

# AN ALTERNATIVE THEORY OF HEALTH CARE DECISION MAKING\*

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## **ABSTRACT**

Starting from a finite or countable set of states of health, and assuming the existence of an objective transitive preference relation on that set, we propose a way of performing interpersonal comparisons of states of health among individuals of different type. Then, a way of evaluating health streams for an individual is proposed. Finally, we rationalize a way of ranking alternative situations (life histories in terms of health) for a given group of individuals. Our approach is both Type-sensitive and Time-sensitive.

Keywords: QALY; States of Health; Comparison; Types; Cohort; Streams.

# 1. INTRODUCTION

Utility based outcome measures in medical decision making and health economics are widely recognized as to be useful to model health related behaviour. The alternative approach of attaching monetary values of the health improvements by means of the so called *willingness to pay (WTP)*-*willingness to accept (WTA)* procedures presents some practical difficulties due to the experimental disparities between WTP-WTA measures. Previous difficulties stimulated the so called *cost utility* analysis, characterized by the fact that the benefits of health care programs are not expressed in monetary terms, but in utility terms. Several utility indices has been used with this purpose, by far the most popular one being the QALY (quality adjusted life years) index. Since its introduction in the seventies, the number of practical applications of QALY decision making increased rapidly in the eighties. At the same time, this approach was strongly criticized [see Mehrez & Gafni (1989), Loomes & McKenzie (1989) or Mooney & Olsen (1991)]. The main points of criticism for the QALY index can be summarized as follows: (1) The validity and reliability of the different methods of eliciting quality of life valuations; (2) The problem of interpersonal comparability and the aggregation procedure; (3) If values used in social decision making should be some aggregate of individual valuations.

In this paper we offer an alternative proposal. We start by considering that there is a set of well-defined *states of health*,  $\mathbb{S}$ . Furthermore, we shall assume that there is a social preference relation on  $\mathbb{S}$  satisfying completeness and transitivity. The *best* state of health,  $b$ , and the *worst* state of health are well specified, and are the best and worst elements of the aforementioned ordering.

Then, we introduce a way of *making interpersonal and intertemporal comparisons of states of health*. We claim that identical states of health are not equally socially valuable, irrespective of individual characteristics and/or irrespective of age [see Williams (1988), or Murray and López (1996)]. A proper way of grouping characteristics induces a classification of the population into social *types*. Furthermore, individuals are also characterized by their cohort, namely, the year in which they are born. In this way, every individual is identified by means of two labels: her type and her cohort.

If data on the distribution of states of health in a population is available, and if these data are rich enough, it can be used to perform comparisons of states of health [see Herrero and Pinto (1997)], by using the particular distribution of the population.

Once types and cohorts are specified, we may perform intertemporal comparisons of states of health for a particular individual, as well as interpersonal comparisons of states of health across individuals at a particular moment of time. This way of comparison can be extended as to allow for ranking alternative streams of states of health for a particular individual, and also to rank alternative *situations*, namely, alternative lifetime streams in terms of health for a group of agents.

In a companion paper [see Herrero & Pinto (1997)], we introduce the basic model and consider societal evaluations in a static framework. In this paper, we extend the model allowing for evaluations in a dynamic context.

## 2. STATES OF HEALTH. SOCIAL ORDERING ON THE STATES OF HEALTH.

Let us consider a set  $\mathbb{S}$  of states of health. A particular state of health is defined by means of a vector of characteristics [see, for instance, the EUROQOL questionnaire, or alternatively, we may think of a set of functionings, a la Sen, see Sen (1985), Pereira (1993) or Herrero (1996); that set of functionings convey to a certain *capability set*, associated to a particular state of health]. Suppose furthermore that  $\mathbb{S}$  is a finite or countable set.

Assume now that there exists a preference relation  $\Pi$ , defined over  $\mathbb{S}$ . For any two states of health,  $x, y \in \mathbb{S}$ ,  $x\Pi y$  means that state  $x$  is considered better than state  $y$ . If, for two states  $x, y \in \mathbb{S}$ , it is not true that  $x\Pi y$ , we say that  $y$  is at least as good as  $x$ , and we write  $y\Xi x$ . If, simultaneously,  $x\Xi y$  and  $y\Xi x$ , then we say that  $x$  and  $y$  are *similar or equally good*, and write  $x\Upsilon y$ .

We will ask  $\Pi$  to satisfy also some additional requirements:

**(i) Preference**, namely that  $\Pi$  is asymmetric and negatively transitive, i.e., for any  $x \in \mathbb{S}$ , it is not true that  $x\Pi x$ , and for any  $x, y, z \in \mathbb{S}$ , if  $x\Xi y$  and  $y\Xi z$ , then  $x\Xi z$

**(ii) Existence of extremes**, namely there are two states,  $w, b \in \mathbb{S}$ , such that for any  $x \in \mathbb{S}$ ,  $b\Xi x\Xi w$ .

Previous requirements indicate, (i) that  $\Pi$  is an *ordering*, and (ii) that this ordering has a minimum and a maximum, that is, there exists a particular state of health (maybe several, equally good) which is better than any other, the *perfect*

*state of health, best*, and there is another state of health (maybe several, equally good), which are worse than any other, the *worst state of health*, possibly death.

It is important to stress that so far  $\Pi$  is purely ordinal, namely, for two given states of health,  $x, y \in \mathbb{S}$ , we may say if  $x$  is better than  $y$  ( $x\Pi y$ ), or if  $y$  is better than  $x$  ( $y\Pi x$ ), or if they are equally good ( $x\Upsilon y$ ), but there are no cardinal valuations. So, if we consider four states of health, and it turns out that  $x\Pi y$ , and  $z\Pi t$ , we cannot measure the increment in going from  $y$  to  $x$  in relation with the increment in going from  $t$  to  $z$ .

Previous requirements guarantee that  $\Pi$  can be represented by means of a *utility function*  $v : \mathbb{S} \rightarrow \mathbb{R}$ , such that  $x\Pi y$  iff  $v(x) > v(y)$ . Furthermore, any monotone transformation of  $v$  is also a utility representation of  $\Pi$ . In consequence, we may choose a particular representation such that  $v(b) = 1$ , and  $v(w) = 0$ , and therefore, any state of health,  $x \in \mathbb{S}$ , will be associated with a number  $v(x)$ , such that  $0 \leq v(x) \leq 1$ . Again, it has to be noticed that those numbers have only ordinal significance [cf. Kreps (1988), Chapter 3].

### 3. THE MODEL

Let us consider a society made out of individuals, living a maximum of  $T$  periods each, and such that they born in different moments of time. Every individual is identified by the year he/she is born, namely his/her cohort. Let us call  $C^r$  the cohort of year  $r$ . At period  $t$  society considers the population  $N_t$ , consisting of all cohorts from period  $t - T$ , up to  $t$ , namely  $N_t = \cup_{r=t-T}^t C^r$ .

Assume, furthermore, that we have the population divided into types,  $N = T_1 \cup T_2 \cup \dots \cup T_\tau$ , in such a way that individuals belong to one and only one type each during all her lifetime. Types can be defined by using those characteristics society considers relevant (e.g., gender, ethnicity, etc.).

If two individuals belong to the same type, they are considered as *socially similar*. Individuals belonging to different types are considered as *socially different*.

Since cohorts live in different periods, it turns out that at any time, the set of individuals of type  $i$  may vary. By adding up the information on types with the division of the population by cohorts, we denote  $T_i^r$  the set of individuals of type  $i$  in cohort  $r$ .

At time  $t$ , we have the *state of health at time  $t$  function*,  $h^t : N_t \rightarrow \mathbb{S}$ .

Consider now the following functions, where  $i = 1, \dots, \tau$ , and  $r = t - T, \dots, t$ :

$$f_i^r : \mathbb{S} \longrightarrow \mathbb{N}, \text{ such that } f_i^r(x) = \# \{a \in T_i^r \mid h^t(a) = x\}, \text{ and}$$

$$F_i^r : \mathbb{S} \longrightarrow \mathbb{N}, \text{ such that } F_i^r(x) = \# \{a \in T_i^r \mid x \Xi h^t(a)\}$$

Namely,  $f_i^r(x)$  is the number of individuals of type  $i$  in cohort  $r$ , whose state of health at time  $t$  is  $x$ , and  $F_i^r(x)$  stands for the number of individuals of type  $i$  in cohort  $r$ , whose state of health at time  $t$  is worse than or as good as  $x$ .

Notice now that  $F_i^r$  is a *utility function* for  $\Pi$ , for every  $i = 1, 2, \dots, \tau$ , . for all  $r$ . It can be considered as a *cardinal utility function*. In such a case, it turns out that, by considering

$$g_i^r : \mathbb{S} \longrightarrow [0, 1], \text{ such that } g_i^r(x) = \frac{F_i^r(x)}{\#T_i^r} \text{ and}$$

$$G_i^r : \mathbb{S} \longrightarrow [0, 1], \text{ such that } G_i^r(x) = \frac{F_i^r(x)}{\#T_i^r},$$

$G_i^r$  is also a utility function for  $\Pi$ , and  $F_i^r$  and  $G_i^r$  represents identical *cardinal preferences*, since  $G_i^r = \lambda F_i^r(x)$ , where  $\lambda = [\#T_i^r]^{-1}$ . Notice, nevertheless, that  $G_i^r, G_j^r, G_i^{r'}$  and  $G_j^{r'}$  may represent different cardinal preferences in spite of the fact that they represent identical ordinal preferences ( $\Pi$ ).

Consider now the following definition [cf. Roemer (1993), (1996)].

**Definition 1.** *Two individuals  $a, c \in N_t$ ,  $a \in T_i^r$ ,  $c \in T_j^{r'}$ , have a comparable state of health at time  $t$ , whenever  $G_i^r[h^t(a)] = G_j^{r'}[h^t(c)]$ .*

Previous definition indicates that we consider two individuals belonging to different types and to different cohorts as having a *comparable state of health at time  $t$* , whenever the percentage of individuals in their types and cohorts having a state of health worse than or equal to them is the same. This idea can be interpreted as saying that, by means of the utility functions  $G_i^r$  we associate cardinal numbers to states of health at time  $t$ , *in a type/cohort-dependent way*, by using the distributions of states of health in population  $N_t$ .

Note that if  $a, c \in T_i^r$ , then  $a, c$  have a comparable state of health at time  $t$  iff  $h^t(a) = h^t(c)$ . In consequence, our criterion is an extension of the usual valuation for individuals of the same type and age.

If state  $w$  is defined in such a way that  $F_i^r(w) = 0, \forall i, r$ , then  $G_i^r(w) = 0, G_i^r(b) = 1$ , for any  $i, r$ . Notice also that the valuations  $G_i^r$ , correspond to a cross-section time, namely they correspond to time  $t$ . That is, we have a theory of comparability of states of health wich turns out to be both *type-cohort sensitive*

and *time-sensitive*, in the sense that utility functions  $G_i^r$  can vary from one period of time to another.

Consider now the problem of comparing states of health, *for a particular individual*, in two different moments of time. Namely, assume that individual  $a \in T_i^r$ , that is,  $a$  is an individual of type  $i$ , born in time  $r$ . Consider then two different moments of (social) time within this individual's life, namely take  $t$  and  $t'$  such that  $t, t' \in \{r, r + 1, \dots, r + T - 1\}$ . Consider now the valuation  $G_i^r$  in both time  $t$  and  $t'$ , and call them  $G_i^{r,t}, G_i^{r,t'}$ , respectively. Then we have the following definition:

**Definition 2.** *Individual  $a$  estate of health at time  $t$  is comparable to individual  $a$  estate of health at time  $t'$  whenever  $G_i^{r,t}[h^t(a)] = G_i^{r,t'}[h^{t'}(a)]$ , provided that  $a \in T_i^r$ ,  $t, t' \in \{r, r + 1, \dots, r + T - 1\}$ .*

Note that, in previous definition, we consider the distributions of states of health both in  $t$  and  $t'$  in order to perform comparisons. That is, *strictus sensu*, we cannot compare the state of health of individual  $a$  in  $t$  with her state of health in  $t'$  *before we have the relevant information*, that is, before a moment of time greater than or equal to  $\max\{t, t'\}$ . In order to avoid previous problem, let us consider the following definition:

**Definition 3.** *Individual  $a$  estate of health at time  $t$  is comparable to individual  $a$  estate of health at time  $t'$  (in  $t''$ ), whenever  $G_i^{r, \min\{t, t''\}}[h^t(a)] = G_i^{r, \min\{t', t''\}}[h^{t'}(a)]$ , provided that  $a \in T_i^r$ ,  $t, t', t'' \in \{r, r + 1, \dots, r + T - 1\}$ .*

Previous definition allows us to compare states of health for an individual in two different moments of her lifetime, in all circumstances. We use available (past) information in order to compare (today) past states of health. For future states of health we forecast by using today's distribution in order to evaluate the future.

## 4. RANKING PROFILES OF STATES OF HEALTH AT AN INDIVIDUAL LEVEL

Let us now consider the problem of ranking profiles of states of health for a particular individual.

Consider individual  $a \in C^r$ , whose stream of states of health is  $\mathbf{x} = (x_1, \dots, x_T)$ . If we consider this at time  $t = r + i - 1$ , previous stream can be subdivided as  $\mathbf{x} = [(x_1, \dots, x_i)(x_{i+1}, \dots, x_T)]$  where  $(x_1, \dots, x_i)$  represents states of health the individual enjoyed in the past, and  $(x_{i+1}, \dots, x_T)$ , indicates the states of health this particular individual may enjoy in the future. For that particular individual, we may look at health streams as alternatives, and then consider lotteries over health streams. In dealing with the past, no change for that individual may be introduced, whereas for future states, we may consider several alternative streams, with different probabilities each.

Consider now the set  $\Omega$  of lotteries over uncertain streams  $\mathbf{x} = (x_{i+1}, \dots, x_T) \in \mathbb{S}^{T-i}$ . Elements in  $\Omega$  are probability distributions of finite support on  $\mathbb{S}^{T-i}$ , namely  $L \in \Omega$  is such that  $L : \mathbb{S}^{T-i} \rightarrow \mathbb{R}_+$ , where  $L(\mathbf{x}) \neq 0$  only for a finite set of elements  $\mathbf{x}$ , and  $\sum_{\mathbf{x} \in \mathbb{S}^{T-i}} L(\mathbf{x}) = 1$ .

If  $L, M \in \Omega$ , and  $\lambda \in [0, 1]$ , define  $[\lambda L + (1 - \lambda)M](\mathbf{x}) = \lambda L(\mathbf{x}) + (1 - \lambda)M(\mathbf{x})$ . Thus,  $[\lambda L + (1 - \lambda)M] \in \Omega$ .

For a states of health stream  $\mathbf{x} = (x_{i+1}, \dots, x_T) \in \mathbb{S}^{T-i}$ , and a state of health  $z \in \mathbb{S}$ , denote by  $(\mathbf{x}^{-k}, z) = \mathbf{y} \in \mathbb{S}^{T-i}$  the stream such that  $y_j = x_j$ , whenever  $j \neq k$ , and  $y_k = z$ . That is,  $(\mathbf{x}^{-k}, z)$  coincides with  $\mathbf{x}$  in all periods but period  $k$ , and the state of health in period  $k$  is just  $z$ . Also, call  $\mathbf{w}$  and  $\mathbf{b}$  the streams such that in all periods, the state of health of individual  $a$  is  $w$ , respectively,  $b$ .

We shall now consider the existence of a binary relation  $P_a$  defined over  $\Omega$ , understood as a *strict preference relation*, in such a way that  $R_a$  and  $I_a$  are, respectively, the *weak preference relation* and the *indifference relation* associated to  $P_a$ .

Notice that  $P_a$  also induces a binary relation on  $\mathbb{S}^{T-i}$ , since any stream of states of health  $\mathbf{x} \in \mathbb{S}^{T-i}$  can also be interpreted as a degenerated lottery in  $\Omega$ , where  $\mathbf{x}(\mathbf{x}) = 1$ , and  $\mathbf{x}(\mathbf{y}) = 0$ , for all  $\mathbf{y} \neq \mathbf{x}$ .

Let us now consider the following assumptions:

**VNM.-** (i)  $P_a$  is a preference relation on  $\Omega$ , namely, it is asymmetric and negatively transitive. (ii) For all  $L, M, N \in \Omega$ , and for all  $\lambda \in (0, 1]$ , if  $LP_aM$ , then  $[\lambda L + (1 - \lambda)N]P_a[\lambda M + (1 - \lambda)N]$ . (iii) For all  $L, M, N \in \Omega$ , if  $LP_aMP_aN$ , then there exist  $\lambda, \mu \in (0, 1)$ , such that  $[\lambda L + (1 - \lambda)N]P_aMP_a[\mu L + (1 - \mu)N]$

**Time Additive Independence.-** For any  $\mathbf{x}, \mathbf{y} \in \mathbb{S}^{T-i}$ ,  $\mathbf{x} = (x_{i+1}, \dots, x_T)$ ,  $\mathbf{y} = (y_{i+1}, \dots, y_T)$ , any  $k = i + 1, \dots, T$ , if we call  $\mathbf{z} = (\mathbf{x}^{-k}, y_k)$ ,  $\mathbf{v} = (\mathbf{y}^{-k}, x_k)$ , and  $L(\mathbf{x}) = L(\mathbf{y}) = \frac{1}{2}$ ,  $M(\mathbf{z}) = M(\mathbf{v}) = \frac{1}{2}$ , then  $LI_aM$ .



**Neutrality for comparable states of health for individual a.**- For any two streams,  $\mathbf{x} = (\mathbf{w}^{-j}, x_j)$ ,  $\mathbf{y} = (\mathbf{w}^{-k}, y_k) \in \mathbb{S}^{T-i}$ , if , for individual  $a$ , state  $x_j$  enjoyed in year  $k$  is comparable to state  $y_k$  enjoyed in year  $k$ , then  $\mathbf{x}I_a\mathbf{y}$ .

VNM is nothing but the basic assumptions in the Von Neumann-Morgenstern expected utility theory.

Time Additive Independence asks for states of health in the different periods to be additive independent, namely preferences depend only on the marginal probability distribution and not on the joint distribution.

Finally, Neutrality for Comparable states of health for individual  $a$ , says that individual  $a$  is indifferent between enjoying a state of health in one year or a comparable state in another year.

Then, we obtain the following result:

**Theorem 1.** Under VNM, Time Additive Independence and Neutrality for Comparable states of health for individual  $a$ , (i) There exists a function  $u_a : \mathbb{S}^{T-i} \rightarrow \mathbb{R}$  such that  $LP_aM$  iff  $\sum_{\mathbf{x} \in \mathbb{S}^{T-i}} L(\mathbf{x})u_a(\mathbf{x}) > \sum_{\mathbf{x} \in \mathbb{S}^{T-i}} M(\mathbf{x})u_a(\mathbf{x})$ . Furthermore,  $u_a$  is unique up to positive linear transformations. (ii) There exist  $u_k^a : \mathbb{S} \rightarrow \mathbb{R}$ ,  $k = i + 1, \dots, T$ , such that:  $u_a(\mathbf{x}) = \sum_{k=i+1}^T u_k^a(x_k)$ ; (iii)  $u_k^a$  can be chose so that  $u_k^a(x_k) = G_k^t(x_k)$ , for all  $k = i + 1, \dots, T$ .

Proof: (i) is a direct consequence of Von Neumann-Morgenstern expected utility theorem. Cf. Kreps (1988, Theorem 5.4).

Time Additive Independence indicates that attributes  $x_{i+1}, x_{i+2}, \dots, x_T$  are additively independent, since it implies that preferences over lotteries on  $x_{i+1}, x_{i+2}, \dots, x_T$  depend only on their marginal probability distributions and not on their joint probability distribution. Thus, we may apply Keeney and Raiffa (1976, Theorems 5.1 and 6.4), in order to obtain the representation result in (ii) [cf. Bleichrodt, Theorem 3.2]. Additionally,  $u_a$  and  $u_k^a$  can be normalized so that  $u_a(\mathbf{w}) = u_k^a(w) = 0$ ,  $u_k^a(b) = 1$ ,  $u_a(\mathbf{b}) = T - i$ .

The values of  $\Gamma_k$  can be obtained as follows:

$$\begin{aligned} u_a(\mathbf{b}) &= \Gamma_{i+1}u_{i+1}^a(b) + \dots + \Gamma_Tu_T^a(b) = \Gamma_{i+1} + \dots + \Gamma_T \\ u_a(\mathbf{w}^{-k}, b) &= \Gamma_ku_k^a(b) = \Gamma_k. \end{aligned}$$

Then, Neutrality for Comparable States of Health for individual  $a$ , indicates that  $\Gamma_k = \Gamma_j$ , for all values of  $k, j$ , and since  $u_a(\mathbf{b}) = \Gamma_{i+1} + \dots + \Gamma_T = (T-i)\Gamma_{i+1} = T - i$ , it follows that  $\Gamma_k = 1$  for all  $k$ .

Consider now the streams  $\mathbf{x} = (\mathbf{w}^{-j}, x_j)$ ,  $\mathbf{y} = (\mathbf{w}^{-k}, x_k)$ . Neutrality for Comparable States of Health implies that  $u_j^a(x_j) = u_k^a(y_k)$  iff  $G_j^t(x_j) = G_k^t(x_k)$ . In consequence,  $G_j^t$  and  $u_k^a$  represent identical preferences, and thus, they are related by a positive affine transformation. Furthermore,  $G_k^t(w) = u_k^a(w) = 0$ , and  $G_k^t(b) = u_k^a(b) = 1$ , and thus,  $G_j^t = u_k^a$ . ♠

**Remark.** The rationalization we made in this section to rank uncertain streams of health *for a particular individual* can be extended straightforwardly to include also previous (past enjoyed) states of health. In so doing, we may also consider lotteries over past states, and the combination of previous Assumptions convey to the following valuation function:

$$u_a(\mathbf{x}) = \sum_{k=1}^i G_k^{t+k}(x_k) + \sum_{k=i+1}^T G_k^{t+i}(x_k)$$

## 5. EVALUATION OF HEALTH STREAMS FOR A GROUP

Consider now a group of agents,  $A \subset N_t$ , and call  $c : A \rightarrow \{t - T, \dots, t - 1, t\}$  the mapping associating to every agent in  $A$  her corresponding cohort.

At time  $t$ , any agent in  $A$  has an expected stream of life in terms of the state of health they will enjoy. If  $c(a) = i$ , agent  $a$  stream is a vector  $\mathbf{x} = [(x_1^{t-i+1}, \dots, x_i^t)(x_{i+1}^{t+1}, \dots, x_T^{t+T-i})] \in \mathbb{S}^T$ , such that the subvector  $(x_1^{t-i+1}, \dots, x_i^t)$  corresponds to states of health the agent enjoyed in the past (and today), whereas  $(x_{i+1}^{t+1}, \dots, x_T^{t+T-i})$  stands for a forecast (expected) stream of states of health the agent is going to enjoy in future periods.

A *situation* for group  $A$  in time  $t$  is then a mapping  $s : A \rightarrow \mathbb{S}^T$ , such that  $s(a) = \mathbf{x} = [(x_1^{t-i+1}, \dots, x_i^t)(x_{i+1}^{t+1}, \dots, x_T^{t+T-i})]$ , stands for the stream enjoyed by agent  $a$ ,  $a \in A$ .

Consider now the set  $\Omega$  of lotteries over situations  $s : A \rightarrow \mathbb{S}^T$ . Call  $\sigma$  the set of all situations for group  $A$  in time  $t$ . Elements in  $\Omega$  are probability distributions of finite support on  $\sigma$ , namely  $L \in \Omega$  is such that  $L : \sigma \rightarrow \mathbb{R}_+$ , where  $L(s) \neq 0$  only for a finite set of situations  $s$ , and  $\sum_{s \in \sigma} L(s) = 1$ .

If  $L, M \in \Omega$ , and  $\lambda \in [0, 1]$ , define  $[\lambda L + (1 - \lambda)M](s) = \lambda L(s) + (1 - \lambda)M(s)$ . Thus,  $[\lambda L + (1 - \lambda)M] \in \Omega$ .

We shall now consider the existence of a binary relation  $P$  defined over  $\Omega$ , understood as a *strict preference relation*, in such a way that  $R$  and  $I$  are, respectively, the *weak preference relation* and the *indifference relation* associated

to  $P$ . That is, for any  $L, M \in \Omega$ ,  $LRM$  iff it is not true that  $MPL$ , and  $LIM$  iff, simultaneously, both  $LRM$  and  $MRL$ .

Notice that  $P$  also induces a binary relation on  $\sigma$ , since any situation  $s \in \sigma$  can also be interpreted as a degenerated lottery in  $\Omega$ , where  $s(s) = 1$ , and  $s(s') = 0$ , for all  $s' \neq s$ .

Let us now consider the following assumptions:

**VNM** (i)  $P$  is a preference relation on  $\Omega$ , namely, it is asymmetric and negatively transitive, (ii) For all  $L, M, N \in \Omega$ , and for all  $\lambda \in (0, 1]$ , if  $LPM$ , then  $[\lambda L + (1 - \lambda)N]P[\lambda M + (1 - \lambda)N]$ .(iii) For all  $L, M, N \in \Omega$ , if  $LPMPN$ , then there exist  $\lambda, \mu \in (0, 1)$ , such that  $[\lambda L + (1 - \lambda)N]PMP[\mu L + (1 - \mu)N]$

VNM are the basic assumptions in the Von Neumann-Morgenstern expected utility theory.

Let us call  $W, B \in \sigma$  the streams  $W(a) = \mathbf{w}$ , for all  $a \in A$ , and  $B(a) = \mathbf{b}$ , for all  $a \in A$ , respectively.

For a situation  $s \in \sigma$ , an stream  $\mathbf{x} \in \mathbb{S}^T$  and an agent  $a \in A$ , let us denote  $(s^{-a}, \mathbf{x}) = q$ , the situation in  $\sigma$  such that  $q(a') = s(a')$  if  $a' \neq a$ ,  $q(a) = \mathbf{x}$ .

Consider now the following assumption:

**Agent Additive Independence:** For any  $s, r \in \sigma$ , any agent  $a \in A$ , if we call  $q = (s^{-a}, r(a))$ ,  $p = (r^{-a}, s(a))$ , and  $L(s) = L(r) = \frac{1}{2}$ ,  $M(q) = M(p) = \frac{1}{2}$ , then  $LIM$ .

Agent Additive Independence asks for situations to be additive independent with respect to agents, namely, preferences depend only on the marginal probability distribution of agents streams, and not on the joint distribution.

**Neutrality.-** For any  $a, a' \in A$ ,  $(W^{-a}, \mathbf{b})I(W^{-a'}, \mathbf{b})$ .

Neutrality says that in a situation such that all individuals in  $A$  but one are at the worst possible state of health in all their life stream, and the only agent not in such a stream is, on the contrary, at the best possible state of health in all her lifetime, society is indifferent with respect to whom is the favored agent.

Then, we obtain the following result:

**Theorem 2.** Under VNM. Agent Additive Independence and Neutrality, (i) There exists a function  $u : \sigma \rightarrow \mathbb{R}$  such that  $LPM$  iff  $\sum_{s \in \sigma} L(s)u(s) > \sum_{s \in \sigma} M(s)u(s)$ ,

and such  $u$  is unique up to positive linear transformations, (ii) There exist  $u_a : \mathbb{S}^T \rightarrow \mathbb{R}$ ,  $a \in A$ , such that:  $u(s) = \sum_{a \in A} u_a[s(a)]$ , (iii)  $u_a$ ,  $a \in A$  are normalized so that  $u_a(\mathbf{w}) = 0$ ,  $u_a(\mathbf{b}) = T$ ;  $u$  is normalized so that  $u(W) = 0$ ,  $u(B) = (\#A)T$

Proof: (i) is a direct consequence of Von Neumann-Morgenstern expected utility theorem. Cf. Kreps (1988, Theorem 5.4). (ii) Agent Additive Independence indicates that attributes  $s(a)$ ,  $a \in A$  are additively independent. Thus, we may apply Keeney and Raiffa (1976, Theorems 5.1 and 6.4), in order to obtain the following representation result [cf. Bleichrodt, Theorem 3.2].

$u(s) = \sum_{a \in A} \Gamma_a u_a[s(a)]$ ; furthermore, we are free to normalize  $u$  and  $u_a$  as we wish. If we normalize  $u_a$  such that  $u_a(b) = T$ ,  $u_a(w) = 0$ , for all  $a \in A$ , then,

$$\begin{aligned} u(B) &= \sum_{a \in A} \Gamma_a u_a(b) = [\sum_{a \in A} \Gamma_a] T \\ u(W^{-a}, b) &= \Gamma_a u_a(b) = T \Gamma_a. \end{aligned}$$

Notice that under Neutrality,  $\Gamma_a = \Gamma_{a'}$ , for all  $a, a' \in A$ . Furthermore, since  $u(B) = [\#A]T = [\#A]T\Gamma_a$ , it follows that  $\Gamma_a = 1$ , for all  $a \in A$ , and the desired representation follows. ♠

Consider now the following assumption:

**Congruence.-** For any two situations,  $s, s' \in \sigma$ , if  $s(a') = s'(a')$ , for all  $a' \neq a$ , then  $sPs'$  iff  $s(a)P_a s'(a)$ .

Congruence asks the aggregate preference relation to coincide with individual  $a$ 's preference whenever this is the only individual in  $A$  affected in choosing between two different situations.

Then, we obtain the following result:

**Theorem 3.** Under VNM, Agent Additive Independence, Neutrality and Congruence,  $u(s) = \sum_{a \in A} u_a[s(a)]$ , and  $u_a[s(a)] = \sum_{k=c(a)}^{c(a)+T-1} G_{\tau(a)}^{c(a), \min\{k, t\}} [s_k(a)]$ .

Proof: Congruence indicates that  $u_a$  turns out to be a cardinal representation of  $P_a$ . In consequence,  $u_a[s(a)] = \lambda \left[ \sum_{k=c(a)}^{c(a)+T-1} G_{\tau(a)}^{c(a), \min\{k, t\}} [s_k(a)] \right] + \mu$ , where  $\lambda, \mu$  are real numbers,  $\lambda > 0$ . Now, notice that  $u_a(\mathbf{w}) = 0$  implies that  $\mu = 0$ . Furthermore,  $u_a(\mathbf{b}) = T = \lambda T$ , and in consequence,  $\lambda = 1$ . ♠

## 6. FINAL REMARKS

In this paper we offer an alternative way of comparing the benefits of health care programs. Our approach is justified more from a societal perspective than from an individualistic perspective. First, the existence of a well-defined set of states of health, independent of the characteristics (types) of the individuals, and of an ordering (social) on that set, assumes that society is able to both identify and rank different states of health, *irrespective of personal circumstances*. In a second place, the comparability criterion is made out of statistical information, once society identifies types to classify the population. Third, the way of ranking individual streams of states of health has nothing to do with individual preferences, and again relies on objective information. Finally, the ranking of alternative situations for a group, again can be made using only available data.

Our procedure, then, conveys to a way of evaluating alternative situations (possibly related to alternative health care policies), by using only objective data. What we did in this paper is also to provide with a theoretical justification of the properties social preferences may fulfill in order to be consistent with our formulation. Since we start from a principle of comparability and cardinality, it is clear that the social valuation function has to be of the *utilitarian type* [see D'Aspremont & Gevers (1977)]. Letting aside technicalities, we may concentrate on the adequateness of the comparability criterion, on the one hand, and on the assumptions of Neutrality and Consistency, on the other hand. It seems to us that they make perfect sense in a societal evaluation procedure.

Note that if we consider that all individuals in society belong to the same type, and comparability of states of health for a particular individual in two different moments of time is considered as time-independent, our formulation would be consistent with the traditional QALY valuation, from a planner's perspective.

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