ON THE CHOICE OF A POWER INDEX*

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A B S T R A C T

In this paper, we review and compare the main power indices to be found in the literature, that is to say, the Shapley-Shubik index, the Banzhaf index, the Johnston index, the Deegan- Packel index, and the Holler-Packel index. Three different approaches are used. First, the power indices are compared on the basis of a priori properties that they should satisfy. Second, a player's power is interpreted as the probability of her or his vote being necessary to pass a proposal. We present two models consistent with this approach: a model in which this probability arises from the assumptions made on the coalition formation and Straffin's model, in which it arises from the assumptions made on the players' voting behavior. Third, a player's power is defined as her or his probability of playing a crucial role. Some power indices are derived from different assessments concerning the probability of playing such a role. Illustrations of the choice of an index are given for actual decision-making processes.

Keywords: Collective Decision-Making; Power Indices.

1 INTRODUCTION

Simple superadditive games are often used to model decision-making processes. In such models, it is assumed that the procedure to make a decision is basically the following. A proposal is made to the voting body. Each voter says either "yes", or "no", but never abstains[1]. According to the set of rules that prevails in the voting body, the proposal is accepted or rejected. Therefore a decision-making process is fully specified once the voters and the decision-making rules are given. Thus, in what follows, by a "decision-making process", we mean a set of rules that specifies the way in which a given voting body must proceed to make decisions, stating in particular which groups of voters can make a decision if they agree, whatever the others do. So, we never consider any particular application of the decision-making rules to any proposal at stake.

In this context, measures have been developed in order to assess the a priori distribution of power in the voting body. That is, the distribution of power among the voters for a given set of decision rules. These measures only take into account the information embodied in the decision-making process modelled as a simple superadditive game. Of course, the voters' personal characteristics and preferences, or their interpersonal relations influence the particular outcome of any particular issue which is voted upon. But such information is deliberately left aside[2]: power measures are intended to quantify the a priori implications of decision-making rules, not any particular application of them.

These measures are however useful to highlight inherent characteristics of decision-making processes. For instance, they show that power is not directly proportional to voting weights, as the intuition would suggest at first sight. This can be illustrated in the following example. In a voting body of three voters having respectively 4, 2 and 1 votes, it is easy to see that the first voter can act as a dictator if half of the votes are required to make a proposal pass, in the sense that her or his support is necessary and sufficient to make any proposal pass. The ratio of voting power is then 1:0:0, while the ratio of voting weight is 4:2:1. In the same voting body, with the same distribution of votes, the a priori distribution of power will be different if 7 votes are required to make a proposal pass. In such a case, all voters' support is necessary to make a proposal pass and then the ratio of voting power is 1:1:1. This simple example also shows that the number of votes required to make a proposal pass has to be taken into account to assess the distribution of power.

The main power indices to be found in the literature are the Shapley-Shubik index, the

[1] See Felsenthal and Machover (1998) for a model that allows abstention.

[2] Some models have however been developed to include the relations between the players. This is done by incorporating coalitions structures into the games (see for instance Myerson (1977) or Owen (1977, 1982)) or restricting the coalitions that are allowed (see for instance Edelman (1997) or Bilbao et al. (1998)).

Banzhaf index, the Johnston index, the Deegan-Packel index, and the Holler-Packel index. Although all these measures have been defined in the framework of simple superadditive games, the results they give may differ quite widely. So far, no agreement has been reached among the scholars concerning the choice of the most suitable power index, and the picture given by the literature may be confusing. For instance, in most papers concerning the European Union decision-making process, the choice of the power index used is not theoretically justified. In other papers, the choice of the power index is not even made: results are given for different power indices. Some arguments in favor of the Banzhaf index can however be found in the literature. For instance, Felsenthal and Machover (1997) claim that the Banzhaf index is the most appropriate power index in the European context. Their argumentation is based on two different definitions of power, namely, the power to influence the outcome and the power to share a purse. Laruelle and Widgrén (1998) use Straffin's probabilistic model (1977) to justify the choice of the Banzhaf index for a normative analysis of the vote distribution in the European Council of Ministers. These two papers also illustrate the fact that power indices can be interpreted in more than one way.

The aim of this paper is to provide some ground on which to found the choice of a power index. To achieve this aim, three different approaches are used to compare the above-mentioned indices. First, the indices are compared on the basis of a priori properties. The axioms characterizing the power indices are reviewed, and their interpretation is given and discussed in the context of decision-making processes. Other a priori desirable properties are also considered: in such a case, we check whether the above mentioned indices satisfy them or not. Second, a player's power is interpreted as her or his probability of being a swinger in the coalition that passes a decision. The power indices can then arise either from different assumptions made on the players' voting behavior, or from different assumptions on the coalition formation. Third, a player's power is defined as her or his probability of playing a crucial role in the decision-making process.

The rest of the paper is organized as follows. In Section 2, we provide the cooperative game theoretical background. In Section 3, we present the different power indices and their original motivation. In Section 4, the power indices are compared on the basis of a priori properties that they should satisfy. In Section 5, a player's power is interpreted as her or his probability of being a swinger. In Section 6, a player's power is interpreted as her or his probability of playing a crucial role. Section 7 concludes with some remarks and some lines for further research.

2 GAME THEORETICAL BACKGROUND

A cooperative transferable utility (TU) game is a pair (N, v), where $N = \{1, ..., n\}$ denotes the set of players and v a real-valued function defined on the subsets of N to R, assigning the value 0 to the empty set. Any non empty subset of N is called a coalition. An arbitrary coalition is denoted by S, s representing its number of players. A game is symmetric if the value of a coalition only depends on its size. The monotonicity condition requires that $v(S) \leq v(T)$ whenever $S \subseteq T$. A game is superadditive if $v(S) + v(T) \leq v(S \cup T)$ for all coalitions S and T such that $S \cap T = \emptyset$.

In $(0, 1)$-games, the function v only assigns the values 0 and 1. In such games, we refer to coalitions with the property $v(S) = 1$ as winning, while to those with $v(S) = 0$ as losing. Let $\mathcal{W}(v)$ denote the set of all winning coalitions. A player i is a swinger in a winning coalition S if his or her removal from the coalition makes it losing, that is to say, if $v(S) = 1$ and $v(S \setminus \{i\}) = 0$. A swing is a pair (i, S) such that player i is a swinger in coalition S. A minimal winning coalition is a winning coalition in which all players are swingers. Let $\mathcal{M}(v)$ denote the set of all minimal winning coalitions. In the following, we will use $m(v)$ (resp., $m_i(v)$) to denote the total number of minimal winning coalitions (resp., the number of minimal winning coalitions containing player i). A simple game is a (0-1)-game such that v is not identically 0 and is monotonic. A simple game is proper if the complement of a winning coalition is always losing: $v(S) + v(N \setminus S) \leq 1$ for any $S \subset N$. In the context of simple games, this property is equivalent to the superadditivity property. A (0-1)-game is weighted if there exist a quota Q and a weight $w_i > 0$ for each player i $(i = 1, ..., n)$ such that

$$v(S) = \begin{cases} 1 & \text{if } \sum\limits_{i \in S} w_i \geq Q \\ 0 & \text{otherwise.} \end{cases}$$

Note that to represent such a game, it is sufficient to specify the quota and the distribution of weights, that is, Q and $(w_1, ..., w_n)$. A (0-1) weighted game is superadditive if and only if the quota is strictly larger than half of the sum of all players' weights.

A (0-1)-game can model a decision-making process, that is to say, the voting body and the decision-making rules. The set of players represents the voting body. The winning coalitions are defined as those which can make a decision without the vote of the remaining players. A decision-making process generally displays the following properties. The unanimity of players leads to the passage of a proposal, which implies that v is not identically 0. Any subset of a losing coalition cannot be winning, that is to say, the corresponding game is monotonic. Two nonintersecting coalitions cannot make opposite decisions at the same time: the game is thus proper. The first two properties require that the game is simple. In the context of simple games, the third property is equivalent to the superadditivity condition. A decision-making process can thus be modelled as a simple superadditive game. Many decision-making processes can be represented by weighted games[3].

3 MAIN POWER INDICES

In a seminal paper in game theory, Shapley (1953) axiomatically characterizes the prospect of having to play a TU-game, referred to as the Shapley value. The Shapley-Shubik index is the application of this value to simple superadditive games. Shapley and Shubik (1954, p. 788) justify their index by giving the following interpretation: "Let us consider the following scheme: There is a group of individuals all willing to vote for some bill. They vote in order. As soon as a majority has voted for it, it is declared passed, and the member who voted last is given credit for having passed it. Let us choose the voting order of the members randomly. Then we may compute the frequency with which an individual belongs to the groups whose votes are used and, of more importance, we may compute how often he is pivotal." In a simple superadditive game (N, v), the Shapley-Shubik index is given by $\varphi(v) = (\varphi_1(v), ..., \varphi_n(v))$, where

$$\varphi_i(v) = \sum_{\substack{S \subseteq N \\ S \ni i}} \frac{(s-1)!(n-s)!}{n!} \left[v(S) - v(S \setminus \{i\})\right]. \tag{1}$$

Banzhaf (1965) considers that the Shapley and Shubik's power measure makes use of unreasonable assumptions about the political process, arguing that: "It has not been shown that the order in which votes are cast is significant." According to him, a power index should depend on the number of combinations, rather than on the number of permutations of players. He proposes then the so-called normalized Banzhaf index. Dubey and Shapley (1979) note that the normalization of the Banzhaf index (so that the players' indices add up to 1) is not as innocent as it might seem. They present then what we refer to as the Banzhaf index[4], that they consider in many respects more natural than the normalized version. In a simple superadditive game (N, v), the Banzhaf index is given by $\beta(v) = (\beta_1(v), ..., \beta_n(v))$, where

$$\beta_i(v) = \frac{1}{2^{n-1}} \sum_{\substack{S \subseteq N \\ S \ni i}} \left[v(S) - v(S \setminus \{i\})\right]. \tag{2}$$

[3] Although some decision-making processes do not appear as weighted, they can be represented as weighted games (for instance, the UN Security Council). For a characterization of weighted games, see Taylor and Zwicker (1992).

[4] We give here the usual name of the index, although it was first proposed by Penrose in 1946. More precisely, Penrose's measure was half of the Banzhaf index.

The following normalization:

$$\tilde{\beta}_i(v) = \frac{\beta_i(v)}{\sum\limits_{k \in N} \beta_k(v)} \tag{3}$$

gives the normalized Banzhaf index, that we denote $\tilde{\beta}(v)$.

According to Laver (1978) a measure of power should depend on the number of swingers in the coalition. He argues that: "it is clear that a party's power will be greater if it is the only destroyer of a particular coalition than if that honor is shared with a number of others." In response to Laver's argument, Johnston (1978) proposes a modification of the normalized Banzhaf index, which is referred to as the Johnston index. In a simple superadditive game (N, v), let $\varkappa(S)$ denote the number of swingers in a winning coalition S. We can then calculate $\gamma(v) = (\gamma_1(v), ..., \gamma_n(v))$ where

$$\gamma_i(v) = \sum_{\substack{S \subseteq N \\ S \ni i}} \frac{1}{\varkappa(S)} \left[v(S) - v(S \setminus \{i\}) \right]. \tag{4}$$

Note that the summation is only done on the coalitions in which there is at least one swinger. The Johnston index, that we denote $\tilde{\gamma}(v)$, is obtained by the following normalization:

$$\tilde{\gamma}_i(v) = \frac{\gamma_i(v)}{\sum\limits_{k \in N} \gamma_k(v)}. \tag{5}$$

In his book about coalition formation, Riker (1962) argues that: "In n-person, zero-sum games, where side payments are permitted, where players are rational, and where they have perfect information, only minimal winning coalitions occur[5]" (Riker (1962), p. 32). Deegan and Packel (1978) propose an index based on Riker's argument. Their index can be derived from the assumption that all minimal winning coalitions have the same probability of forming, and the assumption that the swingers in a minimal winning coalition share equally the "spoils". Note that this last assumption can be considered as another response to Laver's argument. In a simple superadditive game (N, v), the Deegan-Packel (1978) index is then given by $\rho(v) = (\rho_1(v), ..., \rho_n(v))$, where

$$\rho_i(v) = \frac{1}{m(v)} \sum_{\substack{S \in \mathcal{M}(v) \\ S \ni i}} \frac{1}{s} \left[v(S) - v(S \setminus \{i\}) \right]. \tag{6}$$

Holler and Packel (1983) argue that the Shapley-Shubik index, the Banzhaf index and the Deegan-Packel index "face the problem of distributing (or assigning) the value of a priori coalitions among their members. There might be no adequate solution to this problem, for the coalition value is a collective good. The private good approach, as implied

[5]Strictly speaking, this principle only applies for strong games, that is to say, for games such that $v(S) + v(N \setminus S) = 1$, which are equivalent to zero-sum games.

in the discussed indices, is hence inappropriate if voting is not only a matter of allocating spoils" (Holler and Packel (1983), p. 22). Therefore they propose a slight modification of the Deegan-Packel index, which can be derived from the assumption that all minimal winning coalitions have the same probability of forming, and the assumption that all swingers in a minimal winning coalition get all "spoils". In a simple superadditive game (N, v), let us call $\sigma(v) = (\sigma_1(v), ..., \sigma_n(v))$ the non-normalized Holler-Packel index, where

$$\sigma_i(v) = \frac{1}{m(v)} \sum_{\substack{S \in \mathcal{M}(v) \\ S \ni i}} [v(S) - v(S \setminus \{i\})] = \frac{m_i(v)}{m(v)}. \tag{7}$$

The Holler-Packel index, that we denote $\tilde{\sigma}(v)$, is obtained by the following normalization:

$$\tilde{\sigma}_i(v) = \frac{\sigma_i(v)}{\sum_{k \in N} \sigma_k(v)} = \frac{m_i(v)}{\sum_{k \in N} m_k(v)}. \tag{8}$$

4 COMPARISON BASED ON A PRIORI PROPERTIES

4.1 Axiomatization of the power indices

A natural way to assess the suitability of an axiomatized measure is to discuss the plausibility of the axioms. In the following, we review the underlying axioms of the different power indices and discuss their justification or interpretation.

The Anonymity axiom is common to all power indices.

Anonymity axiom: A player's measure of power does not depend on her or his name.

For any permutation π of N, $\Phi_{\pi(i)}(\pi v) = \Phi_i(v)$ where $(\pi v)(S) := v(\pi^{-1}(S))$.

The justification of this axiom is that the players' names should not matter when measuring the power, in the sense that interchanging the players' names cannot modify the distribution of power. In particular, it implies that if two players are swingers in the same coalitions, their power is equal.

The Null Player axiom is also common to all power indices.

Null Player axiom: A player who is never a swinger has a measure of power equal to zero.

If $v(S) = v(S \setminus \{i\})$ for all S, then $\Phi_i(v) = 0$.

The justification of this axiom is that what matters is the difference that players' votes make in coalitions, in the sense that only players whose vote makes a difference in some coalition may have some positive power. Therefore a player's power is null unless her or his vote can make winning at least one losing coalition.

The Relative Power axiom is common to all indices but the Banzhaf index:

Relative Power axiom: The players' measures of power add up to 1.

$$\sum_{i \in N} \Phi_i(v) = 1.$$

This axiom is often considered as a particular case of the condition $\sum_{i \in N} \Phi_i(v) = v(N)$ in the class of TU-games, where $v(N)$ is not constrained to be equal to 1. In this wider class of games, this condition guarantees an efficient distribution of $v(N)$, the value of the game, among the players. Such a requirement of efficiency is perfectly justified in contexts where $v(N)$ can be interpreted as a cake to be shared among the players. In the context of decision-making processes modelled as simple superadditive games, the fact that $v(N) = 1$ only means that N, the coalition formed by all players, makes any decision pass, as any other winning coalition does. Therefore the efficiency justification does not make sense in this context: there is no cake of size 1 to share. For this reason, we deliberately avoid giving the usual denomination of "efficiency" to this axiom.

This axiom can however be interpreted. One possible interpretation comes from the idea that power measures are probabilities of playing a relevant role[6] in any decision that has to be made. The axiom states then that in any decision that is made, one and only one player plays such a role (other axioms guaranteeing that the power measures are positive). A second possible interpretation of this axiom comes from the idea that power measures only matter in relative terms: what is important is the players' positions relative to each others. Therefore, the measures of power can be normalized in such a way that they always add up to a constant, in order to make comparison between players easier. The axiom sets up this constant to be equal to 1, which may be considered as an ad hoc normalization[7].

The Banzhaf index satisfies instead the following axiom:

Absolute Power axiom: The Players' measures of power add up to the total number of swings divided by the number of coalitions to which any player belongs.

$$\sum_{i=1}^{n} \Phi_i(v) = \tfrac{1}{2^{n-1}} \, \bar{\eta}(v), \text{ where } \bar{\eta}(v) = \sum_{i=1}^{n} \eta_i(v) \text{ and } \eta_i(v) = \sum_{\substack{S \subseteq N \\ S \ni i}} \left[v(S) - v(S \setminus \{i\}) \right].$$

Dubey and Shapley (1979) use this axiom to give a characterization of the Banzhaf index similar to the Shapley-Shubik index (substituting the Absolute Power axiom to the Relative Power axiom). This is however not sufficient to give any credit to the axiom.

[6] In sections 5 and 6 we examine in detail the meaning of this "relevant" role for the different indices satisfying this axiom.

[7] The justification of this axiom is thus much weaker in this particular context than in others.

One interpretation of this axiom comes from the idea that in order to compare a player's power through different decision-making processes, power has to be measured in absolute terms. That is to say, power measures cannot be normalized any more to add up to a constant: the sum may vary with the decision-making process. The axiom states that the lower the risk of status quo in a decision-making process, the larger is the sum of all players' measures of power. Indeed the total number of swings reflects the ease with which a decision can be made: the larger the number of swings, the easier a decision can a priori be made, and the lower the risk of status quo. Dubey and Shapley (1979, p. 106) propose the following interpretation of the total number of swings: "intuitively speaking, $\bar{\eta}(v)$ reflects the volatility or "degree of suspense" in the decision rule. It gives an indication of the likelihood of a close decision, i.e., one so close that a single voter could tip the scales. As discussed later, it is also a kind of democratic participation index, measuring the decision rule's sensitivity to the desires of the "average voter" or to the "public will"." Note that the above argument only justifies a positive correlation between the total number of swings and the total power to be distributed among the players but not the particular relation that appears in the axiom. In this respect, the coefficient 2^{n-1} may be considered as an ad hoc normalization.

A basic difference between the Relative Power axiom and the Absolute Power axiom is the following: in all symmetric n-players games, any index satisfying the Relative Power and the Anonymity axioms (as the Shapley-Shubik, Johnston, Deegan-Packel and Holler-Packel indices do) distributes equally a "unit of power", while the amount -also equally distributed- by any index satisfying the Absolute Power and Anonymity axioms (as the Banzhaf index does) depends on the decision rule. For instance, the Banzhaf index of each player is larger in a simple majority rule than in a unanimity rule, reflecting that it is a priori harder to make a decision in a unanimity rule. The choice between indices satisfying the Relative Power axiom or the Absolute Power axiom may depend on the purpose of the analysis to be carried out. This can be illustrated in the European decision-making. In the European Council of Ministers, all member states have, in relative terms, the same power under the simple majority of members rule or under the unanimity rule, which can be shown with any index satisfying the Anonymity and Relative Power axioms. The use of the Banzhaf index stresses the so claimed "too high frequency or deadlocks" under the unanimity rule. For instance, in the current 15-Members states Council of Ministers, each member state's Banzhaf index is equal to 0.000061 under the unanimity rule, while it increases to 0.209 under the simple majority rule.

The Transfer axiom is common to the Shapley-Shubik index and the Banzhaf index. Dubey and Shapley (1979) state it in terms of games (defining the games $v \vee w$ and

$v \wedge w$), mentioning that the axiom can be presented in a variety of mathematical guises. Although mathematically elegant, their formulation is not transparent in the context of decision-making processes. It is why we propose here an alternative formulation of the axiom, which clearly specifies the effect of changing the status of a coalition (from minimal winning into maximal losing) on the players' power.

Transfer axiom: The elimination of a minimal coalition from the set of winning coalitions has the same effect on the power measure of any game in which this coalition happens to be minimal winning. Formally, let (N, v) and (N, w) be two simple superadditive games. If $S \in \mathcal{M}(v) \cap \mathcal{M}(w)$ then

$$\Phi(v) - \Phi(\bar{v}^S) = \Phi(w) - \Phi(\bar{w}^S),$$

where for any game (N, v), $(N, \bar{v}^S)$ denotes the game such that $\mathcal{W}(\bar{v}^S) = \mathcal{W}(v) \setminus S$.

The DP-Mergeability is satisfied by the Deegan-Packel index. Here we propose an alternative formulation of the axiom that Deegan and Packel (1978) state in terms of games. First we need the following definition. Two games (N, v) and (N, w) are mergeable if no minimal winning coalition of game (N, v) includes or is included in a minimal coalition of game (N, w).

DP-Mergeability axiom: The elimination of a minimal coalition from the set of minimal winning coalitions has a similar effect (up to some weights) on the power measure of any pair of mergeable games in which this coalition happens to be minimal winning. Formally, let (N, v) and (N, w) be two mergeable, simple and superadditive games. If $S \in \mathcal{M}(v) \cap \mathcal{M}(w)$ then

$$m(v)\Phi(v) - m(\hat{v}^S)\Phi(\hat{v}^S) = m(w)\Phi(w) - m(\hat{w}^S)\Phi(\hat{w}^S),$$

where for any game (N, v), $(N, \hat{v}^S)$ denotes the game such that $\mathcal{M}(\hat{v}^S) = \mathcal{M}(v) \setminus S$.

The HP-Mergeability axiom is satisfied by the Holler-Packel index. Again, we reformulate here the axiom that Holler and Packel (1983) state in terms of games.

HP-Mergeability axiom: The elimination of a minimal coalition from the set of minimal winning coalitions has a similar effect (up to some weights) on the power measure of any pair of games in which this coalition happens to be minimal winning. Formally, let (N, v) and (N, w) be two mergeable, simple and superadditive games. If $S \in \mathcal{M}(v) \cap \mathcal{M}(w)$ then

$$\sum_{i \in N} m_i(v)\Phi(v) - \sum_{i \in N} m_i(\hat{v}^S)\Phi(\hat{v}^S) = \sum_{i \in N} m_i(w)\Phi(w) - \sum_{i \in N} m_i(\hat{w}^S)\Phi(\hat{w}^S).$$

From a technical point of view, the role of the Transfer (or any of the Mergeability axioms) in the axiomatizations of the power indices is the following. The Anonymity, and the Null Player axioms, together with either the Relative or the Absolute Power axiom, determine the evaluation of the distribution of power in all symmetric games (and in the unanimity games in particular). Then the Transfer axiom (or one of the Mergeability axiom) permits to extend this evaluation to all simple superadditive games. In fact, the Transfer axiom plays the role that linearity plays in the axiomatization of the Shapley value. Laruelle and Valenciano (1998) extend the class of simple superadditive games to include lotteries on decision-making processes. In this wider domain, they propose an axiomatization of the Shapley-Shubik and the Banzhaf indices using convex linearity (which does not make sense for simple superadditive games). The justification of the convex linearity axiom is then that a player's power in a lottery on decision-making processes is the expected power in the involved decision-making processes. It is worth noting that in this wider domain, the other indices do not satisfy convex linearity.

With respect to the interpretation in our context, in each of these three axioms, a condition is given concerning the effect on the index of a certain modification of the status of a minimal winning coalition. From the formulation it is clear that only minimal coalitions matter for the Mergeability axioms. With regards to the choice of an index, the Transfer seems a simpler condition if not more appealing. On the other hand, in certain contexts it can be reasonable to pay attention only to minimal winning coalitions.

The axioms that are satisfied by the Shapley-Shubik (φ), the Banzhaf (β), the Johnston ($\tilde{\gamma}$), the Deegan-Packel (ρ), and the Holler-Packel ($\tilde{\sigma}$) indices are summarized in Table 1. Note that these axioms allow to uniquely characterize all indices but the Johnston index that has never been axiomatized[8]. For a proof, see respectively Dubey and Shapley (1979), Deegan and Packel (1978), and Holler and Packel (1983).

[8]Note that the normalized Banzhaf index has never been axiomatized either, although van den Brink and van der Laan (1998) recently proposed an axiomatization of the normalized Banzhaf value.

	φ	β	$\tilde{\gamma}$	ρ	$\tilde{\sigma}$
Anonymity	Yes	Yes	Yes	Yes	Yes
Null Player	Yes	Yes	Yes	Yes	Yes
Relative Power	Yes	No	Yes	Yes	Yes
Absolute Power	No	Yes	No	No	No
Transfer	Yes	Yes	No	No	No
DP-Mergeability	No	No	No	Yes	No
HP-Mergeability	No	No	No	No	Yes

Table 1: Axioms satisfied by the indices.

The following comments can be made. There exists no axiomatic characterization of the Johnston index, which, in this respect, appears to be an ad hoc measure. Concerning the others, if the existence of an axiomatization is mathematically compelling, all axioms (but the Anonymity and the Null Player axioms) are not beyond any criticism in terms of justification. In the context of decision-making processes, the Relative Power axiom is not much less arguable than the Absolute Power axiom. The Transfer axiom and the Mergeability axioms are not justified. Dubey and Shapley (1979) mention that they do not attempt any heuristic justification of the Transfer axiom. Deegan and Packel (1978) admit that the DP-Mergeability axiom has no compelling universality. Holler and Packel (1983) recognize that there is no compelling story for the plausibility of the HP-Mergeability. Holler and Li note that "unfortunately, the axiom of mergeability is not very lucid" (Holler and Li (1995), p. 257). Finally, Holler admits that the HP-Mergeability "looks quite complex and perhaps difficult to justify" (Holler (1998), p. 188).

So, we can conclude that the axiomatic approach only gives some basic clues about the different philosophies embodied in the different indices that may help to choose in each particular application. A relative power index should be chosen when what matters is a player's power relative to the others' while an absolute power index may be more appropriate to compare distributions of power for different decision-making rules. Similarly, depending on the importance given to minimal winning coalitions, the choice might be oriented towards an index satisfying the Transfer axiom or one of the Mergeability axioms. The axiomatic approach does not, however, provide a sufficient basis for a choice irrespective of the decision-making process under consideration.

4.2 Postulates

Felsenthal and Machover (1995) do not consider the above set of axioms as very convincing. According to them, the Transfer axiom might have been introduced on a posteriori grounds, in order to guarantee the uniqueness of the result, rather than on grounds of a priori plausibility and the DP-Mergeability axiom does not afford any independent justification. As for them the literature does not contain any characterization of an index of voting power by means of an intuitively compelling set of axioms, they propose new a priori properties that an index of power should satisfy. Here, we review the properties proposed in Felsenthal and Machover (1995), and Felsenthal et al. (1998) and comment them. We extend their analysis in two ways. First, we do not restrict ourselves to relative indices, which allows us to take into account the Banzhaf index, while they just consider its normalized version. Second, we include the Holler-Packel index in the comparison. Note that we slightly differ with their presentation by expressing the Monotonicity and the Donation postulates in terms of weighted games[9] We make this choice because we consider that what is gained in generality or elegance is lost in transparence and intuition. Similarly, we only consider the Weak Bicameral Postulate, which appears to us as more intuitive than the Bicameral Postulate.

A priori, a reasonable measure of power should display the following properties in a decision-making process:

Bloc Postulate: If two voters always vote together so that they end up in forming a single voter, then the new voter has more power than each of the previous voters. Formally, it means that if player i annexes player j then player i must gain power unless player j had no power at all. Given a simple superadditive game (N, v), let us consider the game $(N \setminus \{j\}, v')$ such that: $v'(S) = v(S \cup \{j\})$ if S contains i, and $v'(S) = v(S)$ otherwise. The Bloc postulates states that:

$$\Phi_i(v^{'}) > \Phi_i(v) \text{ if } \Phi_j(v) > 0. \tag{9}$$

Monotonicity Postulate: In a weighted voting body, a voter with a larger voting weight cannot be worse off than a voter with a smaller voting weight. Formally, in a weighted game (N, v) where $(w_1, ..., w_n)$ denotes the distribution of weights, the Monotonicity Postulate states that:

$$\Phi_i(v) \geq \Phi_j(v) \text{ if } w_i > w_j. \tag{10}$$

Note that in non weighted games, the concept of dominance has first to be introduced in order to state the postulate.

[9] For a generalization of the postulates for simple superadditive games, see Felsenthal and Machover (1995).

Donation Postulate: In a weighted voting body, a voter cannot gain power by distributing some of her or his voting weight to other voters. Formally, let (N, v) be a weighted game, with distribution of weights $(w_1, ..., w_n)$ and quota Q. Now let us consider a weighted game (N, v') with quota Q too, and the distribution of weights $(w_1^{'}, ..., w_n^{'})$ such that $\sum_{j \in N} w_j^{'} = \sum_{j \in N} w_j$. The Donation Postulate states that:

$$\text{if } w_j^{'} \geq w_j \text{ for any } j \neq i \text{ and } w_i^{'} < w_i, \text{then } \Phi_i(v^{'}) \leq \Phi_i(v). \tag{11}$$

In non weighted games, the donation postulate is substituted by the transfer postulate.

Weak Bicameral Postulate: Let us consider a bicameral decision-making system where a bill must be accepted in two non intersecting chambers to be passed. If a decision-making process becomes one chamber of a bicameral decision-making system, the ranking of power in the initial decision-making cannot be reversed in the bicameral decision-making system: a voter who has initially more power than another one remains more powerful in the bicameral decision-making process. Formally, let consider the game (N, v) with $N = N^{'} \cup N^{''}$ and

$$v(S) = \begin{cases} 1 & \text{if } v^{'}(S \cap N^{'}) = 1 \text{ and } v^{''}(S \cap N^{''}) = 1 \\ 0 & \text{otherwise,} \end{cases}$$

where $(N^{'}, v^{'})$ and $(N^{''}, v^{''})$ are two simple superadditive games such that $N^{'} \cap N^{''} = \emptyset$. The Weak Bicameral Postulate states that:

$$\text{If } \Phi_{i^{'}}(v^{'}) < \Phi_{j^{'}}(v^{'}) \text{ then } \Phi_{i^{'}}(v) < \Phi_{j^{'}}(v).$$

Note that the Bicameral Postulate requires that the powers ratio are kept, that is to say:

$$\frac{\Phi_{i^{'}}(v^{'})}{\Phi_{j^{'}}(v^{'})} = \frac{\Phi_{i^{'}}(v)}{\Phi_{j^{'}}(v)},$$

which appears to us more difficult to justify.

Felsenthal and Machover (1995) and Felsenthal et al. (1998) show which properties are satisfied by the Shapley-Shubik index, the normalized Banzhaf index, the Johnston index and the Deegan-Packel index. We complete the analysis by showing the following:

Proposition 1 *The Banzhaf index satisfies (1) the Bloc postulate, (2) the Monotonicity postulate, (3) the Donation postulate and (4) the Weak Bicameral postulate.*

Proof: (1) Let $v, v^{'}, i$ and j be as in the Bloc postulate. Then, we got : $\beta_i(v^{'}) = \beta_i(v) + \beta_j(v)$, as shown in Felsenthal and Machover (1995). The Bloc postulate is thus satisfied.

(2) In a weighted game, if $w_i > w_j$ then player i is a swinger in at least as many coalitions as player j. Thus the Banzhaf index satisfies the Monotonicity postulate. (3) Let (N, v), $(N, v^{'})$ and i be as in the donation postulate. Any swing (i, S) of the game (N, v') remains a swing in the game (N, v). The Donation postulate is thus satisfied. (4) Felsenthal et al. (1998) prove that the normalized Banzhaf index satisfies the Bicameral postulate. From their proof, it is easy to see that the Banzhaf index also satisfies the Bicameral postulate and therefore the Weak Bicameral postulate.

Proposition 2 *The Holler-Packel index violates (1) the Bloc postulate, (2) the Monotonicity postulate, and (3) the Donation postulate. However, it satisfies (4) the Weak Bicameral postulate.*

Proof: (1) In the weighted game with distribution of weights $(4, 1, 1, 1, 1, 1)$ and quota $Q = 6$, consider the annexion of player 3 by player 2, which leads to the distribution of weights $(4, 2, 1, 1, 1)$. According to Holler-Packel index, the annexion decreases player 2's power, from $\sigma_2 = 3/8$ ($\tilde{\sigma}_2 = 6/51$) to $\sigma_2 = 1/4$ ($\tilde{\sigma}_2 = 2/25$). The Bloc postulate is thus violated. (2) Consider the weighted game with distribution of weights $(4, 2, 1, 1, 1, 1)$ and quota $Q = 6$. Although player 2's weight is larger than player 3's weight, Holler-Packel index gives a larger power to player 3 than to player 2: $\sigma_2 = 1/4$ and $\sigma_3 = 1/2$ ($\tilde{\sigma}_2 = 2/25$ and $\tilde{\sigma}_3 = 4/25$). The Monotonicity postulate is thus violated. (3) In the weighted game with distribution of weights $(4, 2, 1, 1, 1, 1)$ and quota $Q = 6$, let player 2 gives "1" to player 3, which leads to the distribution of weights $(4, 1, 2, 1, 1, 1)$. According to Holler-Packel index, the annexion increases player 2's power from $\sigma_2 = 1/4$ ($\tilde{\sigma}_2 = 2/25$) to $\sigma_2 = 1/2$ ($\tilde{\sigma}_2 = 4/25$). The Donation postulate is thus violated. (4) Let (N, v), (N', v') and $(N", v")$ be as in the Weak Bicameral postulate. Note that $S \in \mathcal{M}(v)$ if and only if $S = S' \cup S"$ where $S' \in \mathcal{M}(v')$ and $S" \in \mathcal{M}(v")$. Therefore we have: $m(v) = m(v').m(v")$. Similarly we have that $m_i(v) = m_i(v').m(v")$ for player i belonging to N' and $m_j(v) = m_j(v").m(v')$ for player j belonging to $N"$. Thus $\sigma_i(v) = \sigma_i(v')$ and $\sigma_j(v) = \sigma_j(v")$. The Weak Bicameral postulate is thus satisfied for the non normalized Holler-Packel index (and for the Holler-Packel index).

Table 2 summarizes the postulates that are satisfied by the different indices.

	φ	β	$\tilde{\beta}$	$\tilde{\gamma}$	ρ	$\tilde{\sigma}$
Bloc	Yes	Yes	No	No	No	No
Monotonicity	Yes	Yes	Yes	Yes	No	No
Donation	Yes	Yes	No	No	No	No
Weak Bicameral	No	Yes	Yes	No	No	Yes

Table 2: Postulates satisfied by the indices.

Note that the Shapley-Shubik index and the Johnston index satisfy the Weak Bicameral postulate for weighted games, and that the Banzhaf index, the normalized Banzhaf index and the Holler-Packel index also satisfy the Bicameral Postulate.

According to Felsenthal and Machover, a power index that violates more than one postulate should be disqualified as a suitable measure of power: "the fact that a given index suffers from one or more of these phenomena [that is to say, violates at least one postulate] constitutes a strong prima-facie argument for rejecting that index altogether" (Felsenthal and Machover (1995), p. 224). This would thus suggest that the normalized Banzhaf index ($\tilde{\beta}$), the Johnston index ($\tilde{\gamma}$), the Deegan-Packel index (ρ) and the Holler-Packel index ($\tilde{\sigma}$) cannot reasonably measure power because they violate more than one a priori desirable property. Similarly, the Shapley-Shubik (φ) index cannot be considered as a suitable power index for decision-making processes that cannot be represented as weighted games. In this respect, the Banzhaf index (β) appears to be the best power index as it does not violate any postulate. Note that this approach only disqualifies power indices, and does not state that the indices that satisfy all postulates are genuinely appropriate measures of power.

5 PROBABILITY OF BEING A SWINGER

So far we have dealt with power indices loosely interpreted as evaluations or assessments of the a priori capacity of each player to influence the outcome in a decision process. In this section and the next one we examine an alternative and complementary approach to ground the choice of an index that hinges upon the precise interpretation of each index as an expectation. The direct intuition provided by their specific interpretation may help to make a choice in each specific context.

In general terms the power of a player can be interpreted as the expectation of this player of playing a certain relevant role in the decision process. Let us state this more precisely. A simple game is a summary model of a set of rules to make decisions by a group of agents. This same decision procedure will usually be applied repeatedly to decide upon different issues. Then the power of a player can be interpreted as her or his a priori probability of playing a relevant role. As we will see in this section and in the next one, all indices reviewed in this paper fit into this general interpretation. From this point of view, different indices arise either from: (i) different specifications of what is precisely meant by "playing a relevant role"; and/or (ii) different assumptions about the formation of coalitions.

In this section we discuss which indices fit into the following interpretation. In any given decision process a certain set of players -or coalition- shares player i's view about the particular issue at stake. In this particular process player i is considered to exert some power if her or his vote is necessary to pass her or his preferred outcome, that is, if she or he is a swinger in the coalition of players sharing her or his view. This corresponds to what Felsenthal et al. (1998) define as "I-power", which is "proportional to her or his ability to influence the outcome of a vote" (Felsenthal et al., 1998, p. 101).

Formally, if $p_i(S)$ denotes the probability of coalition S being the set of players sharing player i's view on the issue at stake, player i's power is given by:

$$\sum_{\substack{S \subset N \\ S \ni i}} p_i(S) \left[v(S) - v(S \setminus \{i\}) \right].$$

Different power indices arise from different assumptions made on the coalition formation, that is, how a coalition S, to which player i belongs, forms. The Shapley-Shubik index corresponds to the assumption that an ordering of players is chosen at random (all orderings being equiprobable). Then the coalition formed is the coalition containing player i and the players who precede her or him in the chosen ordering. Therefore the probability of forming a given coalition S to which player i belongs is:

$$p_i(S) = \frac{(s-1)!(n-s)!}{n!}.$$

Another assumption on the coalition formation can be given to interpret the Shapley-Shubik index. A non zero size of coalition is chosen at random (all sizes being equiprobable). All coalitions of this size containing player i are then assumed to be equiprobable, and one of them is formed at random. Therefore the probability of forming a given coalition S to which player i belongs is:

$$p_i(S) = \frac{1}{n} \frac{1}{C_{n-1}^{s-1}} = \frac{(s-1)!(n-s)!}{n!}.$$

The Banzhaf index corresponds to the assumption that all coalitions to which player i belongs are equiprobable, and one of them is formed at random. Thus we have:

$$p_i(S) = \frac{1}{2^{n-1}}.$$

The non normalized Holler-Packel index assumes that only minimal winning coalitions form. All such coalitions are equiprobable, and one of them is formed at random. Thus we have:

$$p_i(S) = \begin{cases} \frac{1}{m(v)} & \text{if } S \in \mathcal{M}(v) \\ 0 & \text{otherwise.} \end{cases}$$

The Deegan-Packel index, the Holler-Packel index and the Johnston index cannot be interpreted in a such a way: there does not exist a clear probabilistic story to derive the probability of forming a coalition in which a given player is a swinger.

Within this framework, the Banzhaf index the Shapley-Shubik index and the non normalized Holler-Packel index can be used as measures of power. The Banzhaf index seems to be the most natural index in the absence of any a priori reason to assign different probabilities to different coalitions (Principle of Insufficient Reason). The other indices may, however, be more appropriate in some situations. For instance, if players vote in the order of their eagerness to support the decision to be made, the Shapley-Shubik index is then the most natural index (in the absence of any a priori reason to assign different probabilities to different orders). The non normalized Holler-Packel index is the natural index to use if voters decide to form exclusively minimal winning coalitions (in the absence of any a priori reason to assign different probability to different minimal winning coalitions).

Straffin (1977, 1988) proposes the following probabilistic model that permits to derive the probability of coalition formation from alternative assumptions on the probability of the players accepting a proposal. Let p_i be the probability that player i accepts a random proposal. Then by assuming that for a random coalition S, the event $A_i = \{i \in S\}$ is independent of the event $A_j = \{j \in S\}$, the probability that a randomly chosen coalition S supports a random proposal can be written as follows:

$$\prod_{i \in S} p_i \prod_{j \in N \setminus S} (1 - p_j).$$

If we multiply these probabilities by $v(S)$, we get the probability that the randomly chosen coalition S makes the proposal pass. Then, summing up over all possible coalitions, we obtain the probability that a specific proposal is accepted. With an arbitrary chosen distribution of probabilities, the probability that player i is a swinger, i.e., player i's power, is then given by:

$$\int_0^1 \cdots \int_0^1 \sum_{\substack{S \subset N \\ i \in S}} \prod_{j \in S \setminus i} p_j \prod_{j \in N \setminus S} (1 - p_j)[v(S) - v(S \setminus i)] \, dp_1 ... dp_n.$$

Different power indices arise from different assumptions made on the distribution of probabilities. The Shapley-Shubik index corresponds to the assumption that the probability of accepting a proposal is identical for all players and uniformly distributed, that is,

$$p_i = t, \text{ for any player } i \text{ and } t \sim U(0,1).$$

In the literature, this assumption is referred to as the homogeneity assumption. The Banzhaf index corresponds to the assumption that the probability that player i accepts a proposal is uniformly distributed, that is,

$$p_i \sim U(0,1), \text{ for any player } i.$$

In the literature this assumption is referred to as the independence assumption. The other power indices have not been justified in this probabilistic model.

Within this framework, the Johnston, the Deegan-Packel and the Holler-Packel indices do not seem to be consistent measures, as there is no probabilistic model to justify them. Concerning the choice between the remaining indices, "the Banzhaf index should be used for situations in which voters vote completely independently, the Shapley-Shubik index for situations in which a common set of values tends to influence the choices of all voters." (Straffin (1977, p. 117)).

6 PROBABILITY OF BEING A CRUCIAL PLAYER

In this section, power is interpreted as the probability of playing a crucial role in a decision-making process, assuming that in any decision made only one player plays such a role[10]. A player's power index is then given by her or his probability of being "the crucial player". Different power indices arise from different assumptions on how the crucial player is chosen.

The Shapley-Shubik index arises from the following model: An ordering of players is chosen at random (all orderings being equiprobable). According to this ordering, players enter one by one up to form a winning coalition. The last player who has entered the winning coalition is the crucial player. The model for the normalized Banzhaf index is that a swing is chosen at random (all swings being equiprobable). The corresponding swinger is the crucial player. The model for the Deegan-Packel index is the following: a minimal winning coalition is chosen at random (all minimal winning coalitions being equiprobable). In this minimal winning coalition, a player is chosen at random to be the crucial player (all players having the same probability of being chosen). The model for the Johnston index is that a winning coalition with at least one swinger is chosen at random (all such coalitions being equiprobable). In this coalition, a swinger is chosen at random to be the crucial player (all swingers having the same probability of being chosen). The model for the Holler-Packel index is that a swing such that the corresponding coalition is minimal winning is chosen at random (all such swings being equiprobable). The corresponding swinger is the crucial player.

Note that these models are similar to lottery models with only one prize winner, the crucial player. Therefore the intuition suggests that chance also plays a role in these models. Indeed, to be the crucial player, one has to be a swinger (one's vote must make a difference) but one has also to be lucky enough to get the prize[11]. Therefore, it can

[10] Recall that in the previous section, several players could be swinger for a given decision. Also note that any measure of power satisfying the relative power axiom can be interpreted in this way.

[11] For an extensive discussion between power and luck, see Barry (1980).

be argued that in this model the indices give a measure of power and luck. From this heuristic interpretation of power (and luck), it can be seen that only indices satisfying the Relative Power axiom can fit in the framework: the Banzhaf index cannot be interpreted in this way.

The differences between the relative indices arise from the underlying lottery models that determine the crucial player. Depending on the particular voting rules, some lottery models are more realistic than others. For instance, let us consider a decision-making process in which voters vote in the order of their eagerness to support a bill. If a priori there is no reason to assume that an ordering of voters is more probable than another one, the Shapley-Shubik index gives the voters' probabilities of being the last voter that has to be convinced to make the decision. Another example is provided by the formation of governments, where there are offices to share. Let us assume that the parties' behavior is "office-seeking" (rather than "policy-seeking"), in the sense that parties are more interested in getting into the government than in trying to enforce their preferred policies[12]. Then, if parties maximize their spoils without taking into account the possible instability of small majorities, only minimal winning coalitions should form. If a priori there is no reason to assume that a minimal coalition has a larger probability to form than another one, and if a priori there is no reason to choose a particular party to lead the government, the Deegan-Packel index gives the parties' probabilities of being chosen to lead the government. No plausible story can be given for the Johnston index, the normalized Banzhaf index and the Holler-Packel index.

7 CONCLUSION

It has often been highlighted in the literature that voting weights do not properly represent the voters' influence in a decision-making process. Moreover choosing arbitrarily the decision-making rule may have unobserved and undesirable consequences. For instance, Brams and Affuso (1976) show that in the original European Council of Ministers (1958-1973), the voting weights and the decision-making rule were such that Luxembourg was a null player: in other words, the decision-making was designed in such a way that Luxembourg's vote could never make any difference in any decision. Power indices are more suitable to catch the a priori relations of power in a voting body than the voting weights.

Of course, power indices only give an a priori assessment of the distribution of power, that does not take into account the voters' preferences or the interpersonal relations between the voters. As already pointed out in Shapley and Shubik (1954, p. 791), "it would

[12]For a description of the "office-seeking" versus "policy-seeking" behaviors, see for instance Laver and Schofield (1990).

be foolish to expect to be able to catch all the subtle shades and nuances of custom and procedure that are to be found in most real decision-making bodies. Nevertheless, the power index computations may be useful in the setting up of norms or standards (...)." For instance, they can be useful to choose how to distribute voting weights in a given voting body in order to satisfy some goals on the a priori power distribution.

The problem is then that the existence of several power indices is confusing as the results that they give may differ widely. In this paper we have presented and compared the main existing power indices in order to give some ground to choose among them. The analysis carried out in the paper allows us to conclude as follows. The axiomatic approach, even if it gives some basic clues about the different philosophies embodied in the different indices, does not provide a sufficient basis for a choice irrespective of the decision-making process under consideration. The approach based on postulates allows to disqualify some power indices, although it must be recalled that the desirability of some properties may depend on the context and on the purpose of the analysis. The probabilistic approach is of great interest: as far as the decision-making process to study presents some characteristics that meet the assumptions of some model, then choice of a power index appears clearly and the chosen index can be precisely interpreted.

Let us summarize here the practical probabilistic interpretation of the different indices. The Shapley-Shubik index gives the players' probabilities of being swinger if one of the following three assumptions can be met: (1) players vote according to "common standard", (2) players are supposed to vote in order (assuming that all orderings are equiprobable), (3) all sizes of coalitions and all coalitions of a given size are equiprobable. The Shapley-Shubik index also gives the players' probabilities of making a proposal pass if it can be assumed that players vote according to their eagerness to support a proposal (considering that all orderings of players are equiprobable). The Banzhaf index gives the players' probabilities of being necessary to make decisions (1) if it can be assumed that all coalitions to which a players belongs a priori equiprobable or (2) if it can be assumed that all voters vote independently of each others. There is no practical probabilistic interpretation of the normalized version of the Banzhaf index. The Johnston index cannot be interpreted in such a way either. The Deegan-Packel index measures the parties' probabilities of leading an office-seeking government, if the three following assumptions are simultaneously met: (1) only minimal winning coalitions form, (2) no minimal coalition has a larger probability to form than another one, (3) no particular party has a larger probability to lead the government. The is no compelling probabilistic interpretation of the Holler-Packel index although its non normalized version gives the voters' probabilities of being swinger if one can simultaneously assume that only minimal winning coalitions form, and that one of them is formed at random (considering that all such coalitions are equiprobable)

Finally some lines for further research can be drawn. As already mentioned, the interpretation of the axioms -and thus their plausibility- depends on the context, or class of games considered. Reasonable axioms in a given context may appear less relevant in other contexts. For instance, if the axiom of efficiency can make sense in the class of TU-games, the corresponding axiom is arguable in the context of decision-making processes. Similarly, no satisfactory justification of the Transfer axiom or of the Mergeability axioms can be given in the context of decision-making problems. In this respect, the extension of the class of simple superadditive games to include lotteries may be promising as it allows a more convincing interpretation of the axiom that corresponds to the Transfer axiom. We may also conjecture that a reformulation of the Mergeability axioms in this wider class of games is of interest.

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