

A discusión

ON PROCUREMENT AUCTIONS OF COMPLEMENTARY GOODS*

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ABSTRACT

We compare sequential and bundle auctions in a framework of successive procurement situations, where current success positively affects future market opportunities and competition varies across projects. We find that - if allocation of the projects has to be ensured – bundle auctions with subsequent resale opportunities yield a lower and less risky overall price, whereas sequential procurement leads to the efficient outcome.

Keywords: Sequential auctions, bundling, stochastic scale effects, procurement.

JEL Classification: D44, H57, D92

1 Introduction

Every year governments and private sector firms place numerous orders by running procurement auctions. For instance, governments of Western nations procure about 10 % of national product annually.¹ For the private sector, Burt, Norquist, and Anklesaria (1990) report that in the United States the fraction of purchased inputs increased from 20 % to 56 % of the selling price of finished goods during the last 50 years. A substantial fraction of this procurement takes place by competitive bidding.

Usually, over time firms participate in more than one procurement auction. In this case, a firm's cost of providing one object is quite likely not independent of the allocation of other production rights. Rather, one often observes that the production right for one object guarantees the firm a comparative cost advantage in a follow-up procurement situation. This may be due to scale effects or due to technology improvements from earlier production processes. Even if a firm does not yet know its exact production cost for a possible follow-up order (for example there may be a considerable time lag between two successive procurement situations and therefore, the project is not yet well defined, or due to other uncertainties), success in a current procurement auction may increase the expected profit from follow-up orders due to an *expected* comparative cost advantage. Rational firms will account for this in their price offers.

In procurement auctions, the usual procedure is the following: First, firms have to apply for participation. At this stage, they have to prove their ability to carry out the project in case they are awarded the production right. One observes that competition for production rights strongly varies across different auctions, that is, some of the objects can be provided by only few firms, others by many (for example, only a few firms could probably build a certain facility, but some more could service the facility once it was built). The auction itself often is a process of underbidding each other's price

¹Cf. McAfee and McMillan (1987).

offers until only one firm is left, which is then awarded the production right. Alternatively, firms submit sealed bids and the bidder who submitted the lowest bid is chosen.

Now, if there is an obvious relation between two projects that have to be carried out successively (for example, building a facility and the service contract for this facility), the government may procure the two contracts sequentially, or as a bundle. Both alternatives have some advantage, although for different reasons: In a sequential auction each item can be procured at a point in time, where firms know their costs exactly (e. g. the project is well defined) and production has to take place. Moreover, in each auction all firms that are qualified to deliver the single object can participate. If a bundle auction is chosen, firms that can only provide one of the items are necessarily excluded. However, recall that the multiproduct firms have a comparative cost advantage for the second item if they are awarded the production right for the first item. Therefore, bids are presumably more aggressive since firms bidding for the bundle base their offers on much lower expected production costs for the second item.²

Furthermore, note that if the buyer chooses to bundle the objects, sale takes place at a point in time where firms are still uncertain with respect to their costs for the second item. Therefore, the second production right may be resold at a later point in time, when the project is adequately defined and costs have been observed by all qualified firms. At this stage, also firms that can only provide the second item (but not the first one, which is why they were excluded from the bundle auction) can participate in the competition for the subcontract. As a matter of fact, subcontracting is a common phenomenon in many procurement situations where contracts oblige the successful firm to provide a variety of goods and services.³ It is often not clear,

²Also, the literature on optimal auctions suggests that bundling may increase the auctioneer's revenue, even in absence of synergies. Cf. Palfrey (1983), Armstrong (2000), and Avery and Hendershott (2000).

³Kamien (1989), as well as Gale, Hausch, and Stegeman (2000) give a variety of examples for horizontal subcontracting.

how the government could prevent subcontracts among the bidders, if further gains from trade are possible. Moreover, the government may not even want to prevent resale, since further expected gains from subcontracting should make the offers in the bundle auction more competitive.

The model analyzed in this chapter is a stylized version of the procurement setting described above. We have two complementary items, where costs for the first item are observed at an earlier point in time (at stage one) than costs for the second item (at stage two). Several multiproduct firms can provide both items, while additional singleproduct firms can only provide the second item. The first (second) item has to be allocated at stage one (two), and production has to take place immediately. However, prior to production, each item can be reallocated among the firms. This implies that the allocation of the first item is irreversible at stage two.

In this framework we compare a sequential and a bundle auction. In order to make the analysis tractable, we assume that each single auction is second-price sealed-bid without reserve price.⁴ The auctioneer's choice not to use a reserve price lower than his valuation for the contract can be motivated by several considerations: Assume that it is common knowledge that the buyer's valuation is higher than cost of any bidder and he cannot carry out production on his own. Then, the threat not to buy if bids are "too high" but still below the buyer's valuation is not credible and bidders will anticipate that there will be a further request to bid in case no transaction took place. Moreover, if the buyer is a public institution, the predominant concern should be that production is carried out if it is efficient, i. e. if production costs fall short of the value attached to the project. Therefore, rejecting prices lower than this value cannot be desirable.⁵

Comparison of the two auction formats yields the following results: It

⁴This is also done by other authors who analyze sequential procurement auctions. See, for example Gale, Hausch, and Stegeman (2000).

⁵In particular with respect to multiple unit auctions, there is a considerable literature looking at efficient auction formats. See the discussion in Krishna (2002), chapters 13 and 14.

turns out that only after the bundle auction there are possible gains from trade at the second stage since the initial allocation of the second production right might turn out to be inefficient once the bidders have observed production costs for the second item at stage two. We find that, while both auctions ensure production of both items, the bundle auction yields a lower overall expected price. This is due to the fact that the winner of the bundle auction has the freedom either to subcontract production of the second item or to carry out production on his own. Moreover, the government can almost never lower the expected price in the bundle auction by preventing subcontracts. Finally, we find a trade off between revenue and efficiency: whereas the bundle auction yields a higher revenue but an inefficient allocation with positive probability, the sequential auction always implements the efficient allocation.

Finally note that our framework deviates from the assumptions commonly made in auction theory in two important ways. First, in most of the models on multi-object auctions, bidders know their individual costs (or valuations) for all objects when the auction takes place. Second, usually the models do not allow for trade after the auction. Still, there are quite a few papers that are related to the topics analyzed here:

Von der Fehr and Riis (1998) and Jeitschko and Wolfstetter (2002) analyze sequential auctions with the same timing of information revelation. Von der Fehr and Riis study how future market opportunities affect the bidding behavior (and thereby the equilibrium price sequence) in sequential second-price auctions, while Jeitschko and Wolfstetter compare sequential first- and second-price auctions in the presence of stochastic economies or diseconomies of scale.

A seminal contribution to the question of bundling versus separate sales is Palfrey (1983). He finds that, even without synergies, a monopolist often prefers bundling as compared to selling the objects in independent auctions. There are also a number of recent theoretical analyses of multi unit auctions. Armstrong (2000) and Avery and Hendershott (2000) derive properties of the

optimal multi unit auction when types are multidimensional.⁶ Both find that the optimal auction favors bundling in a probabilistic sense: a high bid on one product increases the probability of winning another product. However, additional competition for a product reduces the profitability of all bundles including this product for the auctioneer. Levin (1997) and Branco (1997) characterize the optimal multi unit auction in case of synergies. Both authors make the problem tractable by reducing it to a onedimensional mechanism design problem (i. e. each bidder observes only one private signal that determines his valuation for each single object, and for the bundle). Levin considers a model with only multiproduct bidders who may also submit bids on single objects, whereas Branco also considers singleproduct bidders, however, multiproduct bidders are not allowed to bid for single units.

Frequently used auction rules are analyzed and compared by a variety of papers. Mostly they assume one-dimensional types and model synergies by addition of a positive constant.⁷ Menezes and Monteiro (1999) find that in a model with only multiproduct bidders with superadditive valuations a sequential auction and a bundle auction are revenue equivalent.

Only few papers consider the possibility of trade after the auction. Gale, Hausch, and Stegeman (2000) analyze sequential procurement auctions where resale is profitable due to convex cost.⁸ Haile (1999) and Gupta and Lebrun (1998) analyze resale which is due to an inefficient outcome of the initial auction. In Haile this results from noisy signals at the time of the initial auction, in Gupta and Lebrun the initially inefficient allocation is due to asymmetries between bidders.

The paper is organized as follows: In section 2 we state the model. In section 3 we give a description of the different auction games and derive equilibrium bidding strategies and prices in the different auctions. In section

⁶Armstrong assumes only multiproduct bidders, whereas Avery and Hendershott consider one multiproduct bidder competing with several singleproduct bidders.

⁷This model goes back to Krishna and Rosenthal (1996) and was employed, e. g. by Branco (1997a) and Albano, Germano, and Lovo (1999).

⁸This is also the reason for subcontracting in Kamien, Li, and Samet (1989).

4, we compare the sequential and the bundle auction (with resale) and derive a condition under which, after the bundle auction, the auctioneer has no incentive to prevent resale. Section 5 concludes.

2 The Model

Consider a procurement auction of two complementary goods. The (private) costs of providing the two goods are observed successively by the firms, i. e. the firms observe their costs for the first item at an earlier point in time than their costs for the second item. The market for the second item is more competitive. In particular, m multiproduct firms can provide both objects, while $n - m$ additional singleproduct firms can only provide the second object.

Let $Q = (Q_1, \dots, Q_m)$ denote the multiproduct firms' costs for providing the first item. Costs Q_i , $i = 1, \dots, m$, are distributed on the interval $[Q, \bar{Q}]$ with c.d.f. G_i and density g_i . A multiproduct firm that provides the first item has a (stochastic) comparative cost advantage for the second item. We call this firm "incumbent" (I), while the multiproduct firms that do not provide the first item are called "contestants" (C).

All n firms can provide the second item, however, at stochastically different costs. The vector of production costs for the second item is a random vector $X = (X_1, X_2, \dots, X_m, \dots, X_n) \in [0, 1]^n$. We order the components of X such that X_1 denotes the incumbent's, X_2, \dots, X_m the contestants', and X_{m+1}, \dots, X_n the singleproduct bidders' cost for the second item. Costs X_i , $i = 1, \dots, n$, are distributed according to c.d.f. F_i with density f_i . We assume that all contestants are symmetric with respect to their cost for the second item, i. e. the random variables X_2, \dots, X_m follow the same distribution as a reference variable X_C that is distributed according to c.d.f. F_C with density f_C . Furthermore we model the incumbent's comparative cost advantage by assuming that X_1 is lower than X_C in the sense of first order stochastic dominance. Finally, we assume that the components of (Q, X)

are independent. We denote by $Q_{(j)}$ and $X_{(j)}$ the j th order statistic of the random variables in Q and X , respectively, where we order from lowest to highest cost. Furthermore, $X_{(j)}^{\{-1\}}$ and $X_{(j)}^{\{-C\}}$ denote the j th order statistic of all random variables in X except for X_1 (the incumbent's cost) and one representative contestant's cost, respectively.

The government's valuation for the two contracts is assumed to be higher than any firm's cost of providing the two items, so that trade is always desirable. Let us further assume that the government cannot credibly commit not to trade at all and therefore cannot set a reservation price lower than its valuation. This may be the case if it is common knowledge that the government's valuation always exceeds the bidder's costs (i. e. the two items are necessary inputs for a certain project).

In the following, we focus on mechanisms most commonly used in practice, that is, the government either offers both contracts together as a bundle, or sequentially, one after the other. In order to simplify the analysis we assume that for every single transaction the government uses a second-price auction without reserve price.⁹

Throughout the paper we will consider two successive periods. At the beginning of the first period the multiproduct bidders privately observe their cost of providing the first item. Production of the first item has to take place at the end of period one. Then, at the beginning of period two all bidders privately observe their costs of providing the second item. Production of the second item has to take place at the end of the second period.

⁹Jeitschko and Wolfstetter (2002) actually find that with economies of scale the auctioneer is better off selling the second unit second-price and, given he does so, is indifferent between selling the first unit first- or second-price.

3 Equilibria

3.1 The Bundle Auction

If the auctioneer bundles the two objects the auction has to take place in period one, before production of the first item is due. At this point in time, firms know their individual cost for the first, but not for the second item. Since bidders have to prove their ability to carry out every single project that is auctioned off, only the multiproduct firms can participate in the bundle auction. However, in the second period the incumbent's production cost for the second item may turn out to be higher than the cost observed by one or more competitors i , $i = 2, \dots, n$. Therefore, there are potentially positive gains from trade of the second item at stage two. We assume that the incumbent has the power to choose a sales mechanism at stage two, which we will denote by Γ .

3.1.1 The Bundle Auction without Subcontracting

As a first step, we analyze the bundle auction if resale after the auction is not possible. Each multiproduct firm i knows its cost q_i , whereas its cost for the second item is given by the random variable X_1 , since any firm can either win both items, or nothing. Thus, expected total cost of firm i is $q_i + E[X_1]$. In a second price auction bidding truthfully is a weakly dominant strategy. Therefore, the firm with lowest cost for the first item wins the auction and is paid the second lowest bid. We obtain the following

PROPOSITION 1 *In the bundle auction without subcontracting the expected price is*

$$E[P_B] = E[Q_{(2)}] + E[X_1]. \quad (1)$$

Note that *expected* cost for the second item are the same for all firms and thus, the firm with lowest cost for the first item places the lowest bid. Therefore the auction always allocates the first item efficiently.

3.1.2 The Bundle Auction with Subcontracting

If trade after the bundle auction is possible, multiproduct bidders face a positive expected profit from resale, independently of winning or losing in the bundle auction. Since any incumbent faces the same situation at the subcontracting stage¹⁰ we assume that the choice of the resale mechanism Γ does not depend on the outcome of stage one. Hence, for any multiproduct bidder winning the bundle auction gives rise to the same additional profit $\Pi_S^I(\Gamma)$ while losing the bundle auction yields a profit $\Pi_S^C(\Gamma)$ from stage two. Assuming that the second auction yields the above profits as incumbent and contestant, respectively, it is a weakly dominant strategy to bid the perceived cost of providing both items, which differs from a firm's production cost due to the additional expected profits from the subcontracting stage. Bidder i 's expected cost of providing both items are $q_i + E[X_1]$. The expected profit from the subcontracting stage as incumbent, $E[\Pi_S^I(\Gamma)]$, lowers the expected perceived cost of bidder i (i. e. makes winning the bundle auction more valuable). However, losing the auction does not yield zero profits, since as contestant a multiproduct bidder still faces positive expected profits $E[\Pi_S^C(\Gamma)]$ from the subcontracting stage, which makes him less eager to win the bundle auction. Thus, perceived cost of providing both items are $q_i + E[X_1] - E[\Pi_S^I(\Gamma)] + E[\Pi_S^C(\Gamma)]$. At a bid equal to his perceived cost bidder i is just indifferent between winning or losing the bundle auction. This yields the following

PROPOSITION 2 *Assume that at the subcontracting stage the incumbent chooses a mechanism Γ . Then, the expected price in the bundle auction is*

$$E[P_{BS}] = E[Q_{(2)}] + E[X_1] - E[\Pi_S^I(\Gamma)] + E[\Pi_S^C(\Gamma)]. \quad (2)$$

¹⁰Independently of who wins the bundle auction, the continuation game is always played by an incumbent with cost X_1 who chooses the sales mechanism and $n - 1$ bidders with costs X_2, \dots, X_n . Since $Q_1, \dots, Q_m, X_1, \dots, X_n$ are independent, Bayesian updating implies that the bidders do not change their beliefs concerning the distributions of X_1, \dots, X_n , which we assume to be equivalent to the ex ante objective probability distributions.

3.2 The Sequential Auction

3.2.1 Equilibrium Prices

If the two items are auctioned off sequentially, every bidder who is qualified to provide a single object can participate in the respective auction. The auction of the first (second) item takes place in period one (two), after the firms have observed their private cost of providing the item that is auctioned off.

Let us first consider the second auction, which is a single-unit second-price auction without reserve price. Each bidder i , $i = 1, \dots, n$, has observed his cost x_i prior to the auction. In this auction it is a weakly dominant strategy for each bidder to bid his true cost of providing the second item. Therefore, the expected price in the second auction is equal to the expected value of the second highest cost, that is

$$E[P_2] = E[X_{(2)}]. \quad (3)$$

Now we consider the first auction. A multiproduct bidder who wins the first auction has an additional profit, Π_2^I , from the second auction, whenever his cost for the second item is lowest. Thus, in the first period the incumbent's expected profit from the second auction is given by¹¹

$$E[\Pi_2^I] = E[X_{(1)}^{\{-1\}} - X_1; X_1 \leq X_{(1)}^{\{-1\}}]. \quad (4)$$

However, a multiproduct bidder who did not win the first auction (a contestant) also faces a positive profit from the second auction, Π_2^C . In the first period a contestant's expected profit from the second auction is given by

$$E[\Pi_2^C] = E[X_{(1)}^{\{-C\}} - X_C; X_C \leq X_{(1)}^{\{-C\}}]. \quad (5)$$

In appendix A we show that the expected profit from the second auction is higher as incumbent than as contestant, i. e. $E[\Pi_2^I] > E[\Pi_2^C]$, since X_C first order stochastically dominates X_1 . Thus, a bidder i who wins the first

¹¹In order to simplify notation we define $E[V; A] = E[V|A]Prob[A]$.

auction incurs cost q_i but also "wins" an additional expected profit from the second auction, $E[\Pi_2^I] - E[\Pi_2^C]$. Therefore, perceived cost of winning the production right for the first item, $q_i - (E[\Pi_2^I] - E[\Pi_2^C])$, falls short of the real cost of providing the first item, q_i . Iterated elimination of weakly dominated strategies yields that bidders in the first auction bid their perceived cost of winning the first item, that is $b_i = q_i - (E[\Pi_2^I] - E[\Pi_2^C])$.¹² Therefore, the expected price in the sequential auction is given by

$$\begin{aligned} E[P_{SEQ}] &= E[Q_{(2)}] - E[\Pi_2^I] + E[\Pi_2^C] + E[P_2] \\ &= E[Q_{(2)}] - E[X_{(1)}^{\{-1\}} - X_1; X_1 \leq X_{(1)}^{\{-1\}}] \\ &\quad + E[X_{(1)}^{\{-C\}} - X_C; X_C \leq X_{(1)}^{\{-C\}}] + E[X_{(2)}] \end{aligned} \quad (6)$$

Using the decomposition

$$\begin{aligned} E[X_{(2)}] &= E[X_{(1)}^{\{-1\}}; X_1 \leq X_{(1)}^{\{-1\}}] + E[X_1; X_{(1)}^{\{-1\}} \leq X_1 \leq X_{(2)}^{\{-1\}}] \\ &\quad + E[X_{(2)}^{\{-1\}}; X_{(2)}^{\{-1\}} \leq X_1] \end{aligned}$$

allows us to rearrange such that we get

PROPOSITION 3 *The expected overall price in the sequential auction is given by*

$$\begin{aligned} E[P_{SEQ}] &= E[Q_{(2)}] + E[X_1] - E[X_1 - X_{(2)}^{\{-1\}}; X_{(2)}^{\{-1\}} \leq X_1] \\ &\quad + E[X_{(1)}^{\{-C\}} - X_C; X_C \leq X_{(1)}^{\{-C\}}] \end{aligned} \quad (7)$$

3.2.2 Comparative Statics

We observe two contrary effects. On the one hand, loosing the first auction does not yield zero profits. Rather, the unsuccessful multiproduct bidders

¹²Obviously, the first auction bid falls short of the cost of providing the first item if the value of incumbency, $E[\Pi_2^I] - E[\Pi_2^C]$, is positive, which is true in the case of stochastic scale effects. From appendix A it follows immediately that the reverse is true if X_1 first order stochastically dominates X_2 (stochastic diseconomies). These findings mirror the findings of von der Fehr and Riis (1998).

have a second chance to win only the second item at stage 2. A contestant's expected profit from the second auction, which we call the "looser's option value"¹³,

$$LOV = E \left[X_{(1)}^{\{-C\}} - X_C; X_C \leq X_{(1)}^{\{-C\}} \right], \quad (8)$$

makes the multiproduct bidders less eager to win the first auction, and thereby increases the overall price to be paid by the auctioneer in the sequential auction.

On the other hand, intense competition at stage two tends to lower the overall price in the sequential auction [compare the third term in equation (7)]. In the following we refer to this effect as the *value of competition*,

$$VC = E \left[X_1 - X_{(2)}^{\{-1\}}; X_{(2)}^{\{-1\}} \leq X_1 \right]. \quad (9)$$

In order to provide some further intuition for this effect, consider the following decomposition of the price in the sequential auction: Let $E[Q_{(2)}]$ be the amount to be paid for provision of the first item and $LOV = E[\Pi_2^C]$ the cost of conducting a sequential auction. Then

$$E[X_1] - VC = E \left[X_1 \mathbf{1}_{\{X_1 \leq X_{(2)}^{\{-1\}}\}} + X_{(2)}^{\{-1\}} \mathbf{1}_{\{X_{(2)}^{\{-1\}} \leq X_1\}} \right] \quad (10)$$

is the price to be paid for provision of the second item. From this it becomes clear, that the price that has to be paid for the second object is determined by a competitor's cost whenever the incumbent's cost realization at stage 2 is too high. This effect on the price is due to the anticipation of additional payments by the multiproduct bidders that are "competed away" at the first stage of the game.

PROPOSITION 4 [*Comparative Statics*]

(i) *If the number of bidders increases, VC increases and LOV decreases. Therefore, $E[P_{SEQ}]$ decreases.*

(iv) *For every given distributions of X_1 and X_C the order of VC and LOV depends on the intensity of competition at stage two.*

¹³This terminology is due to Von der Fehr and Riis (1998).

PROOF See appendix B. □

We find that fiercer competition in the sense that the number of bidders increases leads to a higher value of additional competition, VC , and, at the same time, lowers the contestants' expected profit from the second auction $E[\Pi_2^C] = LOV$. Clearly, $E[P_{SEQ}]$, as given by proposition 3, decreases. As it turns out (part (ii) of proposition 4), the order of the two effects predominantly depends on the intensity of competition at stage two. That is, for any two random variables X_1 and X_C , by varying the number of bidders and the singleproduct bidders' distributions of cost, one can construct both, scenarios where $VC > LOV$ and scenarios where $VC < LOV$. Note that it follows from propositions 1 and 3 that $VC > LOV$ implies a lower overall price in the sequential auction than in the bundle auction without resale.

REMARK 1 *Lower cost of the incumbent in the sense of FSD lower both, VC and LOV and therefore does not necessarily affect the order of the two effects. However, in the sequential auction a FSD decrease of X_1 raises the value of incumbency $E[\Pi_2^I - \Pi_2^C]$ (which lowers the equilibrium bids in the first auction). Therefore, if an FSD decrease of X_1 lowers $E[X_{(2)}] = E[P_2]$ the overall price $E[P_{SEQ}]$ decreases. We elaborate on this in more detail in appendix C.*

4 Comparison

4.1 Bundling versus Sequential Sales

Now we are in the position to compare the sequential auction and the bundle auction with subcontracting. As a benchmark, we first consider the case that, at the subcontracting stage after the bundle auction, the incumbent uses a second-price auction with a reserve price equal to his observed cost x_1 , which we denote by Γ^2 (proposition 5). Then we will use these results in order to compare the sequential auction and the bundle auction with revenue maximizing subcontracting (theorem 1).

4.1.1 The Benchmark Case

Let us begin by an analysis of subcontracting after the bundle auction by a second-price auction with a reserve price equal to the incumbent's observed cost x_1 . Under these rules it is a weakly dominant strategy for each firm to bid its true cost of providing the second item. Thus, trade occurs whenever the incumbent does not have the lowest cost for the second item. The price is either determined by the second lowest cost among the bidders, or the incumbent's cost, whichever is lower.

This implies that the incumbent realizes a positive profit at the subcontracting stage whenever an unsuccessful bidder's cost determines the price, i. e. his observed cost exceeds the second highest cost among the bidders. In this case, the incumbent saves the difference between his own cost and the price paid for the subcontract. His expected payoff from subcontracting is

$$E [\Pi_S^I(\Gamma^2)] = E \left[X_1 - X_{(2)}^{\{-1\}}; X_{(2)}^{\{-1\}} \leq X_1 \right]. \quad (11)$$

On the other hand, a multiproduct bidder who loses in the bundle auction still has a chance to win at the second stage. In this case he gets paid the lowest cost among the other firms (including the incumbent), which yields an expected profit

$$E [\Pi_S^C(\Gamma^2)] = E \left[X_{(1)}^{\{-C\}} - X_C; X_C \leq X_{(1)}^{\{-C\}} \right]. \quad (12)$$

PROPOSITION 5 *Suppose that after the bundle auction the incumbent uses a second-price auction with a reserve price equal to his observed cost, x_1 , at the subcontracting stage. The following claims hold true:*

(i) *The bundle auction and the sequential auction yield equivalent expected overall prices.*

(ii) *Both mechanisms are efficient.*¹⁴

¹⁴We call an allocation efficient if the firms with the lowest cost realizations produce. Note that cost realizations at the second stage depend on the outcome of the first stage.

(iii) *The price in the bundle auction second order stochastically dominates the overall price in the sequential auction.*

PROOF It follows from proposition 2, that the expected price in the bundle auction is $E[P_{BS}] = E[Q_{(2)}] + E[X_1] - E[\Pi_S^I(\Gamma^2)] + E[\Pi_S^C(\Gamma^2)]$, where $E[\Pi_S^I(\Gamma^2)]$ and $E[\Pi_S^C(\Gamma^2)]$ are given by equations (11) and (12), respectively. Comparison of (11) and (12) with (9) and (8) yields that $E[\Pi_S^I(\Gamma^2)] = VC$ and $E[\Pi_S^C(\Gamma^2)] = E[\Pi_2^C] = LOV$. Therefore, $E[P_{BS}] = E[P_{SEQ}]$, as stated in proposition 3, part (i).

In order to prove part (ii), note that in the sequential auction the second item is sold by an efficient mechanism at a point in time, where private costs are known to the bidders. In the bundle auction, the second item is always allocated efficiently if the incumbent uses a second-price auction with a reserve price equal to his cost realization x_1 . Moreover, also the first item is allocated efficiently in both auctions since, at the first stage, the multiproduct bidders are symmetric except for their cost realization q_i for the first item.

Finally, to prove part (iii) of the theorem, consider the price paid by the government in the two auctions. The price in the bundle auction is given by the random variable

$$\begin{aligned} P_{BS} &= Q_{(2)} + E[X_1] - E[\Pi_S^I(\Gamma^2) - \Pi_S^C(\Gamma^2)] \\ &= Q_{(2)} - E[\Pi_2^I - \Pi_2^C] + E[P_2], \end{aligned} \quad (13)$$

whereas, in the sequential auction, the price is (compare (6))

$$\begin{aligned} P_{SEQ} &= Q_{(2)} - E[\Pi_2^I - \Pi_2^C] + P_2 \\ &= P_{BS} + [P_2 - E[P_2]]. \end{aligned} \quad (14)$$

Thus, whenever the contestant has lower cost of providing the second item than the incumbent, his cost realization as an incumbent would have been even lower. Therefore, ex post it could have been optimal to give the first item to the firm with the higher cost for the first item in order to minimize overall cost. Our notion of efficiency ignores this aspect since the first item has to be allocated in $t = 1$, when costs for the second item are not yet observed.

As we have shown in part (i) of the theorem, $E[P_{SEQ}] = E[P_{BS}]$. Note that $Q_{(2)}$ and P_2 are independent random variables. Therefore, P_{SEQ} is a mean preserving spread of P_{BS} . which is equivalent to P_{BS} second order stochastically dominating P_{SEQ} ,¹⁵ which completes the proof. \square

4.1.2 Optimal Subcontracting

Now observe that the incumbent clearly can do better than by using the benchmark mechanism, since he can provide both items himself (i. e. he is not forced to trade at stage two). This allows him to choose a sales mechanism that guarantees a higher expected profit from resale than the mechanism we used as a benchmark case in the previous section. The characterization of the optimal auction rules in this context goes back to Myerson (1981): Define *virtual cost* of bidder i , $i = 2, \dots, n$ as

$$\gamma_i(x_i) = x_i + \frac{F_i(x_i)}{f_i(x_i)},$$

and assume that virtual costs $\gamma_i(x_i)$ are strict monotone increasing (a sufficient condition is that reverse hazard rates $f_i(x_i)/F_i(x_i)$ are strict monotone decreasing). Then, the following mechanism maximizes the incumbent's expected revenue from subcontracting:

ALLOCATION: The firm with lowest virtual cost is awarded the contract, provided its virtual cost is lower than the incumbent's cost, x_1 .

PAYMENTS are only made to a firm if it is awarded the contract. Then, this firm is paid the highest cost it could have had such that its virtual cost were still lower than the minimum of the lowest virtual cost among its competitors and the incumbent's cost, x_1 .

Now let us first consider the case that at stage two all firms bidding for the subcontract are symmetric. Although we cannot solve explicitly for the expected price in the bundle auction if the incumbent employs the optimal

¹⁵Cf. Mas–Collé, Whinston, and Green (1995), proposition 6.D.2.

mechanism at the second stage we can provide a ranking of the sequential and the bundle auction:

THEOREM 1 *Suppose that X_2, \dots, X_n are identically distributed. Then, if the incumbent maximizes revenue at the subcontracting stage, the overall price is lower in the bundle auction with resale than in the sequential auction.*

PROOF Denote the revenue maximizing mechanism that the incumbent uses at the subcontracting stage by Γ^* and note that $E[\Pi_S^I(\Gamma^*)]$ and $E[\Pi_S^C(\Gamma^*)]$ are uniquely determined by the incumbent's optimization problem. Moreover, since the multiproduct firms are symmetric ex ante, $E[\Pi_S^I(\Gamma^*)]$ and $E[\Pi_S^C(\Gamma^*)]$ are independent of the allocation at the first stage. It follows from proposition 2 that the price paid by the government in the bundle auction is given by

$$E[P_{BS}^*] = E[Q_{(2)}] + E[X_1] - E[\Pi_S^I(\Gamma^*)] + E[\Pi_S^C(\Gamma^*)], \quad (15)$$

whereas the price in the bundle auction if the incumbent uses the benchmark mechanism is given by equation (13). The price difference is

$$\begin{aligned} E[P_{BS}] - E[P_{BS}^*] &= (E[\Pi_S^I(\Gamma^*)] - E[\Pi_S^I(\Gamma^2)]) \\ &\quad + (E[\Pi_S^C(\Gamma^2)] - E[\Pi_S^C(\Gamma^*)]). \end{aligned} \quad (16)$$

Since the only difference between the two mechanisms is the existence of an optimal reserve price, both terms in (16) must be positive, which proves the theorem. \square

It turns out that from a revenue point of view the government benefits from the incumbent's choice of an optimal mechanism. This is due to the fact that winning the bundle auction becomes more valuable to the multiproduct bidders, if they anticipate that, at the subcontracting stage, they use a mechanism that yields a higher expected payoff than a second-price auction with a reserve price equal to their observed cost. Then, perceived costs at the first stage are lower and bidders are more competitive in the bundle auction. Therefore, if the auctioneer himself cannot set a reserve price in order to lower the expected price, running a bundle auction might be a way

of extracting additional profits by "delegating" the second auction to one of the bidders.

However, the use of an optimal mechanism at the second stage implies that the final allocation is not necessarily efficient. In order to increase expected revenue from subcontracting, the incumbent will choose individual reserve prices strictly lower than his own production cost for the second item and, moreover, (in the asymmetric case) discriminate against strong bidders. Thus, the government may pay a lower overall price, however, the allocation is inefficient with some positive probability. Note that the only way of ensuring efficiency is to use a sequential auction, where the government can ensure that the second item is sold by an efficient mechanism at a point in time where bidders have observed their production costs.

REMARK 2 (ASYMMETRIC FIRMS BIDDING FOR THE SUBCONTRACT)

Theorem 1 can be extended to situations, where the random variables X_C, X_{m+1}, \dots, X_n do not follow the same distribution. In this case, the optimal mechanism displays individually different reserve prices, and moreover, discrimination in favor of weak bidders (in order to increase prices paid by strong bidders). If the contestants are discriminated against at stage two (i. e. they are "strong" bidders), theorem 1 extends straightforwardly, if the contestants are "weak" compared to the singleproduct bidders, theorem 1 extends under a very mild condition, which is discussed in appendix D.

4.2 Preventing Subcontracts?

In the previous section we have shown that the government can ensure production at a lower overall price if it "delegates" the allocation of the second item to one of the multiproduct firms instead of running a second auction by itself. However, it is still not clear, whether the bundle auction yields an even lower expected price if the auctioneer prevents trade at the second stage. The following theorem provides a condition under which the auctioneer never wants to prevent subcontracting after the bundle auction from a

revenue point of view:

THEOREM 2 *The price in the bundle auction with subcontracting is lower than the price in the bundle auction without subcontracting if*

$$E_{x_1} \left[\frac{d^2}{dp^2} \ln \left(\int_0^p F_C(y) dy \right) \Big|_{p=p^*(x_1)} \right] \leq 0, \quad (17)$$

where $p^*(x_1) = x_1 - F_C(p^*(x_1))/f_C(p^*(x_1))$.

PROOF See appendix E. □

Condition (17) implies that the possibility of subcontracting leads to a lower price in the bundle auction even in the worst possible case where only two multiproduct bidders compete in both auctions so that, at the second stage, there is only one bidder to whom the incumbent could possibly subcontract production of the second item. Therefore, if (17) holds, subcontracting after the bundle auction is always desirable from the auctioneer's point of view. A sufficient condition for (17) to hold true is obviously that for every $p \in [0, 1)$

$$\frac{d^2}{dp^2} \ln \left(\int_0^p F_C(y) dy \right) = \frac{d}{dp} \left(\frac{F_C(p)}{\int_0^p F_C(y) dy} \right) \leq 0. \quad (18)$$

Note that in case only one contestant bids for the subcontract, $F_C(p)$ is the probability of trade at a price p , whereas $\int_0^p F_C(y) dy$ is the contestant's expected revenue if the price is p . Thus, the bundle auction with subcontracting yields a lower expected price whenever the ratio of the probability of trade and the contestant's expected payoff at a price p is decreasing. As we illustrate in example 1, inequality (18) holds for a wide class of distributions that also satisfy the requirement that reverse hazard rates $f(x)/F(x)$ are strict monotone decreasing:

EXAMPLE 1 *Condition (17) holds for all Beta-distributions of the form $F(x) = x^\alpha$, $\alpha \geq 0$. We get*

$$\pi^I(p^*) = \frac{F(p^*)^2}{f(p^*)} = \frac{1}{\alpha} p^{*\alpha+1} \quad \text{and} \quad \pi^C(p^*) = \int_0^{p^*} F(x) dx = \frac{1}{\alpha+1} p^{*\alpha+1},$$

which immediately yields that $\pi^I(p^*) \geq \pi^C(p^*)$, if $\alpha + 1 \geq \alpha$, which is always true for all $\alpha \geq 0$.

Also note that the condition in theorem 2 has been derived for the case where only two multiproduct bidders compete at both stages. The presence of additional bidders makes the possibility of subcontracting even more attractive, not only for the incumbent, but also for the auctioneer, since the additional profits are partly competed away at the first stage. Then, resale might be desirable even if (17) does not hold.

One might ask under which circumstances it can be desirable for the auctioneer to prevent subcontracting from a revenue point of view. In case only two multiproduct bidders compete at both stages, the proof of theorem 2 implies that

$$E_{x_1} \left[\frac{d^2}{dp^2} \ln \left(\int_0^p F_C(y) dy \right) \Big|_{p=p^*(x_1)} \right] > 0, \quad (19)$$

is a necessary and sufficient condition for a lower expected price in the bundle auction without subcontracting, i. e. $E[P_B] < E[P_{BS}]$. Note that $\ln(\cdot)$ is concave, while $\int_0^p F_C(y) dy$ is necessarily convex, since $f_C(p) \geq 0$. Therefore (19) holds true if the concave transformation $\ln H(p)$ of $H(p) = \int_0^p F_C(y) dy$ remains a convex function for a sufficiently large subset of the relevant prices $p = p^*(x_1)$, with $x_1 \in [0, 1)$. Note that $\ln H(p)$ cannot be convex everywhere on $[0, 1)$ since this would require that $H(p) = \exp(c(p))$, where $c(p)$ is convex. It is easy to show that this cannot be true since the first derivative of $\exp(c(p))$ cannot be a cumulative distribution function if $c(p)$ is convex. Also note that convexity of $\ln H(p)$ prevails more easily for higher values of p since the second derivative of $\ln(\cdot)$ is decreasing.

Now suppose that $\ln H(p)$ is convex at a certain price \hat{p} . Then we know from (36) that both, $f_C(\hat{p})$ and $\int_0^{\hat{p}} F_C(y) dy$ must be strictly positive and one of the expressions, or both, must be relatively large. Therefore, with positive probability, x_C is below \hat{p} . Since X_1 first order stochastically dominates X_2 , x_1 is below \hat{p} with even higher probability. Now recall that $p^*(x_1) < x_1$.

Therefore, whenever $\ln H(p)$ is convex at a price \hat{p} , prices below \hat{p} occur with positive probability due to the distribution of X_1 .¹⁶

Moreover, in case of additional competition (19) does not remain sufficient for $E[P_B] < E[P_{BS}]$. From propositions 4, part (iv), and 5, part (i), we know that if competition is fierce enough, the price in the bundle auction with subcontracting is lower even if trade at the second stage takes place by a second-price auction without reserve price. Therefore, we can conclude that a necessary condition for the desirability of preventing subcontracts is that additional competition is below that critical level and (19) holds true.

5 Discussion and Concluding Remarks

In this paper, we have compared a sequential and a bundle procurement auction of two complementary contracts, where competition for the second contract is more intense and trade among the bidders after the auction may occur. The two objects have to be awarded in period one and two, respectively (e. g. because they are necessary inputs for a certain production process), which is why the auctioneer cannot commit to a reserve price.

We have found that in the sequential auction firms bid below cost in the first auction since they anticipate an additional expected profit from a comparative cost advantage in the second auction.¹⁷ In this respect, our findings are closely related to the results in von der Fehr and Riis (1998), who also show that bids fall short of cost if the incumbent's expected profit from the second auction exceeds the "looser's option value". While von der Fehr and Riis elaborate in much more detail on the evolution of prices,

¹⁶Note that this makes it impossible to construct a simple example where (19) holds true by choosing a function $F_C(p)$ such that $\ln H(p)$ is convex on a subset of $[0, 1)$ and then choosing X_1 such that only those prices have positive probability.

¹⁷These results also contribute to explaining some stylized facts. Rese and Engel (2002) report that firms often supply products like elevators, paper machines, or gas turbines below cost and argue that this is due to the possibility to induce a comparative cost advantage for subsequent (e. g. service) contracts.

the aim of this paper has been to compare sequential sales to bundling the two objects. As it has turned out, in the bundle auction firms even bid below total expected cost as *incumbent*, if the additional expected profit from incumbency is positive. This is the case if the expected profit from subcontracting the second item is higher than the "loser's option value", which we have shown to hold true under mild assumptions (compare theorem 2). In both auctions, the incumbent faces the risk of making an overall loss, since he gambles on the value of incumbency in his first stage bid. However, it is unclear whether the incumbent's payoff is more risky in the bundle or the sequential auction. The overall price to be paid by the auctioneer for both objects is less risky in the bundle auction, since here the incumbent is paid the *expected cost* of providing the second item at stage one, where provision of the second item is delegated to him.

Comparison of the two auction formats yields that the expected overall price is lower in the bundle auction. Here, compared to the sequential auction, the fact that the incumbent "procures" the contract for the second object at stage two results in a lower overall price to be paid. The incumbent's additional profit from subcontracting is to a large extent competed away at the first stage of the game (in the bundle auction), where it benefits the auctioneer. The incumbent's power to lower the price for the second contract stems from the fact that he can always ensure production by carrying out the project on his own. Therefore, he can credibly threaten not to subcontract in case the price is too high (but still below his own production cost).

While we did not discuss whether the auctioneer has the power to prevent subcontracting, we have provided a condition that ensures that preventing subcontracts never pays off for the auctioneer from a revenue point of view. We conclude that in the present framework the bundle auction with the possibility of subcontracting seems the appropriate format if the auctioneer's concern is revenue, since the expected gain from optimal subcontracting at the second stage leads to significantly more competitive bids in the bundle

auction at the first stage.

However, only the sequential auction always allocates the two contracts efficiently (i. e. the firms with lowest observed costs are awarded the production rights). The bundle auction cannot achieve efficiency since reallocation among the bidders at the second stage is not being realized by an efficient mechanism, or, if the auctioneer prevents subcontracting, the initial allocation of the second production right might turn out to be inefficient once firms have observed their cost for the second item at stage two.

Given the results summarized above, the choice of mechanism clearly depends on the objectives of the auctioneer. If efficiency is the predominant concern (which is plausible if the auctioneer is a public authority), the sequential auction is the appropriate mechanism among the mechanisms considered here. If the auctioneer maximizes revenue (e. g. a private sector firm), a bundle auction is the better choice, and the possibility of trade after the auction likely even decreases the price to be paid by the auctioneer.

Let us conclude with two considerations beyond the analysis of this paper. First, the choice of mechanism may have an impact on the competition for the second production right: Suppose that the singleproduct bidders' distributions of costs are not worse than the contestants'. Then, the additional competitors face the highest expected profits in the sequential auction. Under the optimal subcontracting mechanism they are discriminated against if their cost distributions are more favorable than the contestants', and, moreover, they face a reserve price strictly lower than the incumbent's cost of providing the second item. If the auctioneer even prevents subcontracting after the bundle auction, their expected profit is zero. Therefore, if one endogenizes participation, competition for the second item should be at a high level in the sequential auction, less intense in the bundle auction with resale, and zero (by definition) if resale is prevented.

Second, consider the case that the incumbent's comparative advantage is endogenous, i. e. has to be induced by specific investment as suggested by Rese and Engel (2002). Then, there will be no incentive to incur such

cost in the bundle auction (where, by specific investment, the incumbent would only decrease his expected profits from subcontracting), while the sequential auction might give rise to "wasteful" expenditures that only aim at discriminating against potential competitors in the second auction.

A Positive Value of Incumbency

In order to prove that $E[\Pi_2^I] > E[\Pi_2^C]$ we define the vector of all bidders' expected cost for the second item except the incumbent's and one (representative) contestant's cost by $\tilde{X} := (X_i)$, $i \neq 1, 2$. We denote by $\tilde{X}_{(1)}$ the lowest cost among those bidders and by $\tilde{F}_{(1)}$ ($\tilde{f}_{(1)}$) the corresponding c.d.f (density function). Now we can decompose as follows:

$$E[\Pi_2^I] = E\left[X_C - X_1; X_1 \leq X_C \leq \tilde{X}_{(1)}\right] + E\left[\tilde{X}_{(1)} - X_1; X_1 \leq \tilde{X}_{(1)} \leq X_C\right] \quad (20)$$

$$\text{and} \quad (21)$$

$$E[\Pi_2^C] = E\left[X_1 - X_C; X_C \leq X_1 \leq \tilde{X}_{(1)}\right] + E\left[\tilde{X}_{(1)} - X_C; X_C \leq \tilde{X}_{(1)} \leq X_1\right].$$

First, we derive $E[\Pi_2^C]$. We get

$$\begin{aligned} E\left[X_1 - X_C; X_C \leq X_1 \leq \tilde{X}_{(1)}\right] &= E\left[E\left[(X_1 - X_C)\mathbf{1}_{\{0 \leq X_1 - X_C \leq 1\}}\mathbf{1}_{\{X_1 - X_C \leq \tilde{X}_{(1)} - X_C \leq 1\}} \mid X_1, X_C\right]\right] \\ &= E\left[(X_1 - X_C)\mathbf{1}_{\{0 \leq X_1 - X_C \leq 1\}}E\left[\mathbf{1}_{\{X_1 \leq \tilde{X}_{(1)} \leq 1\}} \mid X_1, X_C\right]\right] \\ &= E\left[E\left[(X_1 - X_C)\mathbf{1}_{\{0 \leq X_1 - X_C \leq 1\}}(1 - \tilde{F}_{(1)}(X_1)) \mid X_1\right]\right] \\ &= E\left[(1 - \tilde{F}_{(1)}(X_1)) \int (X_1 - u)\mathbf{1}_{\{0 \leq X_1 - u \leq 1\}} dF_C(u)\right] \\ &= E\left[(1 - \tilde{F}_{(1)}(X_1)) \int_0^{X_1} (X_1 - u) dF_C(u)\right] \\ &= \int_0^1 (1 - \tilde{F}_{(1)}(v)) \int_0^v (v - u) dF_C(u) dF_1(v) \\ &= \int_0^1 (1 - \tilde{F}_{(1)}(v)) \left[vF_C(v) - \int_0^v u dF_C(u)\right] dF_1(v) \\ &= \int_0^1 (1 - \tilde{F}_{(1)}(v)) \left[vF_C(v) - \int_0^v F_C(u) du\right] dF_1(v) \\ &= \int_0^1 \left[(1 - \tilde{F}_{(1)}(v)) \int_0^v F_C(u) du\right] dF_1(v), \end{aligned}$$

and, following the same calculations,

$$E\left[\tilde{X}_{(1)} - X_C; X_C \leq \tilde{X}_{(1)} \leq X_1\right] = \int_0^1 \left[(1 - F_1(v)) \int_0^v F_C(u) du\right] d\tilde{F}_{(1)}(v),$$

Thus,

$$\begin{aligned} E[\Pi_2^C] &= \int_0^1 \left[(1 - \tilde{F}_{(1)}(v)) \int_0^v F_C(u) du\right] f_1(v) dv \\ &\quad + \int_0^1 \left[(1 - F_1(v)) \int_0^v F_C(u) du\right] \tilde{f}_{(1)}(v) dv. \end{aligned} \quad (22)$$

Integration by parts of the second term in (22) yields

$$\begin{aligned}
& \int_0^1 \left[(1 - F_1(v)) \int_0^v F_C(u) du \right] \tilde{f}_{(1)}(v) dv \\
&= \left| (1 - F_1(v)) \int_0^v F_C(u) du \tilde{F}_{(1)}(v) \right|_0^1 \\
&\quad - \int_0^1 \tilde{F}_{(1)}(v) \left[(1 - F_1(v)) F_C(v) - f_1(v) \int_0^v F_C(u) du \right] dv \\
&= - \int_0^1 \tilde{F}_{(1)}(v) \left[(1 - F_1(v)) F_C(v) - f_1(v) \int_0^v F_C(u) du \right] dv.
\end{aligned} \tag{23}$$

Inserting (23) in (22) gives

$$\begin{aligned}
E[\Pi_2^C] &= \int_0^1 \int_0^v F_C(u) du dv - \int_0^1 \tilde{F}_{(1)}(v) (1 - F_1(v)) F_C(v) dv \\
&= \left| F_1(v) \int_0^v F_C(u) du \right|_0^1 - \int_0^1 F_1(v) F_C(v) dv \\
&\quad - \int_0^1 \tilde{F}_{(1)}(v) (1 - F_1(v)) F_C(v) dv,
\end{aligned} \tag{24}$$

which yields

$$E[\Pi_2^C] = \int_0^1 (1 - \tilde{F}_{(1)}(v)) (1 - F_1(v)) F_C(v) dv. \tag{25}$$

Following the same calculations, we get

$$E[\Pi_2^I] = \int_0^1 (1 - \tilde{F}_{(1)}(v)) F_1(v) (1 - F_C(v)) dv. \tag{26}$$

Clearly, if X_C is greater than X_1 in the sense of first order stochastic dominance, it holds that $(1 - F_1(v)) F_C(v) < F_1(v) (1 - F_C(v))$ for every $v \in [0, 1]$, which proves the assertion.

□

B Proof of Proposition 4

In order to proof the proposition we need the following

LEMMA 1 *Consider two random vectors, X and (X, Y) where all components are independent and denote the second order statistic of X and (X, Y) , respectively, by $X_{(2)}$ and $(X, Y)_{(2)}$. It holds that $X_{(2)} \geq (X, Y)_{(2)}$ in the sense of first order stochastic dominance.*

PROOF Denote by $X_{(i)}$ and $Y_{(i)}$ the i th order statistic of X and Y , respectively. Now consider the following decomposition:

$$(X, Y)_{(2)} = X_{(1)} \mathbf{1}_{\{Y_{(1)} \leq X_{(1)} \leq Y_{(2)}\}} + X_{(2)} \mathbf{1}_{\{X_{(2)} \leq Y_{(1)}\}} + Y_{(1)} \mathbf{1}_{\{X_{(1)} \leq Y_{(1)} \leq X_{(2)}\}} + Y_{(2)} \mathbf{1}_{\{Y_{(2)} \leq X_{(1)}\}}$$

Pointwise comparison of the $X_{(2)}$ and $(X, Y)_{(2)}$ in the different events yields that $X_{(2)}$ must be at least as big as $(X, Y)_{(2)}$, but possibly is smaller. \square

Now consider the losers' option value LOV . As we know from equation (8), it holds that $LOV = E[\Pi_2^C]$. Therefore, it follows from equation (25) in appendix A that

$$LOV = \int_0^1 (1 - \tilde{F}_{(1)}(v))(1 - F_1(v))F_C(v)dv. \quad (27)$$

The value of additional competition in the second auction is given by equation (9). We get

$$\begin{aligned} VC &= E \left[X_1 - X_{(2)}^{\{-1\}}; X_{(2)}^{\{-1\}} \leq X_1 \right] \\ &= E \left[E \left[(X_1 - X_{(2)}^{\{-1\}}) \mathbf{1}_{\{0 \leq X_1 - X_{(2)}^{\{-1\}} \leq 1\}} | X_1 \right] \right] \\ &= E \left[\int (X_1 - u) \mathbf{1}_{\{0 \leq u \leq 1\}} dF_{(2)}^{\{-1\}}(u) \right] \\ &= E \left[\int_0^{X_1} (X_1 - u) dF_{(2)}^{\{-1\}}(u) \right] \\ &= \int_0^1 \int_0^v (v - u) dF_{(2)}^{\{-1\}}(u) dF_1(v) \\ &= \int_0^1 \left(v F_{(2)}^{\{-1\}}(v) - \int_0^v u dF_{(2)}^{\{-1\}}(u) \right) dF_1(v) \\ &= \int_0^1 \left(v F_{(2)}^{\{-1\}}(v) - |u F_{(2)}^{\{-1\}}(u)|_0^v + \int_0^v F_{(2)}^{\{-1\}}(u) du \right) dF_1(v) \\ &= \int_0^1 \int_0^v F_{(2)}^{\{-1\}}(u) du dF_1(v) \\ &= |F_1(v) \int_0^v F_{(2)}^{\{-1\}}(u) du|_0^1 - \int_0^1 F_{(2)}^{\{-1\}} F_1(v)(v) dv \\ &= \int_0^1 F_{(2)}^{\{-1\}}(v) dv - \int_0^1 F_{(2)}^{\{-1\}} F_1(v)(v) dv, \end{aligned}$$

which yields

$$VC = \int_0^1 (1 - F_1(v)) F_{(2)}^{\{-1\}}(v) dv. \quad (28)$$

- (i) We know from lemma 1 that $\tilde{F}_{(1)}(v)$ and $F_{(2)}^{\{-1\}}(v)$ increase $\forall v \in [0, 1]$ if we increase the number of bidders by adding contestants or singleproduct bidders with arbitrary distributions. Therefore, VC , as given by (28), increases and LOV , as given by (27), decreases. Since $E[P_{SEQ}] = E[Q_{(2)}] + E[X_1] - VC + LOV$ this yields a decrease of the overall price in the sequential auction, which proves part (i) of proposition 4.
- (ii) In order to prove part (ii) of the proposition, we show that even if we only have two multiproduct bidders and one singleproduct bidder the order of VC and LOV

depends on the distribution of the singleproduct bidder's cost. Let us denote this singleproduct bidder's cost by X_S with distribution F_S and density f_S . For the case of three bidders we get

$$VC = \int_0^1 F_S(v)(1 - F_1(v))F_C(v)dv. \quad (29)$$

and

$$LOV = \int_0^1 (1 - F_S(v))(1 - F_1(v))F_C(v)dv. \quad (30)$$

Now assume $0 < F_C(v) < F_1(v) < 1 \forall v \in (0, 1)$ and consider a sequence (X_{S_n}) such that $\lim_{n \rightarrow \infty} F_{S_n}(v) = 1 \forall v \in [0, 1]$. Denote the price difference between the two auctions if $X_S = X_{S_n}$ by $VC_n - LOV_n$. It holds for all $v \in [0, 1]$ that

$$\lim_{n \rightarrow \infty} ([F_{S_n}(v) - (1 - F_{S_n}(v))](1 - F_1(v))F_C(v)) = (1 - F_1(v))F_C(v).$$

By dominated convergence,

$$\begin{aligned} \lim_{n \rightarrow \infty} (VC_n - LOV_n) &= \lim_{n \rightarrow \infty} \int_0^1 [F_{S_n}(v) - (1 - F_{S_n}(v))](1 - F_1(v))F_C(v)dv \\ &= \int_0^1 (1 - F_1(v))F_C(v)dv > 0 \end{aligned}$$

Therefore, if X_S is sufficiently low, the value of additional competition is higher than the loser's option value and the price in the sequential auction is lower than the price in the bundle auction without resale.

Now consider a sequence X_{S_m} such that $\lim_{m \rightarrow \infty} F_{S_m}(v) = 0 \forall v \in [0, 1]$. Then, it holds for all $v \in [0, 1]$ that

$$\lim_{m \rightarrow \infty} ([F_{S_m}(v) - (1 - F_{S_m}(v))](1 - F_1(v))F_C(v)) = -(1 - F_1(v))F_C(v).$$

By dominated convergence,

$$\begin{aligned} \lim_{m \rightarrow \infty} (VC_m - LOV_m) &= \lim_{m \rightarrow \infty} \int_0^1 [F_{S_m}(v) - (1 - F_{S_m}(v))](1 - F_1(v))F_C(v)dv \\ &= - \int_0^1 (1 - F_1(v))F_C(v)dv < 0 \end{aligned}$$

Therefore, if X_S is sufficiently large, the loser's option value is higher than the value of additional competition, and therefore, the price in the sequential auction is higher than in the bundle auction without resale.

We can conclude that there exist critical values \underline{X}_S and \overline{X}_S , such that $VC > LOV$ if $X_S <_{FSD} \underline{X}_S$ and $VC < LOV$ if $X_S >_{FSD} \overline{X}_S$, which completes the proof. \square

C The Impact of Lower Cost as Incumbent

Let us have a closer look at the impact of a FSD decrease of X_1 on VC and LOV . If X_1 decreases in the sense of FSD, $1 - F_1(v)$ decreases $\forall v \in (0, 1)$. Therefore, it follows from equations (28) and (27) that both, VC and LOV , decrease. The effect on $E[P_{SEQ}]$ is unclear. However, it follows from equation (6) that

$$E[P_{SEQ}] = E[Q_{(2)}] - E[\Pi_2^I] + E[\Pi_2^C] + E[P_2]. \quad (31)$$

From equation (26) and (25) it follows that $E[\Pi_2^I]$ and $E[\Pi_2^C]$ increase and decrease, respectively, if X_1 increases. Therefore, if a FSD decrease of X_1 leads to a decrease of $E[P_2] = E[X_{(2)}]$, the overall price in the sequential auction decreases.

D Asymmetric Firms Bidding for the Sub-contract

In the case of asymmetric bidders, the optimal mechanism Γ^* imposes an individualized reserve price on every bidder that is strictly lower than the incumbent's cost and, moreover, strong bidders are discriminated against in favor of weak bidders in order to lower the expected price to be paid in case trade takes place. Now, let Γ^r denote the direct revelation mechanism that imposes the optimal reserve price $r_i = x_1 - F_i(r_i)/f_i(r_i)$, $i = 2, \dots, n$, on every bidder at the second stage but, except for this, allocation and transfers are the same as in the second-price sealed bid auction. Then, $\delta_i^* = E[\Pi_S^i(\Gamma^*)] - E[\Pi_S^i(\Gamma^r)]$ is the additional expected profit bidder i , $i = I, C$, obtains from optimal discrimination among the bidders on top of this (which may be negative).

Now we decompose $E[\Pi_S^I(\Gamma^*)]$ and $E[\Pi_S^C(\Gamma^*)]$ into the profits from only setting the optimal reserve prices, $E[\Pi_S^i(\Gamma^r)]$, $i = I, C$, and the additional expected profits from optimal discrimination on top of this, δ_i^* , $i = I, C$. Then we have $E[\Pi_S^i(\Gamma^*)] = E[\Pi_S^i(\Gamma^r)] + \delta_i^*$, $i = I, C$, and we can rewrite equation (16) as follows:

$$\begin{aligned} E[P_{BS}] - E[P_{BS}^*] &= (E[\Pi_S^I(\Gamma^r)] - E[\Pi_S^I(\Gamma^2)]) \\ &\quad + (E[\Pi_S^C(\Gamma^2)] - E[\Pi_S^C(\Gamma^r)]) \\ &\quad + (\delta_I^* - \delta_C^*). \end{aligned} \quad (32)$$

Obviously, the introduction of optimal reserve prices increases the incumbent's expected profit and lowers the expected profit as contestant compared to a second-price auction without reserve prices. Therefore, the first two terms in equation (32) are positive. Now consider the additional profits from optimal discrimination. If a contestant

has a favorable distribution of cost as compared to the singleproduct bidders (i. e. he is discriminated against by the optimal mechanism), δ_C^* is negative, and thus, $\delta_I^* - \delta_C^*$ is positive. Therefore, $E[P_{BS}] - E[P_{BS}^*] = E[P_{SEQ}] - E[P_{BS}^*] \geq 0$, which proves that theorem 1 extends straightforwardly in this case.

Now consider the case that a contestant has an unfavorable distribution of cost (i. e. he is favored by the optimal mechanism). Then, δ_C^* is positive. However, since the first two terms and δ_I^* are positive, one could still argue that theorem 1 is likely to hold true: In order to reverse the result, a contestant's profit from optimal discrimination (as a weak bidder) would have to be higher than the incumbent's additional profit from switching from the second price auction to the optimal mechanism and, on top of this, would have to compensate the bidder's own loss in expected revenue caused by the introduction of a reserve price. \square

E Proof of Theorem 2

Recall that the price in the bundle auction with subcontracting by an optimal mechanism is given by

$$E[P_{BS}^*] = E[Q_{(2)}] + E[X_1] - E[\Pi_S^I(\Gamma^*)] + E[\Pi_S^C(\Gamma^*)]. \quad (33)$$

From comparison of $E[P_B]$ as given in proposition 1 and $E[P_{BS}^*]$ it follows that the expected price in the bundle auction with resale is lower whenever $E[\Pi_S^I(\Gamma^*)] \geq E[\Pi_S^C(\Gamma^*)]$, i. e. a multiproduct bidder's expected profit from subcontracting at the second stage is higher as incumbent than as contestant.

In order to prove the theorem, we have to consider the worst possible scenario. Suppose there are only two multiproduct bidders competing at both stages, such that, at the second stage, there is only one bidder (a contestant) to whom the incumbent could possibly subcontract production of the second item. In this case, the incumbent's problem reduces to setting the optimal take-it-or-leave-it price at which the contestant is willing to trade whenever his cost is lower. Otherwise, no trade takes place. Let $\pi_S^I(p, x_1)$ denote the incumbent's expected profit from posting the price p at the second stage, after he has observed his cost for the second item, x_1 . We get $\pi_S^I(p, x_1) = (x_1 - p)F_C(p)$. Maximization with respect to p yields $p^* = x_1 - F_C(p^*)/f_C(p^*)$. Inserting p^* into $\pi_S^I(p, x_1)$ yields

$$\pi_S^{I*} = \frac{F_C(p^*)^2}{f_C(p^*)}. \quad (34)$$

Then, the contestant's expected profit from the subcontracting stage is given by

$$\begin{aligned}\pi_S^{C^*} &= (p^* - E[X_C | X_C \leq p^*]) F_C(p^*) \\ &= \int_0^{p^*} F_C(y) dy.\end{aligned}\tag{35}$$

Therefore, $\pi_S^{I^*} \geq \pi_S^{C^*}$ whenever $F_C(p^*)^2 / f_C(p^*) \geq \int_0^{p^*} F_C(y) dy$, or, equivalently,

$$f_C(p^*) \int_0^{p^*} F_C(y) dy - F_C(p^*)^2 \leq 0.\tag{36}$$

Note that

$$\frac{d}{dp} \left(\frac{F_C(p)}{\int_0^p F_C(y) dy} \right) = \frac{f_C(p) \int_0^p F_C(y) dy - F_C(p)^2}{\left(\int_0^p F_C(y) dy \right)^2},\tag{37}$$

which proves that from

$$\frac{d}{dp} \left(\frac{F_C(p)}{\int_0^p F_C(y) dy} \right) = \frac{d^2}{dp^2} \ln \left(\int_0^p F_C(y) dy \right) \leq 0\tag{38}$$

at $p = p^*(x_1)$ it follows that $\pi_S^{I^*} \geq \pi_S^{C^*}$ for the particular realization of the incumbent's cost, x_1 , that corresponds to the optimal reserve price $p^*(x_1)$.

At the first stage, however, the incumbent is still uncertain with respect to his cost at the second stage. Thus, $E[\Pi_S^I(\Gamma^*)] \geq E[\Pi_S^C(\Gamma^*)]$ will hold whenever the incumbent's expected profit from the second stage is greater than the contestant's. Taking the expectation of (38) with respect to x_1 yields condition (17) as given in the theorem.

Finally note that additional bidders at the subcontracting stage can only raise the value of incumbency and thereby lower the multiproduct bidders' bids in the bundle auction with subcontracting, which completes the proof. \square

F References

- Albano, G.-L., F. Germano, and S. Lovo (2001). A Comparison of Standard Multi-Unit Auctions with Synergies, *Economics Letters* 71, 55 – 60.
- Armstrong, M. (2000). Optimal Multi-Object Auctions, *Review of Economic Studies* 67, 455 – 481.
- Avery, C. and T. Hendershott (2000). Bundling and Optimal Auctions of Multiple Products, *Review of Economic Studies* 67, 483 – 497.
- Branco, F. (1997a). Sequential Auctions with Synergies: An Example. *Economics Letters* 54, 159 – 163.
- Branco, F. (1997b). Multi-object Auctions with Synergies, Working Paper.
- Burt, D. N., W. Norquist, and J. Anklesaria (1990). Zero-Base Pricing, Chicago: Irwin.
- Gale, I. (1990). A Multiple Object Auction with Superadditive Values, *Economics Letters* 34, 323 – 328.
- Gale, I., D. Hausch, and M. Stegeman (2000). Sequential Procurement Auctions with Subcontracting, *International Economic Review*, 41, 989 – 1020.
- Haile, P. (1999a). Auctions with Private Uncertainty and Resale Opportunities, *Journal of Economic Theory* 108, 72 – 110.
- Haile, P. (1999b). Auctions with Resale, mimeo, University of Wisconsin.
- Harstadt, R. M. and M. H. Rothkopf (1995). Combinatorial Auctions with Synergies, mimeo.

- Jeitschko, T. D. and E. Wolfstetter (2002). Scale Economies and the Dynamics of Recurring Auctions, *Economic Inquiry* 40, 403-414.
- Kamien, M., L. Li, and D. Samet (1989). Bertrand Competition with Subcontracting, *Rand Journal of Economics* 20, 553 – 567.
- Krishna, V. (2002). Auction Theory, Academic Press, San Diego.
- Krishna, V. and R. Rosenthal (1996). Simultaneous Auctions with Synergies, *Games and Economic Behavior* 17, 1 – 31.
- Levin, J. (1997). On Optimal Auctions for Complements, *Games and Economic Behavior* 18, 176 – 192.
- Mas–Collel, A., M. D. Whinston, and J. R. Green (1995). Microeconomic Theory, Oxford University Press, New York, Oxford.
- McAfee, R. and J. McMillan (1987). Auctions and Bidding, *Journal of Economic Literature* 35, 699 – 738.
- Menezes, F. and P. Monteiro (1999), Synergies and Price Trends in Sequential Auctions, *Review of Economic Design* 8, 85 – 98.
- Myerson, R. B. (1981). Optimal Auction Design, *Mathematics of Operations Research*, 6 (13), 619 – 632.
- Palfrey, T. R. (1983). Bundling Decisions by a Multiproduct Monopolist with Incomplete Information, *Econometrica* 51 (2), 463 – 483.
- Rese, M. and B. Engel (2002). The Sustainability of Exploitation Positions. Towards an Integration of New Institutional Economics, Austrian Economics, and Resource Based View, Working Paper, Humboldt University.
- Rothkopf, M. H. , A. Pekec, and R. M. Harstadt (1995). Computationally Manageable Combinatorial Auctions, RUTCOR research report RR 13-95.

von der Fehr, N.-H. M. and C. Riis (1998). Option Values in Sequential Markets, Discussion Paper, Department of Economics, University of Oslo.