0^{aa}, \underline{d}_0^a]$ ;*
- iv) either single alliance or double alliance for  $d \in (\underline{d}_0^{aa}, \bar{d}_0^{aa}]$ . Single alliance for  $\alpha \in (\max\{\underline{\alpha}(d), \alpha_2(d)\}, \alpha_3(d))$ ;*
- v) double alliance for  $d < \underline{d}_0^{aa}$ .*

*Note that  $\bar{d}_0^a \equiv 0.829$ ,  $\underline{d}_0^a \equiv 0.817$ ,  $\bar{d}_0^{aa} \equiv 0.725$ ,  $\underline{d}_0^{aa} \equiv 0.707$ .*

<sup>25</sup>This is because the upper bound for  $\alpha$  comes from positivity of marginal costs. Since now marginal costs are constant and positive, such a condition is not required. Following the notation in Appendix 1,  $\underline{\alpha}(d)$  corresponds to  $B1$  for  $d < \frac{1}{2}$  and to  $B3$  for  $d > \frac{1}{2}$ . At  $d = \frac{1}{2}$  there is a discontinuity and  $B1 = B3 = 1$ .

**Proof.** See Appendix 2. ■

As we did when there exist economies of traffic density, a detailed analysis of the *double alliance* case allows us to classify them in the following corollary.

**Corollary 2** *Suppose  $\theta = 0$ . Then, for any market  $\{d, \theta = 0, \alpha\} \in R$ , whenever double alliance is the equilibrium in alliance formation, these alliances can be classified as follows:*

- i) double alliance involving a prisoner's dilemma for  $d > 0.5$ ;*
- ii) either double alliance involving a prisoner's dilemma or double alliance profitable per se for  $d \in (d_0^{pd}, 0.5]$ . In particular, double alliance involving a prisoner's dilemma will be the equilibrium for  $\alpha \geq \alpha_4(d)$ ;*
- iii) double alliance profitable per se for  $d < d_0^{pd}$ ;*

Note that  $d_0^{pd} \equiv 0.355$ .

**Proof.** See Appendix 2. ■

Figure 5 summarizes the above proposition and corollary representing the relevant regions for different values of the degree of product differentiation.

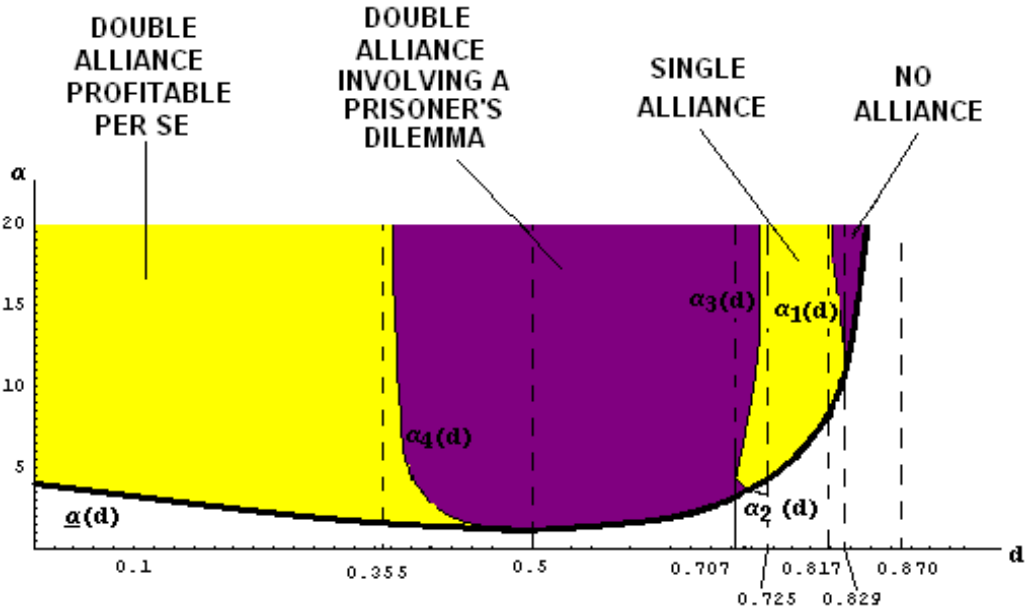


Figure 5: Equilibrium when  $\theta = 0$

In this particular case, one can observe that *asymmetric equilibria* (with just one alliance) arise. Notice too the previously mentioned parsimonious nature of the

transition from the region where the equilibrium is *no alliance* to the one where it is the *double alliance*.<sup>26</sup> This result highlights the positive relationship between product differentiation and strategic complementary integration in our setting. The simplification  $\theta = 0$  results in a fairly easy comparison when analyzing  $\Psi^a(d, \theta = 0, \alpha)$  and  $\Psi^{aa}(d, \theta = 0, \alpha)$ . It amounts to checking the gains or losses in the interline market vis-à-vis those in the short market. As competition intensity gets stronger it is more likely to find an equilibrium with just one alliance. Eventually, if the degree of product differentiation is rather low and market size is rather large one can find the *no alliance* equilibrium. Furthermore, and as happened in the presence of economies of traffic density, carriers may get involved in a prisoner's dilemma situation unless the degree of product differentiation be sufficiently large.

## 5 Concluding Remarks

We have developed a formal model to illustrate how factors such as market size, the degree of product differentiation and the intensity of economies of traffic density, may explain that (complementary) airline alliances do not always emerge in a strategic environment. Specifically, alliances allow carriers to benefit from product complementarities, which together with the presence of increasing returns to traffic density would make one expect them to be privately profitable. Our findings indicate that this is not necessarily so and it seems consistent with some of the observed facts in the industry. This paper raises an interesting empirical question as it suggests to study the significance of the above mentioned factors in assessing the incentives of carriers to form strategic alliances. On the other hand, total travel volumes are higher when either a *single* or a *double alliance* arises as long as there is a minimum degree of economies of traffic density, regardless of market size and the degree of product differentiation. In the current framework, such an increase in travel volumes would account for a greater consumer surplus. Competition authorities should therefore favor complementary alliances since consumers would otherwise be worse off. Furthermore, our model has identified parameter conditions under which al-

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<sup>26</sup>For positive values of  $\theta$ , the relevant region  $R$  has a similar shape but it is bounded from above because positivity of marginal costs requires lower values of  $\alpha$  when  $\theta$  increases. In addition, the regions on the right side in Figure 5 reduce as  $\theta$  takes higher values since increases in the intensity of economies of traffic density favor alliance formation.

liances are privately profitable and, consequently, they would lead to higher welfare levels.

In addition to the proliferation of airline alliances, there are many other features that help to characterize the current and future status of the air transportation landscape such as the surge of low cost carriers, the closing of old hubs and the development of new ones, the expected evolution of regional operators to join networks and so on. Nevertheless, our results are realistic suggesting that the model captures some important aspects of the airline industry. An interesting question to be addressed is to endogenize network formation where the structure herein is a possibility. Further work should explore whether the presence of a low cost carrier in the network facilitates or hinders the profitability of strategic alliances.

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## Appendix 1: Definition of the relevant region $R$ .

A number of restrictions on the parameters  $d, \theta$  and  $\alpha$  have to be observed to ensure positive prices, quantities, marginal costs, margins and the compliance with non-arbitrage conditions are guaranteed in the three scenarios under consideration. Markets defined by a triple  $\{d, \theta, \alpha\} \in R$  guarantee comparable results.

- Bounds on  $\alpha$ . Positivity and non-arbitrage conditions in the three considered scenarios lead to several bounds in  $\alpha$ . After comparing all these bounds and selecting the most stringent ones, we obtain  $\alpha \in (\underline{\alpha}(d, \theta), \bar{\alpha}(d, \theta))$  with  $\bar{\alpha}(d, \theta) = \min(B1, B2)$  and  $\underline{\alpha}(d, \theta) = \max(B1, B3, 1)$  where  $B1 \equiv \frac{4(4-6\theta+d(2\theta-3+d(5\theta-2)))}{d(2\theta-3)(7\theta-2)+2(2+\theta-6\theta^2)+2d^2(2+\theta(10\theta-9))}$ ,

$$B2 \equiv \frac{4\theta((d-1)d-3)-2(d^2-5)}{\theta(10-12\theta+(d-4)d(2\theta-1))} \text{ and } B3 \equiv \frac{4d(3\theta-2+d(2\theta-1))}{4d(2+d)-10+27\theta-4d(7+5d)\theta+6\theta^2(4d(1+d)-3)}.$$

Specifically,  $B1$  comes from ensuring positive equilibrium travel volume in the interline trip for outsiders in the single alliance situation;  $B2$  from positive marginal cost for partners in the single alliance situation;  $B3$  from the fulfillment of a non-arbitrage condition for partners in the single alliance situation.

Notice that  $B1$  can be either a lower or an upper bound.

- An illustrative representation can be displayed in space  $(\theta, d)$  - see Figure 6. To this end, we can compute the bounds on  $\theta$  that come from the difference between  $\bar{\alpha}(d, \theta)$  and  $\underline{\alpha}(d, \theta)$ , i.e., the bounds ensuring the existence of a positive  $\alpha$  such that we can find markets  $\{d, \theta, \alpha\} \in R$ . We obtain  $\theta \in (0, \bar{\theta}(d))$  with

$$\bar{\theta}(d) = \begin{cases} L1 & \text{for } d < \frac{1}{2} \\ L3 & \text{for } d \in (\frac{1}{2}, 0.618] \\ L2 & \text{for } d > 0.618 \end{cases}$$

where  $L1 \equiv \frac{2+3d+2d^2}{2(2+3d+d^2)}$ ,  $L2 \equiv \frac{2d(2+d)-5}{2(d-3+2d^2)}$  and  $L3 \equiv \frac{4+d}{6+4d}$ . The case  $d = \frac{1}{2}$  is a particular case: there is a discontinuity and  $\alpha$  is bounded below by  $B1 = B3 = 1$

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<sup>27</sup>There are 20 bounds on  $\alpha$  to take into account. Let us denote them by  $B(\cdot)$ , putting in the argument the equilibrium condition that gives rise to the bound. The precise expressions can be derived from the equilibrium values provided in the main text. *Pre-alliance*:  $B(q^{na} > 0)$ ,  $B(Q^{na} > 0)$ ,  $B(1 - \theta(q^{na} + Q^{na}) > 0)$ , and  $B(p^{na} - 1 + \frac{\theta(q^{na} + Q^{na})}{2} > 0)$ ; *Single alliance*:  $B(q_p^a > 0)$ ,  $B(q_o^a > 0)$ ,  $B(Q_p^a > 0)$ ,  $B(Q_o^a > 0)$ ,  $B(1 - \theta(q_o^a + Q_o^a) > 0)$ ,  $B(1 - \theta(q_p^a + Q_p^a) > 0)$ ,  $B(p_o^a - 1 + \frac{\theta(q_o^a + Q_o^a)}{2} > 0)$ ,  $B(p_p^a - 2 + \theta(q_p^a + Q_p^a) > 0)$ ,  $B(p_p^a > 0)$ ,  $B(p_o^a > 0)$ ,  $B(P_p^a > 0)$ ,  $B(2p_p^a - P_p^a > 0)$  and  $B(P_p^a - p_p^a > 0)$ ; *Double alliance*:  $B(q^{aa} > 0)$ ,  $B(1 - \theta(q^{aa} + Q^{aa}) > 0)$ , and  $B(P^{aa} - 2 + \theta(q^{aa} + Q^{aa}) > 0)$ . One can observe that  $B(p_p^a - 1 + \frac{\theta(q_p^a + Q_p^a)}{2} > 0)$  and  $B(p^{aa} - 1 + \frac{\theta(q^{aa} + Q^{aa})}{2} > 0)$  simply reduce to  $\alpha > 1$ . After comparing all these bounds and selecting the most stringent ones, we are left with  $B1$ ,  $B2$  and  $B3$  where  $B1 \equiv B(Q_o^a > 0)$ ,  $B2 \equiv B(1 - \theta(q_p^a + Q_p^a) > 0)$  and finally  $B3 \equiv B(2p_p^a - P_p^a > 0)$ .

and above by  $B2 = \frac{38-52\theta}{47\theta-62\theta^2} > 1$ .

Figure 6 below represents  $L1$ ,  $L2$  and  $L3$ . We claim that, for any pair  $\{d, \theta\}$  in the region delimited by  $L1$ ,  $L2$  and  $L3$ , there exist values of  $\alpha \in (\underline{\alpha}(d, \theta), \bar{\alpha}(d, \theta))$  such that we can find markets  $\{d, \theta, \alpha\} \in R$ .

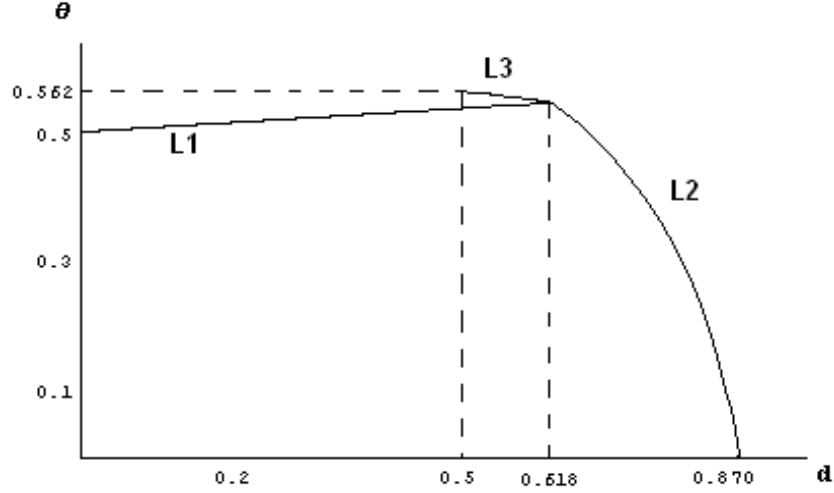


Figure 6: Bounds for  $d$  and  $\theta$  in region  $R$

More precisely,

- For  $d < \frac{1}{2}$  and  $\theta < L1$ , there exist values of  $\alpha \in (B1, B2)$  such that we can find markets  $\{d, \theta, \alpha\} \in R$ .
- For  $d \in (\frac{1}{2}, 0.618]$  and  $\theta \in [L1, L3)$ , there exist values of  $\alpha \in (B3, B1)$  such that we can find markets  $\{d, \theta, \alpha\} \in R$ .
- For  $d > \frac{1}{2}$  and  $\theta < \min(L1, L2)$ , there exist values of  $\alpha \in (B3, B2)$  such that we can find markets  $\{d, \theta, \alpha\} \in R$ .

In addition, we know that  $\bar{\theta} < \frac{2}{3}$  from the second order conditions. This means that economies of traffic density cannot be too high. This makes sense because otherwise marginal costs would become negative.



## Appendix 2: Proofs.

### Proof of Proposition 1.

The difference  $P_p^a - 2p^{na}$  yields an expression whose denominator is negative for  $\{d, \theta, \alpha\} \in R$ . The numerator is positive for  $\alpha > \alpha^* \equiv \frac{4(d\theta-1)}{4d+9\theta+12d\theta^2-6-14d\theta-6\theta^2}$ . We now compare  $\alpha^*$  with the corresponding lower bounds in  $R$ . Thus, for  $d < \frac{1}{2}$ , the difference  $B1 - \alpha^*$  is positive and, for  $d > \frac{1}{2}$ , the difference  $B3 - \alpha^*$  is positive too. Therefore,  $\alpha > \alpha^*$  is always verified in  $R$ . It is straightforward to check that  $\alpha > \alpha^*$  also implies  $Q_p^a > Q^{na}$ ,  $p_o^a < p^{na}$ ,  $Q_o^a < Q^{na}$  and  $q_o^a > q^{na}$ .

As for the fares and travel volumes for the partners' short markets, the difference  $q_p^a - q^{na}$  yields an expression whose denominator is negative for  $\{d, \theta, \alpha\} \in R$ . The sign of the numerator depends on whether market size  $\alpha$  is greater or smaller than  $\frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$ . The function  $\phi_1(d, \theta)$  is positive for  $\theta \in (\theta^-(d), \theta^+(d))$ , where  $\theta^+(d) > \frac{2}{3}$  and  $\theta^-(d) = \frac{3-11d-d^2+4d^3+\sqrt{9-6d-5d^2-6d^3+17d^4}}{4d(2d^2-3)}$ . The function  $\phi_2(d, \theta)$  is positive for values of  $\theta$  above  $\tilde{\theta}(d)$ , which is a decreasing function in  $d$ , it is discontinuous at  $d = \frac{1}{2}$  and it lies above  $\frac{2}{3}$  for  $d > \frac{1}{2}$ . When  $\theta < \tilde{\theta}(d)$  the numerator in  $q_p^a - q^{na}$  is positive; when  $\theta > \tilde{\theta}(d)$  the numerator in  $q_p^a - q^{na}$  is positive for  $\alpha < \frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$ . We have the following cases.

- Case  $d < \frac{1}{2}$ . For every  $\{d, \theta, \alpha\} \in R$ ,
  - i) for  $\theta < \tilde{\theta}(d)$  the numerator in  $q_p^a - q^{na}$  is positive and therefore  $q_p^a - q^{na} < 0$ .
  - ii) for  $\theta > \tilde{\theta}(d)$ ,  $\frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$  is positive and greater than  $B1$ . If  $\alpha < \frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$  the numerator in  $q_p^a - q^{na}$  is positive and hence  $q_p^a - q^{na} < 0$ ; if  $\alpha > \frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$ , then  $q_p^a - q^{na} > 0$ .
- Case  $d = \frac{1}{2}$ . For every  $\{d, \theta, \alpha\} \in R$ , the lower bounds on  $\alpha$  are  $B1 = B3 = 1$  and the upper bound is  $B2 = \frac{38-52\theta}{47\theta-62\theta^2} > 1$ . Since the numerator in  $q_p^a - q^{na}$  is negative for every  $\alpha < \frac{38-52\theta}{47\theta-62\theta^2}$ , which is always the case,  $q_p^a - q^{na}$  is positive.
- Case  $d > \frac{1}{2}$ . For every  $\{d, \theta, \alpha\} \in R$ ,
  - i) for  $\theta < \theta^-(d)$  the numerator in  $q_p^a - q^{na}$  is negative and therefore  $q_p^a - q^{na} > 0$ .
  - ii) for  $\theta \in (\theta^-(d), \theta^+(d))$ ,  $\frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$  is positive but smaller than 1. Therefore, for  $\alpha > \frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$ , the numerator in  $q_p^a - q^{na}$  is negative and  $q_p^a - q^{na} > 0$ .

The difference  $p_p^a - p^{na}$  follows exactly the opposite pattern.

### Proof of Proposition 2.

The difference  $P^{aa} - 2p_o^a$  yields an expression whose denominator is negative for  $\{d, \theta, \alpha\} \in R$ . The numerator is positive for  $\alpha > \alpha^*$ , as previously defined, and it follows straightforward that  $Q^{aa} > Q_o^a$ ,  $P^{aa} < P_p^a$ ,  $Q^{aa} < Q_p^a$ ,  $p^{aa} > p_p^a$  and  $q^{aa} < q_p^a$ .

As for the fares and travel volumes for the partners' short markets, the difference  $q^{aa} - q_o^a$  yields an expression whose denominator is negative for  $\{d, \theta, \alpha\} \in R$ . The sign of the numerator depends on whether market size  $\alpha$  is greater or smaller than  $\frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$ . The function  $\phi_1(d, \theta)$  is positive for  $\theta \in (\theta^-(d), \theta^+(d))$ , where  $\theta^+(d) > \frac{2}{3}$  and  $\theta^-(d) = \frac{6-23d+9d^3+\sqrt{36-84d+49d^2-52d^3+82d^4+d^6}}{4d(5d^2-6)}$ . The function  $\phi_2(d, \theta)$  is positive for values of  $\theta$  above  $\tilde{\theta}(d)$ , which is a decreasing function in  $d$ , it is discontinuous at  $d = \frac{1}{2}$  and it lies above  $\frac{2}{3}$  for  $d > \frac{1}{2}$ . When  $\theta < \tilde{\theta}(d)$  the numerator in  $q^{aa} - q_o^a$  is positive; when  $\theta > \tilde{\theta}(d)$  the numerator in  $q^{aa} - q_o^a$  is positive for  $\alpha < \frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$ . We have the following cases.

- Case  $d < \frac{1}{2}$ . For every  $\{d, \theta, \alpha\} \in R$ ,
  - i) for  $\theta < \tilde{\theta}(d)$  the numerator in  $q^{aa} - q_o^a$  is positive and therefore  $q^{aa} - q_o^a < 0$ .
  - ii) for  $\theta > \tilde{\theta}(d)$ ,  $\frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$  is positive and greater than  $B1$ . If  $\alpha < \frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$  the numerator in  $q^{aa} - q_o^a$  is positive and hence  $q^{aa} - q_o^a < 0$ ; if  $\alpha > \frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$  then  $q^{aa} - q_o^a > 0$ .
- Case  $d = \frac{1}{2}$ . For every  $\{d, \theta, \alpha\} \in R$ , the lower bounds on  $\alpha$  are  $B1 = B3 = 1$  and the upper bound is  $B2 = \frac{38-52\theta}{47\theta-62\theta^2} > 1$ . Since the numerator in  $q^{aa} - q_o^a$  is negative for every  $\alpha < \frac{38-52\theta}{47\theta-62\theta^2}$ , which is always the case, then  $q^{aa} - q_o^a$  is positive.
- Case  $d > \frac{1}{2}$ . For every  $\{d, \theta, \alpha\} \in R$ ,
  - i) for  $\theta < \theta^-(d)$  the numerator in  $q^{aa} - q_o^a$  is negative and therefore  $q^{aa} - q_o^a > 0$ .
  - ii) for  $\theta \in (\theta^-(d), \theta^+(d))$ ,  $\frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$  is positive but smaller than 1. Therefore, for  $\alpha > \frac{\phi_1(d, \theta)}{\phi_2(d, \theta)}$ , the numerator in  $q^{aa} - q_o^a$  is negative and  $q^{aa} - q_o^a > 0$ .

The difference  $p^{aa} - p_o^a$  follows exactly the opposite pattern.

### Proof of Lemma 1.

The denominator in  $\Psi^a(d, \theta, \alpha) = \frac{\pi^a}{2} - \pi^{na}$  is positive for any  $\{d, \theta, \alpha\} \in R$ . The numerator can be written as  $\alpha^2 K_1(d, \theta) + \alpha K_2(d, \theta) + K_3(d, \theta)$  where  $K_1(d, \theta)$  may be either positive or negative. Solving  $K_1(d, \theta) = 0$  for  $\theta$  yields several solutions, from which only one is relevant in  $R$ . Denote this root by  $\tilde{\theta}(d)$  which is increasing in  $d$ . For any  $\{d, \theta, \alpha\} \in R$ , if  $\theta > \tilde{\theta}(d)$ , the function  $K_1(d, \theta)$  is positive and the numerator in  $\Psi^a(d, \theta, \alpha)$  is a convex function in  $\alpha$ . On the other hand, if  $\theta < \tilde{\theta}(d)$ , the function  $K_1(d, \theta)$  is negative and the numerator in  $\Psi^a(d, \theta, \alpha)$  is a concave function in  $\alpha$ . Solving the numerator in  $\Psi^a(d, \theta, \alpha)$  for  $\alpha$  results in  $\alpha^-(d, \theta)$  and  $\alpha^+(d, \theta)$ . Thus, there are two constraints on  $\alpha$  to be met to have a positive numerator in  $\Psi^a(d, \theta, \alpha)$ :  $\alpha \notin (\alpha^-(d, \theta), \alpha^+(d, \theta))$  if  $K_1(d, \theta)$  is positive; and  $\alpha \in (\alpha^-(d, \theta), \alpha^+(d, \theta))$  if  $K_1(d, \theta)$  is negative.

- If  $K_1(d, \theta)$  is positive ( $\theta > \tilde{\theta}(d)$ ), the functions  $\alpha^-(d, \theta)$  and  $\alpha^+(d, \theta)$  are either non real or yield an interval outside region  $R$ . Hence if  $\alpha \notin (\alpha^-(d, \theta), \alpha^+(d, \theta))$  then the numerator in  $\Psi^a(d, \theta, \alpha)$  is positive and hence  $\Psi^a(d, \theta, \alpha) > 0$ .

One can check that  $d = 0.802$  when  $\tilde{\theta}(d) = 0$ . Consequently, since  $\tilde{\theta}(d)$  is increasing in  $d$ ,  $d < 0.802$  is sufficient to ensure  $\Psi^a(d, \theta, \alpha) > 0$ .

- If  $K_1(d, \theta)$  is negative ( $\theta < \tilde{\theta}(d)$ ), it is unclear whether  $\alpha$  belongs to  $(\alpha^-(d, \theta), \alpha^+(d, \theta))$ . Nevertheless, one can check that  $\Psi^a(d, \theta, \alpha)$  is decreasing in  $\alpha$  for  $d > 0.849$ . Therefore, we study  $\Psi^a(d, \theta, \alpha = \underline{\alpha} = B3)$  for  $d > 0.849$ . Solving  $\Psi^a(d, \theta, \underline{\alpha}) = 0$ , we obtain a function  $\hat{\theta}(d, \underline{\alpha})$  that is increasing in  $d$  as can be seen in Figure 7 below (since there is an upper bound for  $\theta$  in region  $R$ ,  $\bar{\theta}(d) \equiv L2$  following the notation in Appendix 1, we include it in the figure):

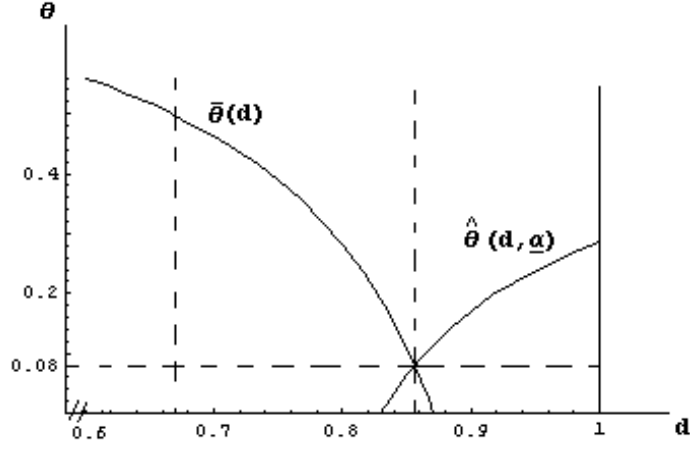


Figure 7 Proof of Lemma 1

For  $\theta > \hat{\theta}(d, \underline{\alpha})$ ,  $\Psi^a(d, \theta, \underline{\alpha}) > 0$  and then  $\Psi^a(d, \theta, \alpha) > 0$  for any  $\alpha$  in  $R$ . Since solving  $\hat{\theta}(d, \underline{\alpha}) = \bar{\theta}(d)$  yields  $\theta = 0.08$ , it is sufficient to require  $\theta > 0.08$  to guarantee  $\Psi^a(d, \theta, \alpha) > 0$  for any  $\{d, \theta, \alpha\} \in R$ .

The value  $d = 0.856$  is obtained by a numerical method when  $\Psi^a(d, \theta, \alpha = \bar{\alpha} = B2) > 0$  since for  $d > 0.849$  the function  $\Psi^a(d, \theta, \alpha)$  is decreasing in  $\alpha$ . Hence, for  $d > 0.856$ ,  $\Psi^a(d, \theta, \alpha = \bar{\alpha}) < 0$  and then  $\Psi^a(d, \theta, \alpha) < 0$  for any  $\{d, \theta, \alpha\} \in R$ .

### Proof of Lemma 2.

The first part of the proof is similar to Lemma 1. As for the sufficient conditions, for any  $\{d, \theta, \alpha\} \in R$ , one can check that  $\Psi^{aa}(d, \theta, \alpha) = \frac{\pi^{aa}}{2} - \pi_o^a$  is increasing in  $\alpha$  for low values of  $d$  in the interval  $d \in (0.707, 0.870]$  and decreasing in  $\alpha$  for high values of  $d$  in this interval. Solving  $\Psi^{aa}(d, \theta, \alpha = \underline{\alpha}) = 0$  and  $\Psi^{aa}(d, \theta, \alpha = \bar{\alpha}) = 0$  yields two functions,  $\hat{\theta}(d, \underline{\alpha})$  and  $\hat{\theta}(d, \bar{\alpha})$  that are increasing in  $d$  as can be seen in Figure 8 below.

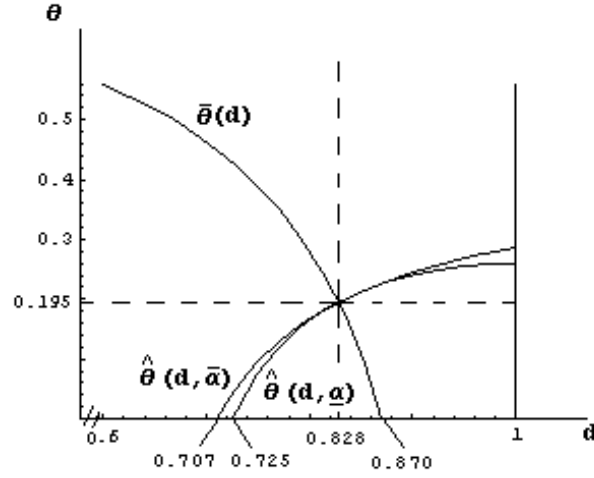


Figure 8: Proof of Lemma 2

Therefore for low values of  $d$  in the interval,  $\theta > \hat{\theta}(d, \bar{\alpha})$  implies  $\Psi^{aa}(d, \theta, \alpha = \bar{\alpha}) > 0$  and hence  $\Psi^{aa}(d, \theta, \alpha) > 0$  for any  $\alpha$  in  $R$ . Solving  $\hat{\theta}(d, \bar{\alpha}) = 0$  we obtain the value  $d = 0.707$ . Hence, for  $d < 0.707$ ,  $\theta > \hat{\theta}(d, \bar{\alpha})$ , we have that  $\Psi^{aa}(d, \theta, \alpha = \bar{\alpha}) > 0$  and then  $\Psi^{aa}(d, \theta, \alpha) > 0$ .

It happens to be the case that  $\hat{\theta}(d, \bar{\alpha}) = \hat{\theta}(d, \underline{\alpha}) = \bar{\theta}(d)$  at  $d = 0.828$  and  $\theta = 0.195$  and  $\Psi^{aa}(d, \theta, \alpha) = 0$  for any  $\alpha$  in  $R$ . Therefore, for  $\theta > 0.195$ , both  $\hat{\theta}(d, \bar{\alpha})$  and  $\hat{\theta}(d, \underline{\alpha})$  are positive, then both  $\Psi^{aa}(d, \theta, \alpha = \bar{\alpha})$  and  $\Psi^{aa}(d, \theta, \alpha = \underline{\alpha})$  are also positive, and hence  $\Psi^{aa}(d, \theta, \alpha) > 0$ . Similarly, for  $d > 0.828$  both  $\hat{\theta}(d, \bar{\alpha})$  and  $\hat{\theta}(d, \underline{\alpha})$  are negative, then both  $\Psi^{aa}(d, \theta, \bar{\alpha})$  and  $\Psi^{aa}(d, \theta, \underline{\alpha})$  are also negative, and hence  $\Psi^{aa}(d, \theta, \alpha) < 0$ .

### Proof of Corollary 1.

One can check that  $\Psi^{pd}(d, \theta, \alpha)$  is decreasing in  $\alpha$  for any  $\{d, \theta, \alpha\} \in R$ . Therefore we study  $\Psi^{pd}(d, \theta, \alpha = \bar{\alpha})$ . Solving  $\Psi^{pd}(d, \theta, \bar{\alpha}) = 0$ , we obtain a function  $\tilde{\theta}(d, \bar{\alpha})$  that is decreasing in  $d$  as can be seen in the figure below.

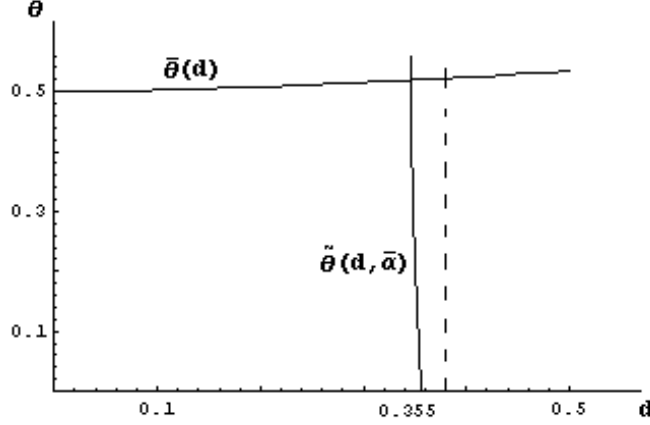


Figure 9: Proof of Corollary 1

For  $\theta < \tilde{\theta}(d, \bar{\alpha})$ ,  $\Psi^{pd}(d, \theta, \bar{\alpha}) < 0$  and then  $\Psi^{pd}(d, \theta, \alpha) < 0$  for any  $\alpha$  in  $R$ . Since solving  $\tilde{\theta}(d, \bar{\alpha}) = 0$  yields  $d = 0.355$ , it is sufficient to require  $d < 0.355$  to guarantee  $\Psi^{pd}(d, \theta, \bar{\alpha}) < 0$  for any  $\{d, \theta, \alpha\} \in R$ .

### Proof of Proposition 4 and Corollary 2.

For any market  $\{d, \theta = 0, \alpha\} \in R$ ,  $\alpha > \underline{\alpha}(d)$  and  $d \in (0, 1)$ , where  $\underline{\alpha}(d) = B1$  for  $d < \frac{1}{2}$  and  $\underline{\alpha}(d) = B3$  for  $d > \frac{1}{2}$ .

1. We first analyze  $\Psi^a(d, \theta = 0, \alpha) = \frac{\pi^a}{2} - \pi^{na}$  and observe that:

- (a) For  $d \in (0, 0.8017]$ ,  $\Psi^a(d, \theta = 0, \alpha) > 0$ .
- (b) For  $d \in (0.8017, 0.829]$ , the sign of  $\Psi^a(d, \theta = 0, \alpha)$  depends on the value of  $\alpha$  in the following way:  $\Psi^a(d, \theta = 0, \alpha) > 0$  for  $\alpha < \alpha_1(d)$ , and  $\Psi^a(d, \theta = 0, \alpha) < 0$  for  $\alpha > \alpha_1(d)$ ,

$$\text{where } \alpha_1(d) = \frac{100 - \sqrt{10} \sqrt{((1-2d)^2(5-2d)^2(d^2-5)^2(8d^2-5)) + 2d(2d-5)(40d-10-5d^3+2d^4)}}{275+2d(d(20+d(120+d(-15+4(-5+d)d)))-250)}.$$

1. (a) For  $d \in (0.829, 870]$ ,  $\Psi^a(d, \theta = 0, \alpha) < 0$ .

2. The analysis of  $\Psi^{aa}(d, \theta = 0, \alpha) = \frac{\pi^{aa}}{2} - \pi_o^a$  yields that,

- (a) For  $d \in (0, 0.707]$ ,  $\Psi^{aa}(d, \theta = 0, \alpha) > 0$ .
- (b) For  $d \in (0.707, 0.7105]$ , the sign of  $\Psi^{aa}(d, \theta = 0, \alpha)$  depends on the value of  $\alpha$  in the following way:  $\Psi^{aa}(d, \theta = 0, \alpha) > 0$  for  $\alpha \notin (\alpha_2(d), \alpha_3(d))$ , and  $\Psi^{aa}(d, \theta = 0, \alpha) < 0$  for  $\alpha \in (\alpha_2(d), \alpha_3(d))$ , where

$$\alpha_2(d) = \frac{8+8d-39d^2+20d^3+2d^4-4d^5+d^6-\sqrt{2}\sqrt{(500d-100-585d^2-700d^3+1796d^4-780d^5-445d^6+380d^7-18d^8-40d^9+8d^{10})}}{22-36d-11d^2+32d^3-6d^4-4d^5+d^6}$$

and  $\alpha_3(d) = \frac{8+8d-39d^2+20d^3+2d^4-4d^5+d^6+\sqrt{2}\sqrt{(500d-100-585d^2-700d^3+1796d^4-780d^5-445d^6+380d^7-18d^8-40d^9+8d^{10})}}{22-36d-11d^2+32d^3-6d^4-4d^5+d^6}$

1. (a) For  $d \in (0.7105, 0.7249]$ , the sign of  $\Psi^{aa}(d, \theta = 0, \alpha)$  depends on the value of  $\alpha$  in the following way:  $\Psi^{aa}(d, \theta = 0, \alpha) > 0$  for  $\alpha > \alpha_3(d)$ , and  $\Psi^{aa}(d, \theta = 0, \alpha) < 0$  for  $\alpha < \alpha_3(d)$ . For these values of  $d$ , we do not need to consider  $\alpha_2(d)$  since  $\underline{\alpha}(d) > \alpha_2(d)$  and  $\alpha$  is always higher than  $\underline{\alpha}(d)$  in  $R$ .

- (b) For  $d \in (0.7249, 0.870]$ ,  $\Psi^{aa}(d, \theta = 0, \alpha) < 0$ .

2. Next we study the sign of  $\Psi^{pd}(d, \theta = 0, \alpha) = \pi^{na} - \frac{\pi^{aa}}{2}$ .

- (a) For  $d \in (0, 0.355]$ ,  $\Psi^{pd}(d, \theta = 0, \alpha) < 0$ .

- (b) For  $d \in (0.355, 0.5]$ , the sign of  $\Psi^{pd}(d, \theta = 0, \alpha)$  depends on the value of  $\alpha$  in the following way:  $\Psi^{pd}(d, \theta = 0, \alpha) > 0$  for  $\alpha > \alpha_4(d)$ , and  $\Psi^{pd}(d, \theta = 0, \alpha) < 0$  for  $\alpha < \alpha_4(d)$ ,

where  $\alpha_4(d) = \frac{8-40d+41d^2-20d^3+4d^4+\sqrt{2}\sqrt{980d-100-3561d^2+6204d^3-5592d^4+2688d^5-656d^6+64d^7}}{22-92d+97d^2-36d^3+4d^2}$ .

- (a) For  $d \in (0.5, 0.725]$ ,  $\Psi^{pd}(d, \theta = 0, \alpha) > 0$ .

The combination of the above analysis characterizes the Nash equilibrium (NE) in alliance formation:

1. (a) For  $d \in (0.829, 0.870]$ , the NE is *no alliance* because  $\Psi^a(d, \theta, \alpha) < 0$ .
- (b) For  $d \in (0.817, 0.829]$ , with  $\alpha < \alpha_1(d)$ , the NE is *single alliance* because  $\Psi^a(d, \theta = 0, \alpha) > 0$  and  $\Psi^{aa}(d, \theta = 0, \alpha) < 0$ ; and with  $\alpha > \alpha_1(d)$ , the NE is *no alliance* because  $\Psi^a(d, \theta = 0, \alpha) < 0$ .
- (c) For  $d \in (0.7249, 0.817]$ , the NE is *single alliance* because  $\Psi^a(d, \theta = 0, \alpha) > 0$  and  $\Psi^{aa}(d, \theta = 0, \alpha) < 0$ .
- (d) For  $d \in (0.7105, 0.7249]$ , with  $\alpha > \alpha_3(d)$ , the NE is *double alliance* because  $\Psi^a(d, \theta = 0, \alpha) > 0$  and  $\Psi^{aa}(d, \theta = 0, \alpha) > 0$ ; and with  $\alpha < \alpha_3(d)$ , the NE is *single alliance* because  $\Psi^a(d, \theta = 0, \alpha) > 0$  and  $\Psi^{aa}(d, \theta = 0, \alpha) < 0$ .

- (e) For  $d \in (0.707, 0.7105]$ , with  $\alpha \notin (\alpha_2(d), \alpha_3(d))$ , the NE is *double alliance* involving a prisoner's dilemma because  $\Psi^a(d, \theta = 0, \alpha) > 0$ ,  $\Psi^{aa}(d, \theta = 0, \alpha) > 0$  and  $\Psi^{pd}(d, \theta = 0, \alpha) > 0$ ; and with  $\alpha \in (\alpha_2(d), \alpha_3(d))$ , the NE is *single alliance* because  $\Psi^a(d, \theta = 0, \alpha) > 0$  and  $\Psi^{aa}(d, \theta = 0, \alpha) < 0$ .
- (f) For  $d \in (0.5, 0.707]$ , the NE is *double alliance* involving a prisoner's dilemma because  $\Psi^a(d, \theta = 0, \alpha) > 0$ ,  $\Psi^{aa}(d, \theta = 0, \alpha) > 0$  and  $\Psi^{pd}(d, \theta = 0, \alpha) > 0$ .
- (g) For  $d \in (0.355, 0.5]$ , with  $\alpha < \alpha_4(d)$  the NE is *double alliance* profitable per se because  $\Psi^a(d, \theta = 0, \alpha) > 0$ ,  $\Psi^{aa}(d, \theta = 0, \alpha) > 0$  and  $\Psi^{pd}(d, \theta = 0, \alpha) < 0$ ; and with  $\alpha > \alpha_4(d)$  the NE is *double alliance* involving a prisoner's dilemma because  $\Psi^a(d, \theta = 0, \alpha) > 0$ ,  $\Psi^{aa}(d, \theta = 0, \alpha) > 0$  and  $\Psi^{pd}(d, \theta = 0, \alpha) > 0$ .
- (h) For  $d \in (0, 0.355]$  the NE is *double alliance* profitable per se because  $\Psi^a(d, \theta = 0, \alpha) > 0$ ,  $\Psi^{aa}(d, \theta = 0, \alpha) > 0$  and  $\Psi^{pd}(d, \theta = 0, \alpha) < 0$ .