

CARTEL SUSTAINABILITY AND CARTEL STABILITY*

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WP-EC 2002-16

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Editor: Instituto Valenciano de Investigaciones Económicas, S.A.

Primera Edición Septiembre 2002

Depósito Legal: V-3617-2002

IVIE working papers offer in advance the results of economic research under way in order to encourage a discussion process before sending them to scientific journals for their final publication.

* I would like to thank my supervisor Ramon Faulí-Oller for his advice and encouragement. Financial support by the I.V.I.E, (*Instituto Valenciano de Investigaciones Económicas*) and the Department of Economical Analysis of Alicante University is gratefully acknowledged.

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A B S T R A C T

The paper studies how does the size of a cartel affect the possibility that its members can sustain a collusive agreement. I obtain that collusion is easier to sustain the larger the cartel is. Then, I explore the implications of this result on the incentives of firms to participate in a cartel. Firms will be more willing to participate because otherwise, they risk that collusion completely collapses, as remaining cartel members are unable to sustain collusion.

KEYWORDS: Collusion, Partial Cartels, Trigger strategies, Optimal Punishment.

1 Introduction

For many years it was widely held among economists that firms could not exercise market power collectively without some form of explicit coordination. However the theory of repeated game has cast some doubt on this approach. Stable arrangements may require little coordination between firms, and possibly none at all. This has raised a dilemma for the design of a policy towards collusion. If the legal standard focuses on explicit coordination, a large number of collusive outcome can fall outside the prohibition, and if it tries to cover collusion without explicit coordination, it will prohibit non-cooperative practises.

Article 81(1) of the Rome Treaty stipulates that agreements or concerted practises between firms which distort competition are prohibited. What is meant by "agreements" and "concerted practises" is not further specified in the treaty¹. However, decisions recently taken by the Commission show that often firms behavior that do not involve a process of coordination are overlooked although they could mean an exercise of market power.

The literature about collusion, mainly deal with two different approaches. The models on *explicit* collusion (Selten (1983) and d'Aspremont et al. (1983)) have mainly focused on the incentives of firms to participate in a cartel agreement. These papers focus on firms "participation constraints" and investigate cartel stability in static models. Two different incentives play a role here. Firms face a trade off between participation and nonparticipation in the cartel, firms have an incentive to join the cartel so as to achieve a more collusive outcome, but on the other hand have an incentive to stay out of the cartel to free-ride on the cartel effort to restrict production.

The models on *implicit* collusion (Friedman (1971)), using a supergame theoretical framework, have focused on the problem of enforcement of collusive behavior. This second strand is a supergame-theoretic approach. It is called *implicit* collusion. This focuses on firms "incentive constraints"². They have studied under which circumstances collusion

¹Although it is tempting to associate "agreements" with explicit collusion and "concerted practices" with implicit collusion.

²"Participation constraints" are firms incentives to join the cartel or the fringe, meanwhile "Incentive constraints" are the incentives to cheat on the cooperation agreement.

can be sustained as an equilibrium of the repeated game. Most research on the field have studied symmetric settings and have focused on the sustainability of the most profitable symmetric equilibrium. The reason to select this equilibrium is that it will be the one that firms will agree to play if they secretly meet to discuss their pricing plans (Mas-Colell et al (1996)).

The main point of the paper is that this argument is compelling but it does not take into account that firms may prefer not to attend this meeting in order not to participate in the coordination to a collusive agreement. This takes us back to the literature on the incentive to participate in a cartel, mentioned above. But now the analysis is richer because one has to study how does this participation incentive interact with the incentive to maintain a collusive agreement (Nocke (1999)). As a first step, I study how does the size of a cartel affect the possibility that its members can sustain a collusive agreement in a supergame theoretical framework. I obtain that collusion is easier to sustain the larger the cartel is. To obtain the result I study the sustainability of partial cartels i.e. cartels that do not include all the firms in a given industry. Partial cartels are often observed in reality, being the OPEC the most well-known example.

The previous result has implications on cartel formation, because it reduces the incentives to free-ride from a cartel by defecting from it. I can illustrate the idea with the following extreme example. I find that for some discount factors, the only sustainable cartel is the cartel that comprises all firms in the industry.

Then all firms have incentives to participate in the cartel, because otherwise collusion completely collapses. This completely eliminates the gains from free-riding at the participation stage. In practice it is easier to fight against explicit than against implicit collusion because the former is easier to prove. The model highlights that policy measures that induce firms to replace explicit with implicit collusion³ to escape antitrust prosecution may have its costs. Fighting against explicit collusion (and forcing firms to collude tacitly) has the positive effect of weaken the incentives to maintain a collusive agreement but the negative effect of making stronger the incentives to participate in a cartel.

³”When the legal advisors of cartel members discovered that Article 85 had to be taken seriously, they had their clients throw their agreements in the waste basket. Simultaneously, the attention of DG\ IV shifted to the detection of tacit collusion, on the assumption that explicit collusion was being replaced by tacit collusion” (Phlips (1994)).

Therefore the total effect on price is uncertain. In the particular model I analyze price is higher with implicit than with explicit collusion.

The structure of the paper is as follows. In the following section, the central model of the paper is set. The sustainability of the partial cartel is analyzed with the "trigger strategies"⁴. In the next section, the participation game is set. Firms decide first whether to join the cartel or not, afterwards the firms infinitely play a quantity game. The main aim of this section is to study the interaction between incentive and participation constraints. Afterwards the sustainability of the partial cartel is analyzed using an optimal penal code to enforce collusion following Abreu (1986). The last section is left to analyze a very simple model constructed so that both cartel size and cost asymmetries among cartel members play a role in determining the stable cartel agreement.

The model predicts that the size of the cartels enforced can be larger in the *implicit* collusion model than in the *explicit* collusion model.

We can think of an interesting example. The introduction of the 1956 Restrictive Practices Act in UK led to the registration and subsequent abolition of explicit restrictive agreements between firms. However, a considerable number of agreements were not registered and continued secretly. Studies of the impact of the 1956 Act showed that in many industries explicit price-fixing agreements had been replaced by informal agreements to exchange information on prices, quantities and conditions of sale, and that in many cases these had serious adverse effects on competition. The most relevant case could be the electrical power industry. A large number of firms were members of the Transformer Maker's Association. Collusion among members of the association had been effective during the 1950's, and competition was slow to emerge after the agreement was formally abandoned following a Court judgment. However, explicit price-fixing was replaced by the exchange of information on prices and tenders, and this helped to sustain prices and margins for several years. Furthermore, several firms that were not members of the association at the time of the Court hearing seem to have adhered to the implicit arrangements.

⁴The sustainability of partial cartels in a dynamic setting is considered by Martin (1990) in a homogeneous firms context. However despite our model, decision is sequential, that is cartel firms act as stackelberg leaders.

2 Partial Cartels

Assume that n firms, where $n > 2$, indexed $i, i = 1, 2, 3, \dots, n$ compete in a market whose demand is given by $P(Q) = 1 - Q$. Cost functions of firms are given by: $c(q_i) = \frac{q_i^2}{2}$, where q_i denotes the production of firm i . Assume firms simultaneously choose quantities ⁵.

A (partial) cartel will be said to be active in this market if there is a group of firms (cartel members) that maximize joint profits and the remaining firms (nonmembers or fringe firms) maximize individual profits. When a cartel of k firms is active, members (m), and nonmembers (nm), simultaneously produce respectively:

$$q_m^k = \frac{2}{nk - k^2 + 3k + 2 + n} \quad (1)$$

$$q_{nm}^k = \frac{k + 1}{nk - k^2 + 3k + 2 + n} \quad (2)$$

In this situation, Profits of members and nonmembers are given respectively by π_m^k and π_{nm}^k . Observe that if $k = 1$, we have standard Cournot competition and $q_m^1 = q_{nm}^1$.

We are going to study under which conditions playing (1) and (2) in each period can be sustained as an equilibrium of a game where the one stage game described above is repeated infinite times. Firms will be assumed to discount the future at a factor of δ . Member firms are denoted with a natural number from 1 to k .

Cartel members will sustain cooperation by using "trigger strategies", that is when cheating firms are punished with infinite reversion to the Nash Cournot equilibrium. Trigger strategies for a partial cartel can be formulated the following way, where $q_{t,i}$ denotes the strategy played by firm i in period t :

Firm $i, i = 1, \dots, k$ plays

$$\left\{ \begin{array}{l} q_{1,i} = q_m^k \\ q_{t,i} = q_m^k \text{ if } q_{l,j} = q_m^k \text{ for any } l < t \text{ for } j = 1, \dots, k \text{ and for } j \neq i \\ q_{t,i} = q_m^1 \text{ otherwise.} \end{array} \right.$$

Firm $i, i = k + 1, \dots, n$ plays

⁵Shaffer(1995) considers the cartel as a Stackelberg leader because of its power to impose its most preferred timing.

$$\left\{ \begin{array}{l} q_{1,i} = q_{nm}^k \\ q_{t,i} = q_{nm}^k \text{ if } q_{t,j} = q_m^k \text{ for any } l < t \text{ for } j = 1, \dots, k \text{ and for } j \neq i \\ q_{t,i} = q_m^1 \text{ otherwise.} \end{array} \right.$$

Nonmember firms play optimally, because the future play of rivals is independent of how they play today and they maximize current profits. Member firms will behave optimally if the discount factor is high enough. To obtain the conditions on the discount factor such that using "trigger strategies" is also optimal for member firms, we have to calculate the profits of a member firm that deviates from the cartel. They will choose:

$$q_d^k = \arg \max_q P((k-1)q_m^k + (n-k)q_{nm}^k + q)q - \frac{q^2}{2}$$

and will obtain π_d^k like the profits obtained in the period of deviation.

Then trigger strategies are optimal for member firms if:

$$\frac{1}{1-\delta} \pi_m^k \geq \frac{\delta}{1-\delta} \pi_m^1 + \pi_d^k$$

If we let $\delta_k = \frac{\pi_d^k - \pi_m^k}{\pi_d^k - \pi_m^1}$, the previous condition can be written in the following way.

If $\delta_k \geq 1$ the cartel of size k can not be sustained for any δ . If $\delta_k < 1$, the cartel can be sustained for $\delta \geq \delta_k$.

Although it may be surprising at first sight that some cartel sizes can not be sustained in equilibrium, it comes from the well-known result in the literature that with Cournot competition, mergers (or any other collusive agreement) of a small number of firms reduces profits because non-participating firms react by increasing their production (see Salant et al.(1983)).

Next proposition shows that the previous intuition extends to any cartel size in the sense that whenever a cartel of size k is sustainable, cartels of larger size are also sustainable⁶.

Proposition 1 *The cutoff discount factor (δ_k) that sustain the strategies described above, is decreasing in the size of the cartel.*

⁶Remark the similarity with the result in Salant et al.(1983) that if a merger of k firms is profitable, a merger with more firms is also profitable.

We have that δ_k can be rewritten like :

$$\delta_k = \frac{1 - \frac{\pi_m^k}{\pi^d}}{1 - \frac{\pi_m^1}{\pi^d}}$$

Therefore variations of k have two different effects. First, $\frac{\pi_m^k}{\pi^d}$ decreases when k increases because deviation profits increase more than profits from being in the cartel of k firms. This would increase δ_k . Second, as k increases, $\frac{\pi_m^1}{\pi^d}$ also decreases because π_m^1 does not depend on k , and deviation profits increase with k . Thus punishment becomes proportionally more painful. This second effect would decrease δ_k .

The result from the Proposition 1 comes from the fact that the second effect dominates the first one.

3 The participation game.

In the previous Section we have obtained conditions on the discount factor under which cartels of different sizes are active. In this Section, we will allow firms to coordinate in the different outcomes by showing their willingness to participate in a collusive agreement. Those decisions will not affect the payoff of firms, but they will only be used as a coordination device: if k firms decide to participate in a cartel agreement, only cartels of size k can be observed in the repeated game.

This pre-communication play is modelled as a stage prior to market competition. The decision of each firm relates to selecting a zero-one variable w_i where:

$$\begin{aligned} w_i & : 1 \text{ iff firm } i \text{ joins the cartel} \\ & : 0 \text{ iff firm } i \text{ joins the fringe} \end{aligned}$$

If k firms announce joining the cartel, the future play is only modified if the discount factor allows a cartel of k firms to be active ($\delta \geq \delta_k$). Otherwise, all firms play the Cournot quantity in all periods. In short, once a cartel of k firms is formed, we will assume that discounted payoffs of member and nonmember firms are respectively given by the following expressions:

$$\Pi_m^k = \begin{cases} \frac{1}{1-\delta} \pi_m^k & \text{if } \delta \geq \delta_k \\ \frac{1}{1-\delta} \pi_m^1 & \text{otherwise} \end{cases} \quad (3)$$

$$\Pi_{nm}^k = \begin{cases} \frac{1}{1-\delta} \pi_{nm}^k & \text{if } \delta \geq \delta_k \\ \frac{1}{1-\delta} \pi_{nm}^1 & \text{otherwise} \end{cases} \quad (4)$$

We are going to look for the Nash equilibrium of the game.

In our model, a cartel of size k is an equilibrium configuration (stable cartel) if the following two conditions are satisfied:

-Internal, stability: Either no collusion or:

$$\Pi_m^k \geq \Pi_{nm}^{k-1} \quad \forall i \in k \text{ and for } k \in (2, \dots, n) \quad (5)$$

-External stability: Either full collusion or:

$$\Pi_m^{k+1} \leq \Pi_{nm}^k \quad \forall i \in k \text{ and for } k \in (1, \dots, n-1) \quad (6)$$

This participation game has been previously analyzed in the literature in cases where firms can sign binding contracts to sustain the outcome of the cartel⁷. In that case collusion is said to be explicit, while in our model is called implicit. With explicit collusion sustainability of cartels is not at issue. Then payoffs of players would be like (5) and (6) taking $\delta_k = 0$. Solving the participation game for the case of explicit collusion will be both a helpful step to solve it in our case and will provide us a benchmark to compare the results.

The key point in the explicit collusion case is that for any cartel size, internal stability is never satisfied. Firms know that the goal of the cartel is to reduce production. Then firms will have incentives to leave the cartel in order to free ride from the output reduction agreed by the remaining cartel members.

Proposition 2 *No cartel is stable, when collusion is explicit.*

⁷See Donsimoni (1985). The only difference is that it considers the Cartel behaves as a Stackelberg leader while in our case the cartel and nonmember firms compete à la Cournot.

We are ready now to determine the Nash equilibrium of the participation game. This game has many equilibria that imply that no cartel is active. For example all firms deciding not to join the cartel is always an equilibrium. For $\delta < \delta_n$ any choice by firms is an equilibrium because the participation decisions are irrelevant because no cartel can be sustained. To clarify the analysis I will focus on the equilibria where cartels are active. In other words we will determine which of the sustainable cartels are also stable. We state the results in the following Proposition:

Proposition 3 *No cartel is active if $\delta < \delta_n$. Whenever $\delta \in (\delta_k, \delta_{k-1})$ and $\delta_k < 1$, a cartel of k firms is active.*

The fact that for $\delta < \delta_n$ no cartel is active comes from Proposition 1. Therefore we have only to explain the second part of the Proposition. For $\delta_{k-1} > \delta \geq \delta_k$ only cartels of size greater or equal than k can be sustained. Cartels of sizes greater than k are not stable, because the result in Proposition 2 applies: internal stability does not hold.

The cartel of size k is internally stable, because firms know that quitting the cartel means that collusion fully collapses and they would obtain the Cournot profits. Therefore the cartel of size k is stable. That is only the smallest cartels among those which can be sustained is stable in the Participation Game.

Once characterized the equilibrium of the participation game, we see that there are two different corollaries we can extract from Proposition 3.

Simply comparing Proposition 2 and Proposition 3 we get the following conclusion:

Corollary 1 *If $\delta \in (\delta_n, 1)$ the size of active cartels is bigger with implicit collusion than with explicit collusion.*

In explicit collusion cartels are always effective because when members collude they have to commit themselves by signing binding contracts. However in implicit collusion as firms do not dispose of any commitment, when $\delta > \delta_n$ as we saw in Proposition 3 a cartel of certain size is stable. It is precisely the success of the cartels what reduces the incentive to participate in them in explicit collusion.

In the previous Section, we checked that cartels were only active if the discount factor was high enough. Therefore prices were increasing in the discount factor. In the present Section, the size of the cartel is determined endogenously. Then price may decrease with

the discount factor, because it reduces the size of stable cartels. The failure of small cartels to be sustainable when δ is low, induces firms to create bigger cartels. This result is recollected in the following corollary:

Corollary 2 *When the size of the cartel is endogenously determined, if $\delta \in (\delta_n, 1)$ price decreases with the discount factor.*

The reason is basically that as long as the cutoff of the discount factor is decreasing with k , for low δ , only bigger cartels are sustained. Thus as δ increases, smaller cartels associated to lower prices are enforced. We saw that for low δ , as long as no agreement is possible, the price is the Nash equilibrium price.

4 Optimal punishment

The literature about *implicit* collusion has treated repeated game models using basically two different types of strategies to enforce subgame perfect Nash equilibria (S.P.N.E.), the "trigger strategies" and the "optimal punishment" strategies⁸. Trigger strategies have been used in the first three sections of the model. We obtained that the cutoff of the discount factor is decreasing with the size of the cartel, and this led us to the results of the third section. We will see in this section, if it is also true when cooperation is sustained by the optimal punishment strategies.

Cooperation is sustained now with strategies where cheating firms are punished with the fastest and most severe possible punishment. Abreu (1986) outlines a symmetric, two-phase output path that sustains collusive outcomes for an oligopoly of quantity setting firms. The output path considered by Abreu has a "stick and carrot" pattern. The path begins with a period of low per-firm output for cartel members (q_m^k). The strategy calls for all cartel members to continue to produce q_m^k , unless an episode of defection occurs. If some firm cheats on the agreement, all cartel firms expand output for one period (q_m^p) and return to the most collusive sustainable output in the following periods, provided that every player of the cartel went along with the first phase of the punishment. On the other hand, fringe firms just individually maximize per period profits. The optimal punishment strategies for a partial cartel can be formulated in the following way, where $q_{t,i}$ denotes the strategy played by firm i in period t :

Firm i , $i = 1, \dots, k$ plays:

$$\left\{ \begin{array}{l} q_{1,i} = q_m^k \\ q_{t,i} = q_m^k \text{ if } q_{t-1,j} = q_m^k \text{ for } j = 1, \dots, k \ \forall t = 2, 3, \dots, \\ q_{t,i} = q_m^k \text{ if } q_{t-1,j} = q_m^p \text{ for } j = 1, \dots, k \ \forall t = 2, 3, \dots, \\ q_{t,i} = q_m^p \text{ otherwise.} \end{array} \right.$$

Firm i , $i = k + 1, \dots, n$ plays:

⁸The latter were firstly set in a seminal paper from Abreu (1986). This strategies became popular in the literature given their optimality and their renegotiation-proofness quality.

$$\left\{ \begin{array}{l} q_{1,i} = q_n^k \\ q_{t,i} = q_n^k \text{ if } q_{t-1,j} = q_m^k, \text{ for } j = 1, \dots, k \quad \forall t = 2, 3, \dots, \\ q_{t,i} = q_n^k \text{ if } q_{t-1,j} = q_m^p, \text{ for } j = 1, \dots, k \quad \forall t = 2, 3, \dots, \\ q_{t,i} = q_{nm}^p \text{ otherwise.} \end{array} \right.$$

$$q_{nm}^p = \arg \max_q P(kq_m^p + (n - k - 1)q_{nm}^o + q)q - \frac{q^2}{2}.$$

Where q_{nm}^o is the quantity produced by the rest of the fringe firms, that in equilibrium would be q_{nm}^p , q_m^k is the quantity that maximizes cartel members joint profits and q_m^p is the quantity that corresponds to the stick stage for cartel members. We see now how do we obtain the corresponding outputs for the strategies to be a S.P.N.E.

Again, member firms will behave optimally if the discount factor is high enough. To obtain the conditions on the discount factor such that optimal punishment is also optimal for member firms, we need again the profits of a member firm that deviates from the agreement. The punishment is considered optimal as long as it sustains the highest range of collusive outcomes among all possible punishment phases, see Abreu (1986), if this condition holds:

$$\pi_m^k(q_m^p) + \frac{\delta}{1 - \delta} \pi_m^k(q_m^k) = 0 \quad (7)$$

That is, whenever any player deviates from the desired equilibrium path, that player can be punished by players switching to the worst possible equilibrium. Thus, discounted profits for the punishment path should equal the Minimax value (0 in our model).

Therefore, (q_m^p) is such that (7) holds. As we did for the trigger strategies, we can define π^d like the profits obtained when firms deviate from q , applying the one-period best response function. We need then conditions for the punishment to be a S.P.N.E. for both stages of the punishment:

$$\pi^d(q_m^p) - \pi_m^k(q_m^p) \leq \delta(\pi_m^k(q_m^k) - \pi_m^k(q_m^p)) \text{ no deviation in the stick stage} \quad (8)$$

$$\pi^d(q_m^k) - \pi_m^k(q_m^k) \leq \delta(\pi_m^k(q_m^k) - \pi_m^k(q_m^p)) \text{ no deviation in the carrot stage} \quad (9)$$

From (8) and (7) we obtain that no deviation in the stick stage is only possible if $\pi^d(q_m^p) = 0$, since otherwise a firm can deviate in the first period and keep doing so every

time the punishment is reimposed. Hence the total output produced by $(k - 1)$ firms must be large enough that $P((k - 1)q_m^p) \leq 0$, which set a lower bound on the quantity produced in the stick stage, independent of δ , and which also means that $q_{nm}^p = 0$.

From (9) and (7) we obtain that no deviation in the carrot stage is obtained if $\frac{1}{1-\delta}\pi^k(q_m^k) \geq \pi^d(q_m^k)$.

Summarizing, the way to build up the optimal punishment consists of taking the lowest bound on q_m^p from the condition $\pi^d(q_m^p) = 0$ as the value assigned in the punishment phase to q_m^p . From (8) we obtain the threshold of the discount factor for the stick stage. Afterwards from the condition $\frac{1}{1-\delta}\pi^k(q_m^k) \geq \pi^d(q_m^k)$ we obtain the threshold of the discount factor for the carrot stage.

We call δ_a , and δ_b the discount factors that make (8) and (9) hold with equality respectively. Therefore, from (7) and (8) we have that δ must be no smaller than δ_a . and from (7) and (9) that δ must be no smaller than δ_b . So, the strategies described sustain a S.P.N.E if:

$$\delta \geq \max\{\delta_a, \delta_b\} \tag{10}$$

It is easy to see that δ_a is decreasing with k as long as is easier to punish as the size of the cartel increases, that is, the condition $P((k - 1)q_m^p) \leq 0$ more easily holds.

On the other hand δ_b increases with k . As we see in (10), the envelope from above of both, represents the cutoff of the discount factor that sustain the strategies described before as an equilibrium, that is δ . When the number of firms is small, the first effect dominates.

Thus, we obtain the following result:

Proposition 4 *The cutoff discount factor that sustain the strategies with the optimal punishment, is decreasing in the size of the cartel, if the number of firms n , is low enough.*

5 Asymmetric case

We will consider in this section a very simple example with heterogeneous firms to see whether and how the incentives to join the cartel or the fringe depend on cost asymmetry comparing also both types of collusion.

As it is commonly believed, asymmetry hurts collusion. It becomes clear when the cartel is formed by all firms of the industry (see Rothschild, R. (1999)). However we will see in this section whether asymmetry can help or not to enforce partial cartels.

We consider a market comprising 3 firms indexed by i , $i = 1, 2, 3$. If q_i denotes the production of firm i , the cost functions of firms are given by: $c_i(q_i) = \frac{q_i^2}{2}$ for $i = 1, 2$ and $c_i(q_i) = dq_i^2$ if $i = 3$, where $d > 0$. Then firm 3 is the less efficient firm if $d > \frac{1}{2}$ and the more efficient firm if $d < \frac{1}{2}$.⁹ Firms compete à la Cournot in a market whose demand is given by $P(Q) = 1 - Q$ where $Q = \sum_{i=1}^3 q_i$.

We assume that these market conditions are stationary and firms compete in infinite periods indexed by $t = 1, 2, 3, \dots$. Firms discount the future at a rate of δ . Each cartel member obtains the profits of selling and producing the output quota assigned by the cartel. This means that we exclude the possibility of side-payments among the members of the cartel.

We assume that the cartel maximizes joint profits.

5.1 Equilibrium

In this Section, we look for the size and the nature of the cartel that can be implemented in our simplified example. For that purpose we firstly determine the sustainability of the cartel in the repeated game and afterwards, following the same structure of the previous section, we set the stable cartels of the *explicit* collusion model. Finally, the last result establishes the equilibrium of the participation game.

⁹Cost functions are also assumed quadratic in the asymmetric case to rule out uninteresting cases, as long as it would be clearly practical for the firm with the lowest cost to produce the entire cartel output if marginal costs were constant.

5.1.1 Cartel sustainability.

We are going to study which strategies can sustain the different cartels as equilibria of the infinitely repeated game. We look for the equilibria when firms play "trigger strategies". As we said they consist of the immediate and unreversible switch to the non-collusive solution once cheating is discovered, so that the threat consists of reversion to Nash-Cournot forever if someone breaks the agreement.

To study sustainability we have to see the conditions referred before about $\gamma(k)$. We get that the threshold calculated to sustain the several cartel configurations are decreasing with d for firms 1 and 2 and increasing for firm 3.

The following Proposition summarizes the results for the three possible market configurations.

Proposition 5 *The partial asymmetric cartel is never sustainable. The partial symmetric cartel is sustainable if $d > 0.77$ and $\delta \in (\delta_{2,1}, 1)$. The full cartel is sustainable if $0.41 < d < 1.118$ and $\delta \in (\max[\delta_{3,1}, \delta_{3,3}], 1)$.*

Where $\delta_{n,i}$ = Discount factor needed by the firm i , to sustain a cartel of n members¹⁰.

We see that full collusion requires a certain degree of homogeneity among firms. Meanwhile a partial cartel will only be stable when the fringe is relatively inefficient compared to the cartel .

5.1.2 Cartel stability.

We will see now the equilibrium of the participation game described in section three for the case of our simplified example, and for that we will set first the equilibrium of the explicit collusion model.

Applying the concepts of internal and external stability defined before, the following Proposition specifies the type of cartels that are stable.

Proposition 6 *When collusion is explicit, if $d < 0.77$ no cartel is stable and if $d > 0.77$ only the symmetric partial cartel is stable.*

¹⁰Notice here that as long a partial asymmetric cartel is never sustainable, $\delta_{2,1}$ refers always to a partial symmetric cartel. That is the discount factor needed to sustain an agreement between firm 1 and 2.

What we can see here is that the stability of the cartels depend basically on two things; first in a cartel there can not be a too inefficient firm as long as it would be punished with a very low production quota inside the cartel. Second, if the fringe is very efficient it can take advantage of the low production of the cartel. That is the free riding of the fringe on cartel production can make the cartel unprofitable.

Last Proposition leads us to the following result:

Proposition 7 *The full cartel is stable if either $0.41 < d < 0.77$ and $\delta \in (\max[\delta_{3,1}, \delta_{3,3}], 1)$ or $0.77 < d < 0.85$ and $\delta \in (\delta_{3,1}, \delta_{2,1})$. Partial symmetric cartel is stable if $d > 0.77$ and $\delta \in (\delta_{2,1}, 1)$.*

That means therefore that full collusion can only be stable when there exists certain degree of homogeneity among firms. The partial cartel can only be stable when is formed by efficient firms in the cartel and the inefficient firm remains in the fringe.

From comparing Proposition 8 with Proposition 10, one can check that if either $0.41 < d < 0.77$ and $\delta \in (\max[\delta_{3,1}, \delta_{3,3}], 1)$ or $0.77 < d < 0.85$ and $\delta \in (\delta_{3,1}, \delta_{2,1})$ cartel size is greater when collusion is implicit than when it is explicit. This result may seem a bit surprising at first sight, because explicit collusion makes cartels always successful. Indeed, when δ is low, cartels are completely ineffective with implicit collusion. However, it is again the success of cartels what reduces the incentives to participate in them. For the range of parameters just mentioned, full cartel is stable with implicit collusion, because firms know that if they do not participate, again, no degree of collusion is possible, because partial cartels are not sustainable. On the other hand, full cartel is not stable with explicit collusion, because each wants to abandon the cartel in order to free-ride from a partial cartel.

We also have that when the asymmetry within firms is large, that is when d grows, if δ is close to one, asymmetry collapses full collusion, but enhances formation of partial cartels.

For instance if δ is large enough, when $d = 0.75$, full collusion is stable. If the market becomes more asymmetric, $d = 0.8$, full collusion collapses. However, a partial cartel is enforced.

Therefore it turns out to be the case that asymmetry does not completely collapses collusion if partial cartels can be enforced.

6 Conclusions

The main aim of the paper has been basically to analyze partial collusion under the two main different approaches of the literature. The implicit collusion model approach with two different types of strategies to enforce collusion, showed that the larger the cartel, the easier to sustain. When collusion is explicit, that is firms can meet somehow and can sign a binding contract, then it has been proved that the incentives to free ride the cartel play a central role, therefore only very small cartels can be enforced.

To be able to compare both models, a participation game has been set. In this model the interaction between the incentive and the participation constraints, takes place. The main conclusion has been that implicit collusion can enforce larger cartels than explicit collusion, becoming therefore perhaps of greater concern for antitrust authorities.

It has been noticed in a very simple example that although asymmetry among firms does not help collusion, it can enhance the stability of partial cartels joining homogeneous firms in a collusive agreement.

It has also been noticed that mergers among fringe firms can facilitate collusion, as long as firms that cooperate suffer less free riding when competition outside is less severe. However it has been left for future research a deeper analysis.

7 Appendix

Proof. Proposition 1: We have $\gamma(k) = \frac{\pi^d - \pi^k}{\pi^d - \pi^1}$. If we calculate $\frac{\partial \gamma(k)}{\partial k}$, we have that it is the following expressions in our model:

$$-24 \frac{28n-60k-84nk+48k^2+24n^2k^2-12nk^3+60nk^2-39n^2k+30n^2-12k^3+2n^4+13n^3-3n^2k^3+3n^3k^2-6n^3k+8}{(9k^3-18nk^2-45k^2+5n^2k+2nk-16k+28+28n+7n^2)^2}$$

It is tedious but straightforward to show that, as long as $k \leq n$, we obtain that the derivative is negative. ■

Proof. Proposition 2: The conditions for stability are the following:

Internal stability:

$$2 \frac{2k+1}{(nk-k^2+3k+2+n)^2} \geq \frac{3}{2} \frac{k^2}{(n(k-1)-(k-1)^2+3k-1+n)^2}$$

External stability:

$$(k+1) \frac{\frac{3}{2}k+\frac{3}{2}}{(nk-k^2+3k+2+n)^2} \geq 2 \frac{2k+3}{(n(k+1)-(k+1)^2+3k+5+n)^2}$$

We can show that the expression of Internal stability is decreasing in k . Therefore showing that the condition does not hold at $k = 3$ so it also proves that coalitions of $k \geq 3$ are not stable. When $k = n = 2$ cooperation is sustainable. For $k = 2$, we can see in the internal stability that if $n \geq 3$ there are incentives to leave the cartel. ■

Proof. Proposition 3: The Nash equilibrium of the game is: $w_i = 0 \forall i$, no collusive agreement if $\delta < \delta_n$. If $\delta \in (\delta_n, \delta_k)$ k firms cooperate, for $k = (3, \dots, n)$. If $\delta \geq \delta_2$, 2 firms cooperate whenever $k = 2$ is stable in the *explicit* collusion, and $w_i = 0 \forall i$ otherwise.

If $\delta < \delta_n$ no cartel configuration is sustainable, therefore $w_i = 0 \forall i$. If $\delta \in (\delta_n, \delta_k)$, among all the cartels that are sustainable, (k, \dots, n) and $k \geq 3$, only the smallest could be enforced. This is because two reasons, external stability is hold and increasing in k for $k \geq 2$, therefore no firm wants to join the cartel. When $\delta \in (\delta_n, \delta_k)$, although internal stability tells us if $k \geq 3$ always a firm wants to leave the cartel, a cartel of k is the only stable, as long as $k - 1$ firms is not sustainable, and if a firm leaves the cartel, its profits will turn to Nash-Cournot, which is worse than any cooperation profits. When $\delta \geq \delta_2$ that is all cooperation configuration is possible, the cooperation game becomes the same as *explicit* collusion, thus the results are the same. ■

Proof. Corollary 2: This is straightforward to show, only seeing that the price of the market is decreasing with k . Therefore, as the configuration enforced in the market involves smaller cartels, prices decrease. ■

Proof. Proposition 4: To see this, we should check the participation constraints, first

let see the respective profits of the firms: a) Cartel 3 for $x = \frac{1-d}{2(1+5d)}$ e) Cartel of 2 for $x = \frac{5(1+2d)^2}{8(4+5d)^2}$
b) Cartel 3 for $z = \frac{1}{4(1+5d)}$ f) Cartel 2 for $z = \frac{9(1+d)}{4(4+5d)^2}$ c) Cournot $x = \frac{3(1+2d)^2}{8(3+4d)^2}$ g) Efficient member of Cartel of 2 asym = $8d \frac{1+3d}{(5+16d)^2}$ d) Cournot $z = \frac{1+d}{(3+4d)^2}$ h) Efficient non-member of Cartel of 2 asym = $\frac{3(1+4d)^2}{2(5+16d)^2}$ i) No-efficient member of Cartel of 2 asym = $4 \frac{1+3d}{(5+16d)^2}$. Therefore the conditions for the coalitions to hold both statements are the following: 1) Cartel of 3 if $a > c, b > d, b > f, a > h$ 2) Cartel of 2 if $e > c, f > b, f > d$ 3) Asymmetric Cartel of 2 if $g > c, g > h, i > d$ 1) $\frac{1-d}{2(1+5d)} \geq \frac{3(1+2d)^2}{8(3+4d)^2}$ if $d > 0.23$, $\frac{1}{4(1+5d)} \geq \frac{1+d}{(3+4d)^2}$ if $d < 1.118$, $\frac{1}{4(1+5d)} \geq \frac{9(1+d)}{4(4+5d)^2}$ if $d < 0.337$, $\frac{1-d}{2(1+5d)} \geq \frac{3(1+4d)^2}{2(5+16d)^2}$ if $d > 1.29$ 2) $\frac{5(1+2d)^2}{8(4+5d)^2} \geq \frac{3(1+2d)^2}{8(3+4d)^2}$ if $d > 0.77$ $\frac{9(1+d)}{4(4+5d)^2} \geq \frac{1}{4(1+5d)}$ if $d > 0.337$, $\frac{9(1+d)}{4(4+5d)^2} \geq \frac{1+d}{(3+4d)^2}$ always 3) $8d \frac{1+3d}{(5+16d)^2} \geq \frac{3(1+2d)^2}{8(3+4d)^2}$ if $d > 0.55$ $8d \frac{1+3d}{(5+16d)^2} \geq \frac{3(1+4d)^2}{2(5+16d)^2}$ never, $4 \frac{1+3d}{(5+16d)^2} \geq \frac{1+d}{(3+4d)^2}$ if $d < 0.48$ Therefore, we got the conclusion that only symmetric partial cartel is stable and only for values of d above 0.77. ■

Proof. Proposition 5: Take from the incentive constraints of the cartel 2 and the cartel 3, the function of the discount factor. Cartel 3: For 1 $\delta \geq -\frac{60d+148d^2+160d^3+64d^4+9}{-42d-355d^2-532d^3-124d^4}$ and for 3 $\delta \geq -\frac{320d^4+384d^3+20d^2-78d-9}{-300d^4-360d^3-41d^2+48d+4}$ We can see that this cartel can only be sustained for $d \in (0.41, 1.118)$. We can also easily see that the operative discount factor is the one from 3 for values above 0.5 to 1.118. Take the incentive constraint of cartel 2: $\delta \geq -\frac{9-24d-16d^2}{24d+31d^2}$, we see that it is only possible for values above 0.7, so just comparing the different thresholds we obtain that when both types of cartels can be sustained and that partial asymmetric cartel can never be sustained. ■

Proof. Proposition 6: If $0.41 < d < 0.77$ or if $0.77 < d < 0.85$ and $\delta \in (\delta_{3,1}, \delta_{2,1})$ full collusion is stable as long as is the only that can be sustained whereas if either $0.77 < d$ and $\delta \in (\delta_{2,1}, 1)$ partial symmetric cartel can be sustained and it is stable because even though it could be the case that full collusion was sustainable, firm 3 would always quit, making the cartel unstable. ■

Proof. Proposition 7: We obtain the cutoff δ for both stages of the punishment phase, where the envelope of both will be the significative cutoff that sustain the strategies. It is easy to show that $\delta \geq \frac{(nk-k^2+3k+2+n)^2}{10k^3-19k^2+12k+12+4n+10nk+n^2k^2-2nk^3+4nk^2+2n^2k+n^2+k^4}$ and $\delta \geq \frac{8}{3}(k-1)^2 \frac{k^2-2k+1}{10k^3-19k^2+12k+12+4n+10nk+n^2k^2-2nk^3+4nk^2+2n^2k+n^2+k^4}$ are respectively decreasing and increasing with k . Therefore the minimum value of the decreasing δ will be at $k = n$. So we just have to calculate up to which value the decreasing part is above the increasing part.. Thus the envelope from above of both cutoffs is decreasing with k . So

we get that this happens when the expression $-\frac{1}{3} \frac{-20n-1+2n^4-8n^3}{4n^3-2n^2+4n+3}$ is positive, that is when $n < 5$. ■

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