

# FORECASTING TIME-VARYING COVARIANCE MATRICES IN INTRADAILY ELECTRICITY SPOT PRICES

Ángel León and Antonio Rubia

WP-AD 2002-10

Correspondence to A. Rubia: University of Alicante. Departamento de Economía Financiera, Contabilidad y Marketing, Campus San Vicente del Raspeig, 03071 Alicante. Antonio.Rubia@ua.es

Editor: Instituto Valenciano de Investigaciones Económicas, S.A.

First Edition July 2002.

Depósito Legal V-2751-2002

IVIE working papers offer in advance the results of economic research under way in order to encourage a discussion process before sending them to scientific journals for their final publication.

---

\* We thank Sabino Mastrángelo for his kindness in providing data and also for his comments and suggestions. We have also benefited from discussions with Gabriele Fiorentini, Ignacio Peña, Gonzalo Rubio, Eduardo Schwartz and Enrique Sentana. Financial support from the Ivie is gratefully acknowledged. The content of this paper is the sole responsibility of the authors.

\*\* University of Alicante.

# FORECASTING TIME-VARYING COVARIANCE MATRICES IN INTRADAILY ELECTRICITY SPOT PRICES

Ángel León and Antonio Rubia

## A B S T R A C T

This paper deals with analysing and forecasting intradaily volatility in electricity spot prices. We analyse the hourly spot prices from the Argentine Electricity Market by grouping prices in three daily series (block bids). We estimate the VAR model for the conditional mean structure and several multivariate analysis based on the multivariate GARCH models, specifically the orthogonal GARCH by Alexander (2000) and the constrained multivariate GARCH by Engle and Mezrich (1996). We also measure the forecasting performance of the daily block bid volatilities and covariances under both approaches obtaining similar results. This methodology could be used for managing risk of block bid portfolios and also for the valuation of derivatives on intradaily time-blocks of electricity spot prices.

**Keywords:** Electricity Industry, Intradaily Volatility, value-at-risk models.

**JEL Classification:** C32,C53.

# 1 Introduction

Due to the recent deregulation process in the electricity industry, electricity is traded nowadays as a commodity in many countries. However, this is not a standard commodity since it is a non-storable good. This nature is responsible for the key features that electricity prices display, such as high volatility and infrequent jumps. Since demand needs to be continuously balanced with power supply without the possibility of handling stores, electricity markets are typically organised as competitive pools where all the generated electricity is pooled and scheduled to meet the overall electricity demand. This deregulation process has been accompanied by the introduction of competitive wholesale electricity markets and power derivative contracts in both OTC and exchange-traded markets. These financial tools are useful in managing risk because of the high volatility of electricity prices. Since the deregulation process has been recently carried out over a large number of countries worldwide, a very few markets have an organised derivatives market so far. For example, in the Nordic Power Exchange (NordPool), which is the most advanced power exchange, we can find power contracts of one-hour duration along with block bids in the spot power exchange (Elspot market), meanwhile forwards, futures (Eltermin market) and options (Eloption market) are traded in the financial power exchange.

Block bidding is the more recent innovation in the trading process of power markets. Under this trading modality, prices and volumes are settled in the day-ahead spot market by taking as a reference periods of time covering different intervals within the day, rather than the standard hour-to-hour basis. Block contracts are designed to attract trade by producers and consumers alike. Interest has been greatest among thermal-power producers: block bidding ensures an average price for a specified number of hours, which provides more efficient handling of start-up and shut-down costs, for example, than reliance on hourly bidding<sup>1</sup>.

---

<sup>1</sup>Note that power-thermal generation is much less flexible than hydraulic generation, and power stations work only efficient if they produce uninterrupted. Thus, holding a portfolio of block contracts is specially indicated to these sellers. For

In this paper, we adjust both the conditional mean and variance dynamics of intradaily electricity prices for block bids under a multivariate environment. This analysis suggests an immediate financial application in forecasting conditional covariance matrices, which are useful in managing the portfolio risk through the value-at-risk measure for those market agents holding block contracts portfolios. Since the portfolio value-at-risk analysis requires the correlation estimates as one of the inputs, the multivariate methodology has been selected.

We estimate the conditional mean of intradaily series by means of both the autoregressive vector (VAR) methodology and a deterministic function capturing the strong seasonal behaviour implied in the power commodities. Then, taking the multivariate residual error series from VAR model, we estimate the conditional covariance matrix by the multidimensional GARCH model. More specifically, we estimate the models proposed by Alexander (2000) and Engle and Mezrich (1996), which are easier to estimate than the Engle and Kroner (1995) multivariate simultaneous GARCH model. The latter is extremely difficult to implement because of the large number of parameters to be estimated in practice. With so many parameters, the likelihood function becomes very flat and severe convergence problems are very likely to occur in the optimization routine.

The data used in this paper are based on the hourly electricity prices from the Argentine Wholesale Electricity Market (MEM). The electricity industry in Argentina is the most important one after Brazil and Mexico in the Latin-American area. Thermal and nuclear resources mainly constitute the whole generation resources, while hydraulic resources are about the 46% of the whole production<sup>2</sup>. The MEM has carried out its progressive deregulation process early in the last decade and, therefore, it is possible to get long enough series quoted in a stable and fully competitive environment. This is the reason why this work has focused on this market.

---

further details about this trading modality, see the official Webpage of Nord-Pool (<http://www.norpdpool.com/products>).

<sup>2</sup>A deeper analysis covering all the characteristics of this power market is beyond the scope of this paper. See Mastrangelo (2001) for a review of this topic.

The outline of our paper is as follows. In Section 2 we estimate the VAR structure for the MEM block bid price series. In Section 3 we estimate the multivariate GARCH model under two different methodologies. Section 4 measures the forecasting performance of the two alternative multivariate GARCH models under both the in-sample and out-of-sample analyses. Finally, the main conclusions are summarised in Section 5.

## 2 VAR analysis for block bids

The Argentine wholesale electricity market (MEM) consists mainly of an organised Spot Market where electricity is traded on an hourly basis in the market sessions. The time series we analyse in this paper are several intradaily averages obtained through subsets of the 24 hourly spot price series (measured in dollars per megawatt hour, \$/MWh) that balance the aggregated supply and demand on the Spot Market every hour each day<sup>3</sup>. To avoid market irregularities in the early years, the sample period was restricted to the period from 1/01/1996 to 30/06/2000, i.e., 1,643 daily observations covering each hour of the day<sup>4</sup>. We thereafter denote  $P_{jt}$  as the spot price traded each hour,  $j = 1, \dots, 24$ , during the daily sample period  $t = 1, \dots, T = 1643$ . In this paper, we consider three intradaily blocks of hours. The main structures are called ‘*off-peak hours I*’ (comprising hours of minimum demand, that is, from 24.00 to 5.00) and ‘*peak hours*’ (comprising hours of maximum demand, that is, from 19.00 to 23.00). The third block comprises the leaving hours (i.e., hours from 6.00 to 18.00), which will be referred through the paper as ‘*off-peak hours II*’. Notice that in the NordPool market there are also three block biddings which divide the 24-hour period roughly the same as here, and market agents are allowed to trade electricity by fixing prices and volumes for each block.

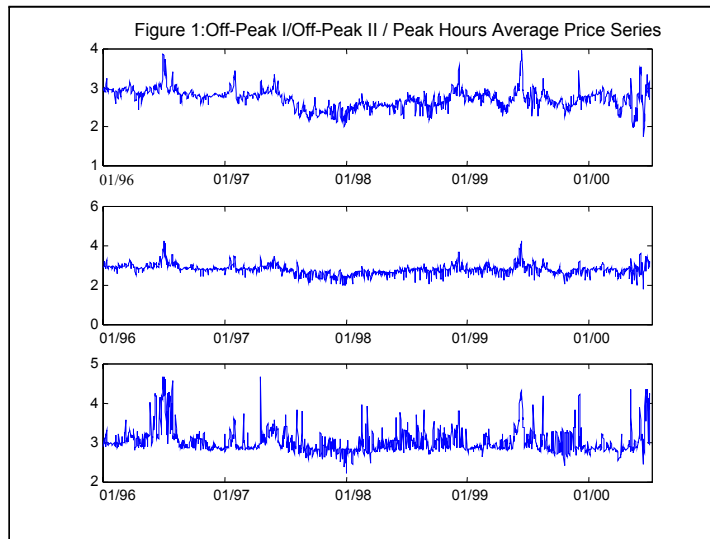
Let  $\mathbf{Z}_t = (\mathbf{z}_{1t}, \mathbf{z}_{2t}, \mathbf{z}_{3t})'$ , where  $\mathbf{z}_{1t} = \ln \left[ \left( P_{24t} + \sum_{j=1,5} P_{jt} \right) / 6 \right]$  denotes the daily observation of the log transformation for the mean corresponding

---

<sup>3</sup>The data used in this paper are available from the official Webpage of the Argentina’s Power Market Operator ([www.//cammesa.com.ar](http://cammesa.com.ar)).

<sup>4</sup>The latest reforms were developed in 1995. The Power Market is considered stabilised since that date. Authors are indebted to Sabino Mastrangelo (CAMMESA) for his valuable suggestions about this issue.

to the off-peak block I,  $\mathbf{z}_{2t} = \ln \left[ \sum_{j=6,18} P_{jt} / 13 \right]$  for the off-peak II block and finally,  $\mathbf{z}_{3t} = \ln \left[ \sum_{j=19,23} P_{jt} / 5 \right]$  for the peak block. We apply the usual log transformation on each average series since the volatility does not remain constant over the sample period. It is well known that a fundamental feature in electricity prices is the existence of a strong intradaily seasonality pattern, that is, prices show a characteristic pattern regarding the particular hour in which is traded. Figure 1 displays the graphics of the above series, showing quite clearly similar, although different, dynamics in the intradaily average prices. For instance, notice that the peak mean price series exhibits a more volatile behaviour than the others series, as well as a larger mean value.



Since we are modelling multivariate time series, the VAR approach is used for  $\mathbf{Z}_t$ . It is worth noting that this method fits the dynamics of a set of endogenous, covariance-stationary variables, which are assumed to be observed simultaneously. The latter statement is of particular application in data from power markets, since intradaily electricity prices are day-ahead prices and are settled at the same time. Regarding to the assumption of stationarity, the seasonal unit roots test by Hylleberg et al. (1990) (HEGY henceforth) was implemented to test for stationarity in the long run (zero frequency) as well as for all the seasonal frequencies related to weekly

seasonality, since daily electricity prices series show a strong weekly seasonal pattern<sup>5</sup>. The test results show that the three series are stationary in both the zero and seasonal frequencies.

Definitively, the autoregressive vector model of order  $p$ , denoted as  $\text{VAR}(p)$ , for a covariance stationary random vector  $\mathbf{Z}_t = (\mathbf{z}_{1t}, \mathbf{z}_{2t}, \dots, \mathbf{z}_{Nt})'$  is represented as follows:

$$\mathbf{Z}_t = \boldsymbol{\mu}_t + \sum_{j=1}^p \mathbf{B}_j \mathbf{Z}_{t-j} + \boldsymbol{\varepsilon}_t \quad (1)$$

where  $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}_{1t}, \boldsymbol{\varepsilon}_{2t}, \dots, \boldsymbol{\varepsilon}_{Nt})'$  is a white noise vector with zero mean and positive-definite covariance matrix  $\mathbb{E}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \Omega$ . The  $\mathbf{B}_j$  are unknown diagonal coefficient matrices and, finally,  $\boldsymbol{\mu}_t$  denotes the deterministic mean vector to which the system reverts. This term might include (deterministic) seasonal and nonseasonal trends. Wolak (1997) applied this methodology on electricity spot prices by fitting VAR models on time series quoted in an hourly and half-hourly basis from several deregulated markets. In our case, we have constrained the dimension of the multivariate system to  $N = 3$ . The identification of model (1) is performed by the usual Schwartz Information Criterion (SIC) on different specifications about  $\boldsymbol{\mu}_t$  and different values of  $p$ . The SIC statistics for the more plausible specifications are shown in Table 1.

**Table 1: SIC Statistics VAR(p):** SIC statistics given different lag-orders of the VAR(p) process (first column) and given the following specifications for the deterministic mean of the process:

$$\begin{aligned} Z_t &= \mu_{kt} + \sum_{j=1,p} B_j Z_{t-j} + \varepsilon_t & \mu_{2t} &= \mu_{1t} + \lambda t \\ \mu_{1t} &= c + \sum_{j=2,7} \gamma_j D_{(day)jt}; & \mu_{2t} &= \mu_{1t} + \lambda t \\ \mu_{3t} &= \mu_{1t} + \alpha \cos(2\pi t/365); & \mu_{4t} &= \mu_{1t} + \beta \cos(2\pi t/365 + \phi) \end{aligned}$$

---

<sup>5</sup>The extension of the HEGY test to the case of weekly seasonality in daily data is due to León and Rubia (2001) and Rubia (2001). The results of applying such test are not presented here but are available from authors on request.

$p$	$\mu_{1t}$	$\mu_{2t}$	$\mu_{3t}$	$\mu_{4t}$
1	-3.684	-3.683	-3.750	-3.745
2	-3.758	-3.754	-3.795	-3.787
3	-3.776	-3.770	-3.800	-3.791
4	-3.748	-3.742	-3.768	-3.758
5	-3.745	-3.738	-3.759	-3.748
6	-3.749	-3.742	-3.758	-3.747
7	-3.807	-3.799	<u>-3.814</u>	-3.802
8	-3.776	-3.768	-3.783	-3.772
9	-3.745	-3.736	-3.751	-3.740
10	-3.731	-3.722	-3.734	-3.723
14	-3.620	-3.611	-3.621	-3.609
21	-3.394	-3.833	-3.397	-3.385

According to Table 1, the VAR process seems to include a significant deterministic seasonal term, fitted by 1/0 dummies in each day of the week (the best model is remarked in that table). The effect of annual seasonality seems to be less important since the gain from including a sinusoidal function on the annual frequency is rather poor, though it seems to be relevant in statistical terms. Including other explanatory terms such as a linear time trend, however, does not seem to be appropriate. Finally, the lag order that performs better results is equal to seven. This order seems to be not casual, since it is likely that the seasonal dynamics of electricity prices includes stochastic terms. So, the selected specification, which clearly reflects the main stylised features in the dynamics of electricity price series, is given by:

$$\mathbf{Z}_t = \mathbf{c} + \delta \cos(2\pi t/3654) + \sum_{j=2}^7 \gamma_j \mathbf{D}_{(day)jt} + \sum_{j=1}^7 \mathbf{B}_j \mathbf{Z}_{t-j} + \boldsymbol{\varepsilon}_t \quad (2)$$

The above regression model is then estimated by maximum likelihood (ML) under the usual assumption of normality in the error term. The goodness of fit (see Table 2), measured by the usual  $\bar{R}^2$ , varies from an 85% for off-peak hours to a 62% in the peak hours. Meanwhile, the fit in the remaining hours is about a 76%. $\varepsilon$



**Table 2: Estimation Results of the VAR(7) model:** Main results from the estimation of the VAR(7) model:

$$Z_t = c + \sum_{j=2,7} \gamma_j D_{(day)jt} + \alpha \cos(2\pi t/365) \sum_{j=1,7} B_j Z_{t-j} + \varepsilon_t$$

Statistics	$z_{1t}$	$z_{2t}$	$z_{3t}$
$R^2$	0.85	0.77	0.63
$\bar{R}^2$	0.85	0.76	0.62
Std. Dev.	0.10	0.13	0.19
<b>Residuals</b>			
Q(100)	108.5 (0.26)	116.8 (0.12)	113.4(0.17)
Q(200)	216.7 (0.20)	203.1 (0.42)	189.2 (0.70)
Q*(100)	766.9 (0.00)	758.0 (0.00)	249.3 (0.00)
Q*(200)	950.6 (0.00)	868.8 (0.00)	379.69 (0.00)

Q(m) and Q\* (m) denote the Ljung-Box statistics (p-value in brackets) for both the standard residual and the squared residuals.

In order to ensure that the past of each variable is really useful in forecasting the remaining variables, the Granger (1969) causality test is applied by performing an F-test on the joint significance of the coefficients of all lags of each time series in the VAR representation. These results are reported in Table 3 showing a clear Granger-causality relationship between off-peak and remaining hours in both directions. The peak price series seems to be little useful when forecasting the other two variables, meanwhile the evidence that both  $z_{1t}$  and  $z_{2t}$  do Granger-cause  $z_{3t}$  seems to be somewhat weak.

**Table 3: Granger Causality Test Statistics:** Statistics of joint significance (p-values in brackets) of the Granger Causality test. The null hypothesis states that the i-th variable of the VAR model (first column) does not Granger-Cause each one of the remaining variables of the system.

	$z_{1t}$	$z_{2t}$	$z_{3t}$
$z_{1t}$	- - -	11.552 (0.00)	1.690 (0.10)
$z_{2t}$	37.006 (0.00)	- - -	1.975 (0.05)
$z_{3t}$	8.054 (0.00)	9.795 (0.00)	- - -

As it could be expected, a sight at the squared residuals in Table 2 reveals a clear evidence for conditional heteroskedasticity through the statistical-significant Ljung-Box tests, denoted as  $Q^*(m)$ . These patterns are treated in greater detail in the next section.

### 3 Modelling conditional covariance matrix

In this section we estimate alternative time-varying covariance matrix models for the residuals from equation (2). Since the multivariate GARCH representation by Engle and Kroner (1995) involve very complex models characterised by the estimation of a large number of parameters as well as heavy restrictions to ensure positive-definite covariance matrices, more suitable alternatives satisfying both a few parameters to estimate and a few constraints to guarantee positive definiteness are widely used in practice. Specifically, we will estimate the conditional covariance matrix (CCM onwards) through the Orthogonal GARCH (OGARCH) model (Alexander, 2000) and the constrained multivariate GARCH (MGARCH) model proposed by Engle and Mezrich (1996). Both models have been developed to cope with the time-dependent volatility of portfolios that include a great number of assets in financial and capital markets. Recently, the OGARCH methodology has also been applied on electricity spot prices and future prices from NordPool in order to obtain the hedging ratio (Byström, 2000).

Rewriting equation (2) in a more general setting to explicitly recognise the time-varying conditional volatility:

$$\mathbf{Z}_t = \mathbb{E}(\mathbf{Z}_t | \Psi_{t-1}) + \boldsymbol{\varepsilon}_t; \quad \boldsymbol{\varepsilon}_t \sim Niid(0, \Omega); \quad \boldsymbol{\varepsilon}_t | \Psi_{t-1} \sim Niid(0, \mathbf{H}_t) \quad (3)$$

where  $\mathbb{E}(\mathbf{Z}_t | \Psi_{t-1})$  denotes the conditional mean, and the unconditional and conditional covariance matrices of the unpredictable term are denoted as  $\Omega = \{\sigma_{ij}\}$  and  $\mathbf{H}_t = \{h_{ij,t}\}$  respectively. The complete estimation is performed in two stages, as follows: In the first stage, we determine the ML estimations of the conditional mean of the process, so the results are just the same than those obtained in Section 2. We then compute the VAR residuals and describe the CCM dynamics of the unpredictable component using both the OGARCH and MGARCH models, by applying the ML procedure once again.

### 3.1 Orthogonal GARCH (Alexander, 2000)

Denote by  $\mathbf{P}$  the  $N \times N$  orthonormal matrix of eigenvectors of  $\Omega$ . Thus, the symmetrical covariance matrix can be decomposed as  $\Omega = \mathbf{P}\Lambda\mathbf{P}'$ , where  $\Lambda$  is the diagonal matrix of eigenvalues of  $\Omega$ . The Orthogonal GARCH is based on the application of the principal component analysis (PCA hereafter) to identify the main sources of variation of the multivariate system associated to each eigenvalue of  $\Omega$ . This allows us to generate a basis of orthogonal factors whose volatility is then individually treated from the univariate perspective. The set of principal components of a multivariate system, say  $\mathbf{Y}_t = (\mathbf{y}_{1t}, \mathbf{y}_{2t}, \dots, \mathbf{y}_{Nt})'$ , is just defined as  $\mathbf{Y}_t = \mathbf{P}'\boldsymbol{\varepsilon}_t$  where  $\mathbb{E}(\mathbf{Y}_t\mathbf{Y}_t') = \Lambda$  due to the orthogonal property of  $\mathbf{P}$ . Under the assumptions of the OGARCH model, the conditional covariance matrix of  $\mathbf{Y}_t$ ,  $\mathbb{E}_{t-1}(\mathbf{Y}_t\mathbf{Y}_t') = \mathbf{P}'\mathbf{H}_t\mathbf{P}$ , is also a diagonal matrix that we denote as  $\mathbf{V}_t$ . Since  $\mathbb{E}(\mathbf{V}_t) = \Lambda$ , we can estimate the CCM of  $\boldsymbol{\varepsilon}_t$  as  $\hat{\mathbf{H}}_t = \mathbf{P}\hat{\mathbf{V}}_t\mathbf{P}'$ , denoted as  $\mathbf{H}_t^O$  and its elements as  $h_{ij,t}^O$  henceforth. This conditional covariance matrix is called OGARCH when the diagonal matrix  $\mathbf{V}_t$  of conditional variances for principal components is estimated by using the univariate GARCH (1,1) method.

The results of the PCA show that the variability of the whole system is mainly explained through a first factor associated with the highest eigenvalue. This factor is able to forecast about a 71% of the total variance. The second factor can explain about a 21% and the last factor only explains the 8% of common variability. The univariate GARCH(1,1) estimations on each principal component are exhibited in Table 4, being all the relevant parameters significant.

**Table 4: Principal Components GARCH estimates:** Main results of the GARCH(1,1) estimation on the  $i$ -th principal component. In brackets, p-values from the robust estimations (Bollerslev and Wooldridge, 1992). The diagnosis of the residual shows the statistics of the normality test of Jarque-Bera (J-B test), and their p-values in brackets.  $Q(m)$  and  $Q^*(m)$  represent the Ljung-Box statistics for the residuals and the squared ones respectively. (p-values in brackets).

	Factor I	Factor II	Factor III
$\omega_i$	0.04 (0.00)	0.001 (0.00)	0.000 (0.00)
$\alpha_i$	0.231 (0.00)	0.094 (0.00)	0.088 (0.00)
$\beta_i$	0.704 (0.00)	0.867 (0.00)	0.887 (0.00)
<b>Residuals</b>			
Average	0.000	0.000	0.000
Std. Dev.	0.210	0.115	0.072
Skewness	1.029	0.714	-0.423
Kurtosis	10.119	5.757	7.344
J-B test	3743 (0.00)	656 (0.00)	1335 (0.00)
$Q(100)$	102.72 (0.40)	138.01 (0.00)	114.74 (0.00)
$Q(200)$	181.05 (0.82)	222.88 (0.12)	231.20 (0.06)
$Q^*(100)$	62.05 (0.99)	124.40 (0.05)	107.01 (0.30)
$Q^*(200)$	113.10 (0.99)	210.69 (0.28)	250.48 (0.00)

### 3.2 Multivariate GARCH model (Engle and Mezrich, 1996)

Useful restrictions are obtained from the multivariate GARCH representation by Engle and Kroner (1995). The simple multivariate GARCH model that is analyzed in this paper is the one discussed by Engle and Mezrich (1996) and Bourgoin (2000). This model proposes the restriction that the long

run covariance matrix equals the sample covariance matrix. Specifically, we estimate the following restricted model in the CCM dynamics:

$$\mathbf{H}_t = (1 - \alpha - \beta) \mathbf{S} + \alpha \boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}'_{t-1} + \beta \mathbf{H}_{t-1} \quad (4)$$

where  $\mathbf{S}$  is the sample covariance matrix and  $\alpha, \beta$  are non-negative parameters constrained to sum to less than one. Note that the number of parameters in this specification is constant and always equal to 2, regardless the dimension of  $\mathbf{Z}_t$ , so that this method is very efficient from a computational point of view. Since the assumption  $\boldsymbol{\varepsilon}_t | \Psi_{t-1} \sim Niid(0, \mathbf{H}_t)$  still holds, the log-likelihood function of the model (dropping the constant term) is given by:

$$L(\alpha, \beta; \mathbf{Z}_t) = -\frac{1}{2} \sum_{t=1}^T (\ln |\mathbf{H}_t| - \boldsymbol{\varepsilon}'_t \mathbf{H}_t^{-1} \boldsymbol{\varepsilon}_t) \quad (5)$$

The results of the MGARCH estimation are also consistent in showing once again the strong persistence of power price volatility. The estimated parameters are  $\hat{\alpha} = 0.081$  and  $\hat{\beta} = 0.871$ , so that the sum is close to one.

## 4 Forecasting conditional covariance matrices

We now compare the forecasting performance for both MGARCH and OGARCH models under both in sample and out of sample environments. The benchmark mean squared error (MSE) is selected to measure forecasting ability. The MSE will be computed in both the univariate and multivariate frameworks as follows:

$$\begin{aligned} MSE_1 &= \frac{1}{T} \sum_{T=1}^T \left( \varepsilon_{i,t} \varepsilon_{j,t} - \hat{h}_{ij,t} \right)^2; \quad i, j = 1, 2, 3. \\ MSE_2 &= \frac{1}{T} \sum_{T=1}^T \left\| \text{vec} \left( \mathbf{S}_t - \hat{\mathbf{H}}_t \right) \right\|^2 \end{aligned} \quad (6)$$

where the  $\text{vec}(\cdot)$  operator stacks the columns of a matrix in a column vector and  $\|\cdot\|$  denotes the Euclidean norm of a vector<sup>6</sup>. Finally,  $\mathbf{S}_t = \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t$  is

---

<sup>6</sup>Notice that  $MSE_2$  is based on matrix metric, which is not subordinate to any vector norm, known as the Euclidean matrix norm of Frobenius.

the realised covariance matrix and the sequence of matrices  $\{\hat{\mathbf{H}}_t\}_{t=1,T}$ , whose elements are denoted by  $\hat{h}_{ij,t}$ , are the estimations of the conditional covariance matrix through both methods, i.e., either  $\mathbf{H}_t^O$  (when the OGARCH model is used) or  $\mathbf{H}_t^M$  (when the MGARCH is applied).

The results about the performance under the in-sample context are shown in Table 5. The MGARCH performance seems to be slightly better than the OGARCH for both  $MSE_1$  and  $MSE_2$  metrics in the covariance cases, although it does not in the variance analysis. Anyway, there seems to be minor differences in the performance of both models.

**Table 5. In-sample MSE:** In-sample MSE (x100) from OGARCH and MGARCH models. Columns show the univariate MSE measurements (i.e.,  $MSE_1$  in (6)) for both the conditional variance ( $h_{ii,t}$ ) and the conditional covariance ( $h_{ij,t}$ ). The last column shows the multivariate MSE metric (i.e.,  $MSE_2$  in (6) for each method).

	$h_{11,t}$	$h_{22,t}$	$h_{33,t}$	$h_{12,t}$	$h_{13,t}$	$h_{23,t}$	$MSE_2$
OGARCH	0.0587	0.0911	0.299	0.0601	0.0858	0.106	0.953
MGARCH	0.0594	0.0915	0.290	0.0601	0.0858	0.105	0.943

The OGARCH forecast of the covariance matrix for period  $t+k$  at time  $t$  is given by the projection  $\mathbb{E}_t(\mathbf{H}_{t+k}^O) = \mathbf{P}\mathbb{E}_t(\mathbf{V}_{t+k})\mathbf{P}'$ , where the diagonal matrix  $\mathbf{V}_{t+k}$  contains the GARCH (1,1) forecasts of the principal components for period  $t+k$  at time  $t$ , that is:

$$\mathbb{E}_t(v_{ii,t+k}) = \hat{\omega}_i \left[ \frac{1 - (\hat{\alpha}_i + \hat{\beta}_i)^{k-1}}{1 - (\hat{\alpha}_i + \hat{\beta}_i)} \right] + (\hat{\alpha}_i + \hat{\beta}_i)^{k-1} v_{ii,t+1} \quad (7)$$

where  $v_{ii,t+1} = \hat{\omega}_i + \hat{\alpha}_i y_{it}^2 + \hat{\beta}_i v_{ii,t}$  is the forecast for period  $t+1$  for the  $i$ -th principal component.

The MGARCH forecasts are determined by the projection  $\mathbb{E}_t(\mathbf{H}_{t+k}^M)$ , whose elements are:

$$\mathbb{E}(h_{ij,t+k}^M) = s_{ij} + (\hat{\alpha} + \hat{\beta})^{k-1} (h_{ij,t+1} - s_{ij}); \quad i, j = 1, 2, 3; \quad k \geq 2 \quad (8)$$

where  $h_{ij,t+1}^M = s_{ij} + \hat{\alpha}\varepsilon_{i,t}\varepsilon_{t} + \hat{\beta}h_{ij,t}$  is the forecast for period  $t + 1$ .

We consider the out-of-sample period from 01/07/2000 to 18/09/2000. Table 6 shows the forecasting performance under both  $MSE_1$  and  $MSE_2$  metrics over different horizons, specifically 1, 5, 10 and 15 days. As in the previous case, the analysis of the forecasting performance does not seem to find significant differences between the OGARCH and MGARCH methods. As a result, the MGARCH model seems to be more suitable than the OGARCH method in the case analysed in this paper, since it yields similar performance results and implies a lesser computational burden.

**Table 6. Out-of-sample MSE:** Out-of-sample MSE from OGARCH and MGARCH models. Columns show the univariate MSE measurements (i.e.,  $MSE_1$  in (6)) for both the conditional variance forecasts ( $h_{ii,t+k}$ ) and the conditional covariance ones ( $h_{ij,t+k}$ ) at time  $t$  for different periods at time  $t + k$  where  $k=1,5,10,15$ . The last column shows the multivariate  $MSE_2$  measurement (see equation (6)) for each method.

	$h_{11,t}$	$h_{22,t}$	$h_{33,t}$	$h_{12,t}$	$h_{13,t}$	$h_{23,t}$	$MSE_2$
OGARCH							
1 day	0.0130	0.0217	0.08044	0.0137	0.0097	0.0188	0.1994
5 days	0.0142	0.0232	0.0829	0.0135	0.0095	0.0186	0.2035
10 days	0.0131	0.0242	0.0862	0.0151	0.0097	0.0185	0.2101
15 days	0.0155	0.0246	0.0855	0.0140	0.0095	0.0180	0.2083
MGARCH							
1 day	0.0140	0.0224	0.0796	0.0134	0.0099	0.0193	0.2013
5 days	0.0147	0.0236	0.0805	0.0136	0.0095	0.0188	0.2028
10 days	0.0148	0.0246	0.0832	0.0141	0.0095	0.0182	0.2062
15 days	0.0152	0.0250	0.0852	0.0141	0.0197	0.0187	0.2103

## 5 Concluding remarks

The aim of this paper is to forecast the multivariate conditional volatility for portfolios containing intradaily spot electricity block bids, after proposing a suitable modelling. These contracts are being successfully implemented in the more recent power markets, since they provide a natural way of facing the uncertainty implied in electricity prices. The method proposed in this paper could be a useful tool in order to manage the risk implied by the high volatility of the intradaily power price. We have applied some methodologies characterized as simple multivariate conditional volatility models by using the OGARCH and MGARCH models. Both models get the estimation of parameters under a feasible computational way. The main conclusion of the forecasting performance between both approaches is that they give similar results.

A priority in this paper has been to provide an intuitive tool which could be really implemented by market agents. Of course, there is an implicate trade-off between simplicity and realism in doing so. This methodology could be appropriate to the development of some extensions trying to cover more complex structures, such as the infrequent extreme jumps, or pricing basket options taking a block bid portfolio as underlying asset. These topics are undoubtedly interesting challenges for further research.



## References

- [1] Alexander, C.O.; (2000) “A Primer on the Orthogonal GARCH Model”, unpublished manuscript, ISMA Centre, University of Reading, UK.
- [2] Bollerslev, T.; Wooldridge, J.M.; (1992) “Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances”, *Econometric Reviews*, 11, pp. 143-172.
- [3] Bourgoin, F.; (2000) “Large Scale Conditional Correlation Estimation”, *Advances in Quantitative Asset Management*, edited by C.L. Dunis, Kluwer Academic Publishers.
- [4] Byström, H.N.; (2000) “The Hedging Performance of Electricity Futures on the Nordic Power Exchange Nord Pool”, Working Paper 2000:15, Lund University, Department of Economics.
- [5] Engle, R.F.; Kroner, K.F.; (1995) “Multivariate Simultaneous Generalized GARCH”, *Econometric Theory*, 11, pp. 122-150.
- [6] Engle, R.F.; Mezrich, J.; (1996) “GARCH for Groups” *Risk Magazine*, 9, pp 36:40.
- [7] Granger, C.W.J.; (1969) “Investigating Causal Relations by Econometric Models and Cross-Spectral Methods,” *Econometrica*, 37, pp.424-438.
- [8] Hylleberg, S.; Engle R.F.; Granger, C.W.J.; Yoo, B.S.; (1990) “Seasonal Integration and Cointegration”, *Journal of Econometrics*, 44, pp. 215-238.
- [9] León, A.; Rubia, A.; (2001) “Comportamiento del Precio y Volatilidad en el Pool Eléctrico Español”, Instituto Valenciano de Investigaciones Económicas, WP-2001-04 ([www.ivie.es](http://www.ivie.es))
- [10] Mastrángelo, S.; (2001) “Legislación y Regulación del Mercado Eléctrico”, en *Introducción al Conocimiento del Mercado Eléctrico Mayorista*, Buenos Aires, Instituto de Tecnología.

- [11] Rubia, A.; (2001) “Testing for Weekly Seasonal Unit Roots in Daily Electricity Demand: Evidence from Deregulated Markets”, Instituto Valenciano de Investigaciones Económicas, WP-2001-21. ([www.ivie.es](http://www.ivie.es))
- [12] Wolak, F.A.; (1997) “Market Design and Price Behavior in Restructured Electricity Markets: An International Comparison” Working Paper PWP-051, University of California Energy Institute.