

**MERGERS BETWEEN ASYMMETRIC FIRMS:
PROFITABILITY AND WELFARE***

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A B S T R A C T

Using only information on the degree of concavity of demand and observable structural variables as the market share of firms, a necessary and sufficient condition for a merger to increase welfare is derived. On the profitability side, we obtain that when market size decreases merger profitability increases.

Keywords: Mergers; Antitrust.

1. INTRODUCTION.

US Merger Guidelines rely on market concentration and market shares of merging partners to infer the effect of a merger on welfare. In a Cournot setting, Farrell and Shapiro (1990), considering very general demand and cost conditions, derived a sufficient condition for a proposed merger to increase welfare that included information on market shares of firms. Levin (1990) assuming that the marginal cost of firms is constant, obtained that any proposed merger with a premerger joint market share lower than the 50% should be approved. The reason why market shares play such a key role is that they are negatively correlated with marginal costs.

In both cases, results are derived assuming that as mergers are proposed they should be profitable. Farrell and Shapiro (1990) notes that it is possible that unprofitable mergers increase welfare, but they add that compulsory action or subsidies intended to carry them through would go against the normal practices of antitrust policy.

However, in other countries Governments pursue an active merger policy. For example, in Japan, the MITI has sponsored mergers in order to achieve the reorganization of industrial sectors (Kester (1991)). Spain has also provided fiscal incentives to encourage mergers (Neven et al. (1993) p.74). In these cases, it would be useful to estimate the welfare effect of mergers independently of their profitability.

In the present paper, assuming that marginal cost is constant but it may differ among firms, I study the welfare effect of mergers without assuming that they are profitable. Furthermore, the condition obtained for a merger to increase welfare is both necessary and sufficient. This condition only depends on the degree of concavity of demand, premerger market concentration and the premerger market share of merging firms. It can be seen as a justification of the use by Merger Guidelines of structural variables. Nevertheless, the implications driven by Merger Guidelines from these variables do not always coincide with the ones obtained in our case. For

example, Merger Guidelines assume that the greater the Herfindahl Index the greater the welfare loss of a merger. In our model, this is only true if demand is convex.

When a merger is not profitable but it increases welfare a policy of subsidies is necessary to carry it through. We derive the lowest subsidy that induces firms to merge. We use the same information as before and information on premerger aggregate output.

As market shares provide a crucial information to determine the effect on welfare of a merger, they will also be very informative on the profitability of a merger. As far as firms are concerned this is not very useful, because they have direct information on costs. However, given that market shares are observable, the result will be applied to derive testable propositions about when are mergers more likely to occur.

In the first place, I obtain that mergers will only be profitable between firms whose sizes are different enough. A merger will only be profitable if the cost savings induced by it are high enough. They are obtained by transferring output from the high cost merging partner to the low cost merging partner and therefore increase with the cost differential.

In the second place, I obtain that when market demand decreases merger profitability increases. We have just seen that merger profitability increases with the cost differential. But this cost differential means nothing in absolute terms and it should be related to price. Then if market size decreases, price decreases and the cost differential becomes relatively greater. This increases merger profitability.

This last result differs from two previous results in the literature. If firms are symmetric and marginal cost is constant, merger profitability does not depend on market size. This explains the generality of the results obtained in this setting (see Salant et al (1983) and its generalization in Cheung (1991) and Faulí-Oller (1997)). On the other hand, van Wegberg (1994) shows that if firms face capacity constraints, mergers become more profitable when market size increases,

because non-participating firms expands output less after merger. My result agrees with the empirical finding that horizontal mergers occur in declining industries as a device to rationalize capacity (Dutz (1989) and Odagiri and Hase (1989)).

In the next Section the sufficient and necessary condition for a merger to increase welfare is derived. In Section III, the lowest subsidy to induce a welfare-increasing merger is derived and in section IV the negative relationship between market size and profitability is proved. Concluding remarks bring the paper to an end.

2. OPTIMAL MERGER POLICY.

I study mergers in a market where firms are assumed to compete à la Cournot in any market structure. They sell an homogenous product whose inverse demand is given by:

$$P(X) = A - \frac{X^{b+1}}{b+1}, \text{ where } A > 0 \text{ and } b > -1.$$

This demand satisfies that $\frac{P''(X)X}{P'(X)}$ (henceforth called the degree of concavity of demand) is constant and it amounts to b . Marginal cost of production is constant, but they may differ among firms. c_i denotes the marginal cost of firm i . We assume that A is high enough such that all firms in the market are active. $b > -1$ guarantees that, given any market structure, the Cournot equilibrium exists and is unique. Note that $b > -1$ is equivalent to the standard assumption:

$$P''(X) X + P'(X) < 0.$$

First of all, I calculate the social welfare given a market structure. Social Welfare is understood as the sum of consumer surplus (CS) and industry profits (Π).

Given a market structure in equilibrium the following must hold for all firm i :

$$P(X) - c_i = -P'(X) x_i.$$

Given that the marginal cost is constant, profits of firms can be written as:

$$\pi_i = (P(X) - c_i) x_i = -P'(X) x_i^2 = X^{b+2} s_i^2, \text{ where } s_i \text{ is the market share of firm } i.$$

Therefore industry profits can be written as:

$$\Pi = X^{b+2} H, \text{ where } H \text{ is the value of the Herfindahl Index in equilibrium.}$$

Consumer surplus only depends on total sales in equilibrium (X) and is given by:

$$CS = \int_0^X P(X) dX - P(X) X = A X - \frac{X^{(b+2)}}{(b+1)(b+2)} - AX + \frac{X^{(b+2)}}{(b+1)} = \frac{X^{(b+2)}}{(b+1)} \left(1 - \frac{1}{(b+2)}\right) = \frac{X^{(b+2)}}{(b+2)}.$$

Therefore Social Welfare (SW) amounts to:

$$SW = X^{(b+2)} \left(\frac{1}{b+2} + H \right)$$

We want to compare the social welfare in two different market configurations:

1. The premerger one.
2. The postmerger one.

In an industry with $n+1$ firms, if a merger between firm i and firm k occurs, we go from structure 1 to structure 2. The average costs of firm i and firm k are respectively given by c_i and c_k . We have that $c_i \leq c_k$. I assume that the merged entity produces at the lowest cost c_i . If premerger and postmerger variables are denoted with superscript \wedge and $*$ respectively we have that a merger increases welfare if:

$$\left(\frac{X^*}{X^\wedge}\right)^{(b+2)} \left(\frac{1}{b+2} + H^* \right) - \frac{1}{b+2} - H^\wedge > 0. \quad (1)$$

This expression is unsatisfactory, because it includes postmerger variables. We proceed to write these postmerger variables as a function of premerger variables.

We use the "infinitesimal merger" technique developed by Farrell and Shapiro (1990). The merger can be usefully interpreted as the sum of infinitesimal reductions in the production of firm k , driving its production to zero, while the other firms adjust their production such that they maximize their profits. Equation (5) in Farrell and Shapiro (1990) traces the evolution of industry sales and individual firm sales given this exogenous reduction in the production of firm k :

$$\frac{dx_j}{dX} = -1 - b s_j, \text{ for } j \neq k.$$

The solution to this differential equation is given by:

$$x_j(X) = K X^{-b} - \frac{X}{1+b}, \text{ where } K \text{ is a constant.}$$

Given the initial condition $x_j^{\wedge} = x_j(X^{\wedge})$, we have:

$$x_j^{\wedge} = K (X^{\wedge})^{-b} - \frac{X^{\wedge}}{1+b}.$$

The constant amounts to:

$$K = (X^{\wedge})^b \left(x_j^{\wedge} + \frac{X^{\wedge}}{1+b} \right)$$

Then, we have

$$x_j^* = x_j(X^*) = \left(x_j^{\wedge} + \frac{X^{\wedge}}{1+b} \right) \left(\frac{X^{\wedge}}{X^*} \right)^b - \frac{X^*}{1+b}. \quad (2)$$

Adding for all $j \neq k$ we get:

$$X^* = \left(X^{\wedge} - x_k^{\wedge} + \frac{n X^{\wedge}}{b+1} \right) \left(\frac{X^{\wedge}}{X^*} \right)^b - \frac{n X^*}{b+1}.$$

Rearranging, we get

$$X^* \left(\frac{n+b+1}{b+1} \right) = X^{\wedge} \left(\frac{n+(1-s_k)(b+1)}{b+1} \right) \left(\frac{X^{\wedge}}{X^*} \right)^b.$$

$$\left(\frac{X^{\wedge}}{X^*} \right)^{b+1} = \frac{n+b+1}{n+(1-s_k)(b+1)} \quad (3)$$

Dividing (2) by X^* and rearranging we obtain:

$$s_j^* = \left(s_j^{\wedge} + \frac{1}{b+1} \right) \left(\frac{X^{\wedge}}{X^*} \right)^{b+1} - \frac{1}{b+1}.$$

Using (3), we obtain

$$s_j^* = \left(s_j^\wedge + \frac{1}{b+1} \right) \left(\frac{n+b+1}{n+(1-s_k^\wedge)(b+1)} \right) - \frac{1}{b+1} = \frac{(n+b+1) s_j^\wedge + s_k^\wedge}{n+(1-s_k^\wedge)(b+1)}. \quad (4)$$

Combining (1), (3) and (4) and manipulating we obtain a necessary and sufficient condition for a merger to increase welfare.

Proposition 1: In an industry with $n+1$ firms, a two-firm merger increases welfare if and only if the following condition holds. It only depends on the premerger Herfindahl Index (H^\wedge), the premerger market share of the inefficient merging partner (s_k^\wedge), and the degree of concavity of demand (b).

$$\begin{aligned} & H^\wedge \left[\left(\frac{n+b+1}{n+(1-s_k^\wedge)(b+1)} \right)^{b/(b+1)} - 1 \right] + \\ & + \left[\frac{n+(1-s_k^\wedge)(b+1)}{n+b+1} \right]^{[(b+2)/(b+1)]} \left(\frac{(s_k^\wedge)^2 (n+1 - (n+b+2)^2) + 2 s_k^\wedge (n+b+1)}{(n+(1-s_k^\wedge)(b+1))^2} \right) + \\ & + \frac{1}{b+2} \left(\left[\frac{n+(1-s_k^\wedge)(b+1)}{n+b+1} \right]^{[(b+2)/(b+1)]} - 1 \right) > 0. \end{aligned}$$

Corollary of Proposition 1: If demand is linear ($b=0$) the previous expression reduces to:

$$s_k^\wedge < \frac{2(n+1)}{2n^2+6n+5}.$$

The premerger Herfindahl Index (H^\wedge) increases the expression in Proposition 1 if demand is strictly concave and decreases it if demand is strictly convex. The greater the concentration in a market the more asymmetric firms are. This asymmetry will have a positive effect on welfare if

big (low-cost) firms are about to increase production more after merger than small (high-cost) firms. Equation (5) in Farrell and Shapiro (1990) stated above, says that this will only be the case if demand is strictly concave and that the opposite will happen if demand is strictly convex.¹ When demand is linear, all firms independently of their costs increase production by the same amount after a merger. This explains why the Herfindahl Index does not play any role in determining the welfare effect of mergers in this particular case.

3.THE SUBSIDIES.

Suppose that a merger increases welfare and at the same time it is profitable for some firm in the market. Then this firm will propose the merger and the antitrust body will approve it. No public intervention is needed in this case. However, if a merger increases welfare but it is not profitable for any firm in the market, it will only take place if it is subsidised. We are going to derive the lowest value of this subsidy, using the same information as before and information on premerger aggregate output.

We are interested in the lowest subsidy, because limiting the transfer to firms may be welcome both on distributive (if the owners of firms are richer than the average tax payer) and efficiency (if the tax system is distorsionary) grounds. To minimize the subsidy the merger should be carried through by the biggest firm, because it has the highest reservation price as the following Lemma shows. One should note that this result will hold for any demand whose degree of concavity is never below -1.

Lemma 1: The bigger a firm the more its profits will increase after a merger.

Proof: Given a market structure, define π_i and π_j as profits of firm i and firm j respectively. Using the FOC of firms we have that $\pi_i = -x_i^2 P'(X) \quad i=j$. Subtracting the FOC of firm j from the one of firm i we get:

¹This argument is already mentioned in Farrell and Shapiro (1990) p.119.

$$c_j - c_i = -(x_i - x_j) P'(X).$$

$$\text{Then we have } \pi_i - \pi_j = -P'(X) (x_i^2 - x_j^2) = -P'(X) (x_i - x_j)(x_i + x_j) = (c_j - c_i) (x_i + x_j) \quad (5)$$

Let superscripts * and ^ denote respectively postmerger and premerger variables. We want to show that after merger the profits of firm i ($\pi_i^* - \pi_i^\wedge$) increase more than the profits of firm j ($\pi_j^* - \pi_j^\wedge$) if firm i is bigger than firm j i.e. if $c_i < c_j$.

Using (5) we have:

$$\pi_i^* - \pi_i^\wedge - (\pi_j^* - \pi_j^\wedge) = (c_j - c_i) (x_i^* + x_j^* - x_i^\wedge - x_j^\wedge).$$

As both firms will increase their production after merger (see Proposition 1 in Corchon (1994)), it will be positive if $c_i < c_j$.

Q.E.D.

Using the results obtained in the previous Section we can obtain the change in profits of firms i and k if they decide to merge. It amounts to:

$$(X^*)^{(b+2)} (s_i^*)^2 - (X^\wedge)^{(b+2)} [(s_i^\wedge)^2 + (s_k^\wedge)^2].$$

Rearranging we have.

$$(X^\wedge)^{(b+2)} \left(\left(\frac{X^*}{X^\wedge} \right)^{(b+2)} (s_i^*)^2 - [(s_i^\wedge)^2 + (s_k^\wedge)^2] \right).$$

Using (3) and (4) we obtain the change in profits as a function of structural variables and the degree of concavity of demand. If it is negative for all firms, its absolute value coincides with the lowest subsidy we are looking for if firm i is the biggest in the market. This result is stated in the following Proposition.

Proposition 2: Suppose that the sale of firm k to a more efficient firm will increase welfare but it is unprofitable. Then the lowest subsidy to induce the merger amounts to the following expression if firm i is the biggest in the market. It only depends on the premerger market share of merging partners (\hat{s}_i , \hat{s}_k), the premerger aggregate output (X^\wedge) and the degree of concavity of demand (b).

$$- (X^\wedge)^{(b+2)} \left(\left[\frac{(n+b+1) \hat{s}_i + \hat{s}_k}{n+(1-\hat{s}_k)(b+1)} \right]^2 \left[\frac{n+(1-\hat{s}_k)(b+1)}{n+b+1} \right]^{[(b+2)/(b+1)]} - (\hat{s}_i)^2 - (\hat{s}_k)^2 \right).$$

4. MERGER PROFITABILITY: THE LINEAR CASE.

Using Proposition 2, the change in profits of firms i and k if they decide to merge for the case of linear demand (b=0) can be obtained. It amounts to:

$$(X^\wedge)^2 \left(\left[\frac{(n+1) \hat{s}_i + \hat{s}_k}{n+1} \right]^2 - (\hat{s}_i)^2 - (\hat{s}_k)^2 \right).$$

Rearranging we obtain,

$$(X^\wedge)^2 \left(\frac{\hat{s}_k}{(n+1)^2} \right) \left(2(n+1) \hat{s}_i - n(n+2) \hat{s}_k \right).$$

It will be positive i.e. the merger will be profitable if:

$$2(n+1) \hat{s}_i - n(n+2) \hat{s}_k > 0.$$

Rearranging we obtain that:

$$\frac{\hat{s}_i}{\hat{s}_k} > \frac{n(n+2)}{2(n+1)}. \quad (6)$$

Given that $\frac{n(n+2)}{2(n+1)} > 1$, this condition means that a merger can only be profitable if it involves firms that are asymmetric enough. To show the sources of these asymmetries, using the equilibrium conditions, one can rewrite the LHS of (6) the following way:

$$\frac{s_i^\wedge}{s_k^\wedge} = \frac{P^\wedge - c_i}{P^\wedge - c_k},$$

where P^\wedge denotes the premerger market price.

In a Cournot setting differences in size are explained by differences in price margins.

Given the price, differences in size are explained by differences in costs. Mergers involve two different effects affecting the private incentives to merge: the anticompetitive effect and the cost minimization effect. The first one drives the price upwards and the second allows cost savings from transferring output from the high cost merging partner to the low cost merging partner. Within a Cournot setting and linear demand we know that the first effect is unable to make a merger profitable. Then a merger will only be profitable if the second effect is large enough. This requires that the cost differential between merger partners be high enough

Given the costs, changes in price affect the differences in size. Then, changes in market size will affect merger profitability through changes in the equilibrium price. When A (the intercept of demand) increases, price increases and differences in size decrease. Therefore, increases in market size reduce the incentive to merge. This result is stated in the following Proposition.

Proposition 3: If demand is linear, when demand increases, merger profitability decreases.

Proof: In an industry with $n+1$ firms, a merger between firm i and firm k such that $c_i < c_k$ is profitable if and only if:

$$\frac{\hat{s}_i}{\hat{s}_k} > \frac{n(n+2)}{2(n+1)}$$

where \hat{s}_i and \hat{s}_k denote the premerger market share of merging firms. In the premerger

equilibrium we have that: $\frac{\hat{s}_i}{\hat{s}_k} = \frac{P^\wedge - c_i}{P^\wedge - c_k}$ and that $P^\wedge = \frac{A + (n+1)\bar{c}}{n+2}$, where P^\wedge is the premerger

price, \bar{c} average cost and A the intercept of demand. Then if A increases, P^\wedge increases and $\frac{\hat{s}_i}{\hat{s}_k}$

decreases.

Q.E.D.

5. DISCUSSION AND CONCLUSION.

The assumption of constant marginal costs imply that when two firms merge the high-cost firm is shut down. Such shut-downs can not be considered rare in real mergers and they are even quite common in industries facing overcapacity problems. In the German steel industry, the 1984 merger of Klockner and Krupp envisaged a restructuring plan "which involved reducing the workforce by 3000, raw steel output by 1m tonnes a year and rolling capacity by 2m tonnes" (Cooke (1986)). Mergers in the food industry, another stagnant market, have been explained by the advantages obtained through concentrating production: "By shutting down the plants of rivals they have taken over and transferring production to their own factories, companies can use spare capacity." (The Economist "The Food Industry survey" December 4th 1993).

In the British machine tool industry, the main purpose of the 1969 merger between Tube Investments and Coventry Gauge was the rationalization of production. "To this end Churchill's Hard Metals division was closed down and Coventry Gauge's smaller production units at Baginton and Yarmouth were closed." (Cowling et al. (1980)). In the following years the production units in Southampton and Halifax were also closed down and their work transferred respectively to the production units in Coventry and Blaydon.

For simplicity I have only studied two-firm mergers. However the methodology developed in Section II can be used to analyse the welfare effect of mergers of any size. For any postmerger market structure the equivalent equations to (3) and (4) can be obtained by simply replacing s_k^{\wedge} with the sum of premerger market shares of selling firms i.e. the ones that will not produce in the postmerger equilibrium. Then it would be possible to compare the welfare in different market structures and determine the optimal market structure. Results can not be obtained without giving specific values to the parameters.

6. REFERENCES.

Cheung, F. K, 1992, "Two remarks on the equilibrium analysis of horizontal merger." *Economics Letters* 40. 119-123.

Cooke, T.E. (1986) Mergers and acquisitions. Basil Blackwell Inc. New York.

Corchon, L.C. (1994) "Comparative statics for aggregative games. The strong concavity case". *Mathematical Social Sciences*, 28, 151-165.

Cowling, K., P. Stoneman, J. Cubbin. (1980) Mergers and economic performance. Cambridge University Press.

Dutz, M.A. (1989) "Horizontal mergers in declining industries" *International Journal of Industrial Organization*, 7 1-10. Special issue on mergers.

Farrell, J. and C. Shapiro (1990) "Horizontal Mergers: an Equilibrium Analysis". *American Economic Review*. March. 107-126.

Fauli-Oller, R. (1997) "On merger profitability in a Cournot setting" *Economics Letters* 54 75-79.

Kester, W.C. (1991) Japanese Takeovers. Harvard Business School Press. Boston.

Levin, D. (1990) "Horizontal mergers: the 50-Percent Benchmark". *American Economic Review* Vol. 80 No. 5. 1238-1245.

Neven, D., R. Nuttall and P. Seabright (1993) Merger in daylight. CEPR.

Odagiri H. and Hase, T. (1989) "Are mergers and acquisitions going to be popular in Japan too ?" *International Journal of Industrial Organization*, 7 49-72. Special issue on mergers.

Salant S.W., S. Switzer , and R.J. Reynolds, 1983, "Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium". *Quarterly Journal of Economics*. May. 185-199.

Van Wegberg, M. (1994) "Why do mergers occur in waves ? Business Cycles, bandwagons and the merger for market power paradox". METEOR Discussion Paper.