CORE SELECTIONS IN ECONOMIES WITH INCREASING RETURNS AND PUBLIC GOODS*

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A B S T R A C T

We consider economies with increasing returns and in which firms follow loss-free pricing rules. In the case of only one firm with an input distributive production set, we obtain an equilibrium which belongs to the Core of the economy and is, hence, a Pareto optimal allocation.

KEYWORDS: Public Goods; Increasing Returns; Input Distributive Production Set; Core.

1 Introduction

One of the first and most studied equilibrium notions in the public goods literature is the Lindahl equilibrium, (investigated by Foley (1970)), its main characteristic being the existence, for each agent, of a personalized price. These individual prices provide unanimity in the demand of public goods and optimality in the convex case. One of the difficulties of the Lindahl equilibrium is that it does not exist in the presence of increasing returns to scale. Many authors have introduced new notions in order to improve the idea of Lindahl equilibrium, but as is well-known, except in special cases such as Mas-Colell and Silvestre (1989) or Diamantaras and Wilkie (1994), firms' behavior as profit maximizers is incompatible with increasing returns to scale in production.

One way to deal with increasing returns to scale is to allow firms to behave differently from profit maximization. In this context, it is useful to model the behavior of firms by means of pricing rules.

There are two different criteria to approach the problem of increasing returns by means of pricing rules, the positive and the normative criteria.

With respect to the latter, in Vohra and Khan (1987) it is proved the existence of an equilibrium, called the Lindahl-Hotelling equilibrium, where firms follow marginal pricing rules.

Bonnisseau (1991) generalizes this concept and shows the existence of Lindahl equilibrium when firms follow loss-bounded pricing rules. In general, the equilibrium obtained in the presence of pricing rules is not a Pareto optimum.

We study the existence of equilibrium when firms follow loss-free pricing rules, which only allow for non-negative profits.

We look for conditions under which this equilibrium satisfies the core property and is, hence a Pareto optimal allocation. To obtain a selection of the core in the presence of increasing returns, we focus on the special case in which there is only one firm with an input distributive set, and it behaves according to profit maximization subject to input constraints.

The result we obtain constitutes the public goods version of Scarf's result in the case of private goods (Scarf (1986)). Our result completes Sharkey's (1989) on non-emptyness of the Core in the sense that we obtain the existence of a Lindahl equilibrium in the Core which maximizes the utility of consumers subject to the budget constraint and which maximizes the profits of the firm subject to input constraints.

The methods presented here also provide an alternative proof of the existence of Lindahl equilibria in the non convex case without making use of an auxiliary economy with an enlarged space of goods as in Bonnisseau (1991). We borrow the formulation of the problem from Khan and Vorha (1985) and introduce pricing rules. In this setting it is possible to adapt the proof of Bonnisseau and Cornet (1988). We provide two examples of pricing rules to which the existence result applies.

2 The Model

First, we introduce some notation. For any $X \subset \mathbb{R}^{k+l}$, we let X_K and X_L denote its projections on the first k and last l coordinates whereas for any $x \in \mathbb{R}^{k+l}$, x_{Ki} will refer to the ith coordinate $(i = 1, \ldots, k)$ and x_{Li} to the $(k+i)$th coordinate. We will denote the $(k+l-1)$ dimensional simplex by Δ, and define $e^{\perp} = \{y \in \mathbb{R}^{k+l}/ye = 0\}$ where $e = (1, \ldots, 1) \in \mathbb{R}^{k+l}$. We identify $e^{\perp}$ with the $(k+l-1)$ dimensional Euclidean space and identify $T = e^{\perp} \times \ldots \times e^{\perp}$ with the Euclidean space $\mathbb{R}^{n(k+l-1)}$. Given $x, y \in \mathbb{R}^s$, we define

$$(x \otimes y) = (x_1 y_1, x_2 y_2, \ldots, x_s y_s)$$

We consider an economy with two types of commodities, l public and k private goods. The set of consumers is $M = \{1, \ldots, m\}$. Each consumer $i \in M$ is characterized by a triple $\{X^i, U_i, \omega^i\}$, where X^i is the consumption set, $U_i : X^i \to \mathbb{R}$ is his utility function and ω^i his initial resources of private goods. There are $N = \{1, \ldots, n\}$ firms. We denote by $Y^j \subset \mathbb{R}^{k+l}$ the production set of firm $j \in N$ and $y = (y^1, \ldots, y^n) \in Y = Y^1 \times \ldots \times Y^n$ is a vector of productions.

A pricing rule is a correspondence which assigns to each element of the boundary of the production set a set of prices. The firms behave following pricing rules, denoting by $\phi_j : \partial Y^j \to \mathbb{R}^{k+l}_+$ the rule that firm j follows.

We define $\phi : \prod_{j=1}^n \partial Y^j \to \Delta^n$ by $\phi(y) = \prod_{j=1}^n (\phi_j(y^j) \cap \Delta)$.

Consider now the following assumptions:

A.1) For each $i \in M$

- **1)** $X^i = \mathbb{R}^{k+l}_+$, where k is the number of private goods and l is the number of public goods. $\omega_i \gg 0$

2) U_i is a continuous function, non decreasing in private goods and strictly increasing in at least one, strictly increasing in public goods and satisfies that given $x, y \in \mathbb{R}_+^{k+l}$ such that $U_i(x) > U_i(y)$ then $U_i(tx + (1-t)y) > U_i(y)$ for all $0 < t < 1$.

3) The wealth of each agent is described with a function $r_i : \mathbb{R}^{k+l} \times Y \to \mathbb{R}$ such that $r_i(p, y) = p_K \omega^i + \sum_{i=1}^n \theta_{ij} p y^j$, where p_K and p_L are the price vectors of private and public goods respectively, and for $i = 1, \dots, m$, $j = 1, \dots, n$, $\theta_{ij} \geq 0$ is the share of agent i in the profits of firm j.

A.2) For each $j \in N$, Y^j is a closed set, $0 \in Y^j$ and $Y^j - \mathbb{R}_+^{k+l} \subset Y^j$ (free disposal).

A.3) The attainable set

$$A = \{(x^1, \dots, x^n; y) \in \prod_{i=1}^m X^i \times Y : \quad x_L^1 = \dots = x_L^m = x_L,$$
$$\text{and } (\sum_{i=1}^m x_K^i, x_L) \leq \sum_{j=1}^n y^j + (\omega, 0)\}$$

is bounded, where $\omega = \sum_{i=1}^m \omega^i \geq 0$ is the total endowment.

A.4) **1)** ϕ is an upper hemicontinuous correspondence, with non-empty, convex and compact values.

2) For every $y \in \prod_{j=1}^n \partial Y^j$ and $q = (q^j)_{j \in N} \in \phi(y)$, $q^j y^j \geq 0$ for each $j \in N$ (Loss-free Rules).

3) Let $y \in \prod_{j=1}^n \partial Y^j$ and $p = (0, p_L) \in \Delta$. If $(p, \dots, p) \in \phi(y)$, then there is $j \in N$ such $p_L y_L^j > 0$.

A public economy is a tuple $E = ((X^i, U_i, \omega^i), (Y^j), (\theta_{ij}), (\phi_j))$ where $i = 1, \dots, n; j = 1, \dots, m$.

We define

$$D = \{\delta = (\delta_1, \dots, \delta_m) \in \prod_{i=1}^m \mathbb{R}_+^l : \quad \sum_{i=1}^m \delta_{ij} = 1 \quad \text{for each } j = 1, \dots, l\}$$

where δ_{ij} is the share of agent i in the price of the jth public good.

Definition 2.1 *Given an economy* $E = ((X^i, U_i, \omega^i), (Y^j), (\theta_{ij}), (\phi_j))$, *a Generalized Lindahl equilibrium is an element* $((x^1, \ldots, x^m), y, p, \delta) \in \prod_{i=1}^m X^i \times \prod_{j=1}^n Y^j \times \Delta \times D$ *such that*

a) x^i *maximizes* $U_i(x)$ *subject to*

$$x \in X^i \text{ and } p_K x_K + (p_L \otimes \delta_i) x_L \leq r_i(p, y)$$

b) For all $j \in N$, $y^j \in \partial Y^j$ *and* $p \in \phi_j(y^j)$.

c) There is some $x_L \in \mathbb{R}^l$ *such that for all* $i \in M, x_L^i = x_L$ *and* $(\sum_{i=1}^m x_K^i, x_L) = (\omega, 0) + \sum_{j=1}^n y^j$.

Note that, the vector $(p_L \otimes \delta_i)$ is the personalized price of agent i.

In the following theorem, we state the existence of a Generalized Lindahl equilibrium. It is less general than the theorem of Bonnisseau (1991) but on the other side we present a direct proof which does not use an enlarged economy.

Theorem 2.2 *The public economy* $E = ((X^i, U_i, \omega^i), (Y^j), (\theta_{ij}), (\phi_j))$ *has a Generalized Lindahl equilibrium if A1-A4 hold.*

The proof of Theorem 2.2 is a variant of the proof of Theorem 3.2 below. For completeness, in the appendix we provide the reader with the required modifications.

We now apply Theorem 2.2 to some pricing rules considered previounly in the literature.

First consider the classical case in which firms maximize profits on a convex set. The next corollary shows the existence of Lindahl Equilibrium, in the context where the production set is convex, there is more than one private and public good and in which there are neither endowments of public goods nor constant returns to scale in the production.

Corollary 2.3 *Let E be an economy which satisfies A.1, A.2 and such that*

(a)For each $j \in N$, Y^j *is convex and* $Y^j \cap \mathbb{R}_+^{k+l} = \{0\}$.

(b) For each $j \in N$, $\phi_j(y^j) = \{p \in \mathbb{R}_+^{k+l} : \quad py^j \geq py \quad \forall y \in Y^j\} \cap \Delta$; *Profit maximization.*

(c) There is $j \in N$ *and* $y^j \in Y^j$ *such that* $y_L^j \gg 0$.

Then, E has a Lindahl equilibrium.

From theorem 2.2 we also derive the two following corollaries

Corollary 2.4 *Let* $E = ((X^i, U_i, \omega^i), (Y^j), (\theta_{ij}), (\phi^j))$ *be an economy which satisfies assumptions A.1-A.3, and consists of two types of firms* $N = I \cup J$ *such that*

a) Every firm $j \in I$ *follows the Average Cost Pricing rule:*
$\phi_j(y^j) = \{p \in \Delta : \quad py^j = 0\}$.

b) J *is non-empty, and for every firm* $j \in J$ *the production set* Y^j *is convex, satisfies* $Y^j \cap \mathbb{R}_+^{k+l} = \{0\}$, *and follows the rule defined by* $\phi_j(y^j) = \{p \in \mathbb{R}_+^{k+l} : \quad py^j \geq py \quad \forall y \in Y^j\} \cap \Delta$. *Moreover, there is* $h \in J$ *and* $y^h \in Y^h$ *such that* $y_L^h \gg 0$.

Then E *has a Generalized Lindahl equilibrium.*

Corollary 2.5 *If assumptions A.1-A.3 hold and there are two types of firms* $N = I \cup J$ *such that*

a) Every firm $j \in I$ *has an output distributive production set*[1] *which satisfies* $Y^j \cap \mathbb{R}_+^{k+l} = \{0\}$ *and follows:*
$\phi_j(y^j) = \{p \in \mathbb{R}_+^{k+l} : \quad py \geq py' \quad \forall y' \in Y \text{ with } y' \leq y^{j+}\} \cap \Delta$ *(Voluntary trading) where* $y^{j+} = \max\{y^j, 0\}$.

b) J *is non-empty, and for every firm* $j \in J$ *the production set* Y^j *is convex, satisfies* $Y^j \cap \mathbb{R}_+^{k+l} = \{0\}$, *and its behavior is modeled by* $\phi_j(y^j) = \{p \in \Delta : \quad py^j \geq py \quad \forall y \in Y^j\}$. *Moreover, there is* $h \in J$ *and* $y^h \in Y^h$ *such that* $y_L^h \gg 0$.

Then, there is a Generalized Lindahl equilibrium in this economy.

3 CORE SELECTION

We focus in the case describe in Sharkey (1989).

There are two types of private goods: capital goods and ordinary commodities. Suppose consumers are indifferent in the consumption of some commodities, the capital goods. Then, these commodities do not affect the marginal calculations of any consumer so the agents fully utilize the set of these commodities in their possession. Let $x \in \mathbb{R}_+^k$ and let x_c denote its

[1] A production set is output distributive if any (nonnegative) weighted sum of feasible production plans is feasible if it involves more outputs than any of the original plans(Dehez and Drèze (1987))

projection onto the space of capital goods, x_o its projection onto the space of ordinary commodities. If $y \in Y$ then $y = (a_c, a_o, b)$ where a_c are capital inputs, a_o ordinary inputs, with $(a_c, a_o) \leq 0$ and b the output vector.

Suppose also, that there is only one firm with an input distributive set Y. Recall that Y is an input distributive set if any non-negative weighted sum $y = (a_c, a_o, b)$ of feasible production plans $y^i = (a_c^i, a_o^i, b^i)$ is feasible, whenever $a_c^y \leq a_c^i$ for all i, i.e., when y uses at least as many inputs of every capital input as every y^i.

Hence, we add to the above assumptions, the following:

A.5) If $x, x' \in \mathbb{R}_+^{k+l}$ are such that $x_o = x'_o$ and $x_L = x'_L$, then $U_i(x) = U_i(x')$ for all $i \in M$

A.6) There only exists one firm, whose production set Y is an input distributive set which satisfies that if $y = (0, a_o, b) \in Y$ then $(a_o, b) \leq 0$. And the firm follows the pricing rule $\phi : \partial Y \to \Delta$ defined by

$$\phi(y) = \{p \in \mathbb{R}_+^{k+l} : \quad py = 0 \text{ and } py \geq py' \quad \forall y' \in Y \text{ with } a'_c \geq a_c\} \cap \Delta.$$

A.7) There is $y = (a_c, a_o, b) \in Y$ such that $y + (\omega, 0) \in \mathbb{R}_+^{k+l}$ and $b \gg 0$.

Assumption A.7 expresses that there is a feasible production of all public goods.

The following lemma is based in a similar one in Villar (1996, Proposition 9.1).

Lemma 3.1 *Let Y be an input distibutive set which satisfies assumption A.2 and if $y = (0, a_o, b) \in Y$ then $(a_o, b) \leq 0$.*

Define now, the following pricing rule $\phi : \partial Y \to \Delta$ by

$$\phi(y) = \{p \in \mathbb{R}_+^{k+l} : \quad py = 0 \text{ and } py \geq py' \quad \forall y' \in Y \text{ with } a'_c \geq a_c\} \cap \Delta.$$

Then, ϕ is an upper hemicontinuous correspondence with non-empty, convex and compact values. Moreover, it is a loss-free pricing rule.

Proof

The pricing rule ϕ is loss-free because $0 \in Y$. Moreover, since Y is an input distributive set, it follows from results by Scarf (1986) that ϕ is non-empty. Clearly, it is a correspondence with convex values. We derive from the upper-hemicontinuity and the fact that is defined on the simplex Δ that it has compact values. We only need to prove that is upper hemicontinuos or it has a closed graph. Let $\bar{y} \in Y$ and consider a sequence (p^α, y^α) converging to $(\bar{p}, \bar{y})$ such that $y^\alpha \in \partial Y$ and $p^\alpha \in \phi(y^\alpha)$ for all α. Suppose $\bar{p} \notin \phi(\bar{y})$ then there exist $y' \in Y$ with $a'_c \geq \bar{a}_c$ such that $\bar{p}y' > \bar{p}\bar{y}$. For α large enough we obtain that $p^\alpha y' > p^\alpha y^\alpha$.

If $\bar{a}_c = 0$ then $a'_c = 0$ and $(a'_o, b) \leq 0$. Therefore, $0 \geq p^\alpha(0, a'_o, b) = p^\alpha y' > p^\alpha y^\alpha = 0$. Which is a contradiction.

Now, define

$a''_{cj} = a'_{cj} + \epsilon$ if $a'_{cj} < 0$ and $a''_{cj} = 0$ otherwise.

$a''_{oj} = a'_{oj} - \delta$ and $b''_j = b'_j - \delta$.

Taking $\epsilon > 0$ and $\delta > 0$ such that $y'' = (a''_c, a''_o, b'') \in Y$ and $\bar{p}y'' > \bar{p}\bar{y}$ (this is possible because Y is comprehensive and close). But $a''_c > a'_c \geq \bar{a}_c$. Now, for α big enough, $(a^\alpha_c, a^\alpha_o, b^\alpha)$ is close to $(\bar{a}_c, \bar{a}_o, \bar{b})$ such that $a''_c \geq a^\alpha_c$ with $p^\alpha y'' > p^\alpha y^\alpha$ which contradicts that $p^\alpha \in \phi(y^\alpha)$. □

The next theorem establishes the existence of a particular core selection.

Theorem 3.2 *An economy $E = ((X^i, U_i, \omega^i), Y, (\theta_i), \phi)$ has a Generalized Lindahl equilibrium if A.1-A.3, A.5-A.7 holds. Moreover, this equilibrium belongs to the Core of the economy.*

Proof

The proof consits of three steps. In the first step we construct a non-empty, convex and compact set. In the second, we define a correspondence from the set obtained in step 1 to itself to which we apply the Kakutani's theorem and the third where we check that the fixed point obtained is a Generalized Lindahl equilibrium by means of five claims.

STEP 1:

Let $Q \in \mathbb{R}_+$, we denote by $C = \{x \in \mathbb{R}^{k+l}_+ : x_c = 0\} \cap [0, Qe]$ the equal truncated consumption set for each agent. From lemma 5.1 of Bonnisseau and Cornet (1988) we define an homeomorphism between $e^\perp$ and ∂Y and from it we have that $y(s) = s - \lambda(s)e$ where $s \in e^\perp$ and $\lambda(s)$ is defined by the homeomorfismo. Also, we define, $\Delta_\epsilon = \{p \in \mathbb{R}^l : \sum_{h=1}^{l} p_h = 1, p_h \geq -\epsilon$ with $h = 1, \ldots, k+l\}$.

Using lemma 5.1 of Bonnisseau and Cornet (1988) and assumption A.3, we may choose $\epsilon > 0$ small enough, Q large enough and a closed cube B centered at 0 in the Euclidean space T satisfying i) and ii) below:

i) If $z \in \hat{Y} + (\omega, 0)$ then $z \ll Qe$, where $\hat{Y}$ is the projection, on the space of production, of the attainable set. As a consequence of this, if $x \in \hat{X}^i$ then $(0, x_o, x_L) \ll Qe$, where $\hat{X}^i$ is the projection, on the space of consumption goods, of the attainable set.

ii) $\{s \in T : y(s) + (\omega, 0) \in \mathbb{R}_+^{k+l}\} \subset int(B)$.

Let $G = \prod_{i=1}^m C \times B \times \Delta_\epsilon \times \Delta \times D$, which is a non-empty, convex and compact set in the product topology.

STEP 2:

First, we define the demand correspondence. Let $(p, \delta, t) \in \Delta_\epsilon \times D \times \mathbb{R}$.

$$B_i(p, \delta, t) = \{x \in C : p_L x_K + (p_L \otimes \delta_i) x_L \leq t\},$$

$$\xi_i(p, \delta, t) = \{x \in B_i(p, \delta, t) : \forall x^i \in B_i(p, \delta, t)\} \quad U_i(x) \geq U_i(x^i),$$

$$\nu_i(p, \delta, t) = \{x \in C : p_K x_K + (p_L \otimes \delta_i) x_L = \inf(p_K C_K + (p_L \otimes \delta_i) C_L)\}.$$

And

$$f_i(p, \delta, t) = \begin{cases} \xi_i(p, \delta, t) & \text{if} \quad t > \inf(p_K C_K + (p_L \otimes \delta_i) C_L) \\ \nu_i(p, \delta, t) & \text{otherwise} \end{cases}$$

Now we are in position to define the correspondence to which we apply Kakutani's theorem. Let $F = \prod_{i=1}^5 F_v : G \to G$ be a correspondence such that:
$F_1(x, s, p, p^1, \delta) = \prod_{i=1}^m f_i(p, \delta, p_K \omega^i + \theta_i p y(s))$ is the demand correspondence.
$F_2(x, s, p, p^1, \delta) = \{\sigma \in B : (p - p^1)(\sigma - \sigma') \geq 0 \quad \forall \sigma' \in B\}$ ensures that in the fixed point all firms agree on the price of the goods.
$F_3(x, s, p, p^1, \delta) = \{q \in \Delta_\epsilon : (q - q')(\sum_{i=1}^m (x_K^i, (\delta_i \otimes x_L^i)) - y(s) - (\omega, 0)) \geq 0 \quad \forall q' \in \Delta_\epsilon\}$ guaranties the feasibility of the equilibrium in the fixed point.
$F_4(x, s, p, p^1, \delta) = \phi(y(s))$ prices have to be acceptable for all firms. And
$F_5(x, s, p, p^1, \delta) = \{\delta \in D : \quad \delta_j \in \arg\max \sum_{i=1}^m \delta_{ij} x_{Lj}^i \quad \forall j = 1, \ldots, l\}$ gets the share of the price of public goods which equalizes the demand of public goods by the agents.

The correspondence F_1 is the classical demand and is upper-hemicontinuous, with non-empty, convex and compact values by assumption A.1, lemma 5.1 of Bonnisseau and Cornet (1988), lemma 1 in Debreu (1962) and the maximum theorem. F_2 is the maximum argument of a linear function on a non empty, convex and compact set. Thus, F_2 has non-empty, convex and compact values. It is upper-hemicontinuous by the maximum theorem. F_3 satisfies these properties by lemma 5.1 of Bonnisseau and Cornet (1988) and the maximum theorem, and so does F_4 by lemma 3.1. To finish, F_5 is again an upper-hemicontinuous function, with non-empty, convex and compact values by the maximum theorem. It follows that we can apply Kakutani's Theorem to the correspondence F to find a fixed point $(x^*, s^*, p^*, p^{*1}, \delta^*)$. That is, the following five equations hold.

$$x^{*i} \in f_i(p^*, \delta^*, r_i^*) \text{ for each } i \in M \tag{3.1}$$

$$(p^* - p^{*1})s^* \geq (p^* - p^{*1})s \quad \forall s \in B \tag{3.2}$$

$$p^*\gamma \geq p\gamma \quad \forall p \in \Delta_\epsilon \tag{3.3}$$

$$p^{*1} \in \phi(y^*) \tag{3.4}$$

$$x_L^{*i} \leq \sum_{h=1}^{m} (\delta_h^* \otimes x_L^{*h}) \text{ for all } i \in M \tag{3.5}$$

where $r_i^* = r_i(p^*, y^*)$, $y^* = y(s^*)$ and

$$\gamma = (\sum_{i=1}^{m} x_K^{*i}, \sum_{i=1}^{m} (\delta_i^* \otimes x_L^{*i})) - (w, 0) - y^*.$$

STEP 3:

From these equations we will derive that $((x^{*i})_{i \in M}, y^*, p^*, \delta^*)$ is a Generalized Lindahl equilibrium for E, by proving the following five claims.

Claim I: $\gamma \in -\mathbb{R}_{++}^l \cup \{0\}$.

First of all, we prove that $p^*y^* \geq 0$. Taking $s = 0$ in inequality 3.2, it is obtained that $(p^* - p^{*1})s^* \geq 0$, hence $p^*s^* \geq p^{*1}s^*$. Moreover, since $y^* = s^* - \lambda(s^*)e$, applying lemma 5.1 of Bonnisseau and Cornet (1988) and taking into account that $p^*e = p^{*1}e$, we obtain that $p^*y^* \geq p^{*1}y^*$. Consequently $p^*y^* \geq 0$ by lemma 3.1 and equation 3.4.

Since $p^*y^* \geq 0$ and $0 \in C_L$ we obtain that $\theta_i p^* y^* \geq 0 \geq \inf((p^*_L \otimes \delta^*_i)C_L)$. Moreover, $r^*_i = p^*_K\omega^i + \theta_i p^* y^* \geq \inf(p^*_K C_K) + \inf((p^*_L \otimes \delta^*_i)C_L) = \inf(p^*_K C_K + (p^*_L \otimes \delta^*_i)C_L)$ for all $i \in M$, because $\omega^i \in C_K$ and $\theta_i p^* y^* \geq \inf((p^*_L \otimes \delta^*_i)C_L)$.

Taking the definition of F_1 into account we see that $p^*_K x^{*h}_K + (p^*_L \otimes \delta^*_h) x^{*h}_L \leq r^*_h$ for all $h \in M$. So,

$$p^*\gamma = \sum_{i=1}^{m}(p^*_K x^{*i}_K + (p^*_L \otimes \delta^*_i)x^{*i}_L - r^*_i) \leq 0$$

and by inequality 3.3 $p\gamma \leq p^*\gamma \leq 0$ for all $p \in \Delta_\epsilon$. This implies that $\gamma \in \Delta^o_\epsilon$, the polar cone of Δ_ϵ, so $\gamma \in -\mathbb{R}^l_{++} \cup \{0\}$.

Claim II: $p^{*1} = p^*$ and $p^* \in \phi(y^*)$.

Since, by claim I, $\gamma \in -\mathbb{R}^l_{++} \cup \{0\}$ and $C \subset \mathbb{R}^{k+l}_+$, we see that $y^* + (\omega, 0) \in \mathbb{R}^{k+l}_+$. From the choice of B, $s^* \in int(B)$. Consequently, by inequality 3.2, the gradient at s^* of the linear function $s \longrightarrow (p^* - p^{*1})s$ is equal to 0, hence $p^* = p^{*1}$, and the claim is proved. In particular, $p^* \geq 0$.

Claim III: $x^{*i} \in \xi_i(p^*, \delta^*, r_i(p^*, (y^{*j})_{j\in N}))$.

Claim II shows that $p^* \in \Delta$. Since $0 \in C$ we have that $\inf(p^*_K C_K + (p^*_L \otimes \delta^*_i)C_L) = 0$ for all $i \in M$. Suppose, for contradiction, that $\sum_{i \in M} r^*_i = p^*_K\omega + p^*y^* = 0$. Since, $p^*y^* \geq 0$ and $\omega = \sum_{i\in M}\omega^i \gg 0$ we obtain that $p^*y^* = 0$ and $p^*_K = 0$. In the proof of claim I, we have obtained that $r^{*i} \geq \inf(p^*_K C_K + (p^*_L \otimes \delta^*_i)C_L) = 0$, for all $i \in M$. As a consequence, we derive that $r^{*i} = 0$, for all $i \in M$. This implies that $p^*\gamma = 0$. But $p^* \in \Delta$ and $\gamma \in -\mathbb{R}^l_{++} \cup \{0\}$. Hence, $\gamma = 0$. Since, the demand of capital goods of all agents is 0 and $\gamma = 0$ we have that $y^* = (\omega_c, a^*_o, b^*)$. To sum up, $p^* = (0, p^*_L) \in \phi(y^*)$, $p^*y^* = 0$ and $p^*y^* \geq p^*y$ for all $y = (a_c, a_o, b) \in Y$ such that $a_c \geq a^*_c = \omega_c$. But this contradicts assumption A.7.

Thus, there exists $h \in M$ such that $r^*_h > 0$. Since $\hat{Y} + (\omega, 0) \subset C$, we have that $y^* + (\omega, 0) \ll Qe$. By claim I,

$$(\sum_{i=1}^{m} x^{*i}_K, \sum_{i=1}^{m}(\delta^*_i \otimes x^{*i}_L)) \leq (w, 0) + y^*,$$

so, $\sum_{i=1}^m x^{*i}_K \leq w + y^*_K \ll Qe_K$. Hence, $x^{*h}_K \ll Qe_K$. We must have that $p^*_K > 0$, because $x^{*h} \in \xi_h(p^*, \delta^*, r^*_h)$ and U_h is strictly increasing in at least one private good. From $\omega^i \gg 0$ for all $i \in M$ and taking into account that

$p^*y^* \geq 0$, it follows that for each $i \in M$ $r_i^* > \inf(p_K^* C_K + (p_L^* \otimes \delta_i^*) C_L)$ and $x^{*i} \in \xi_i(p^*, \delta^*, r_i^*)$.

Claim IV: For some $x_L^* \in \mathbb{R}^l, x_L^{*i} = x_L^*$, for every $i \in M$, and

$$\gamma = (\sum_{i=1}^{m} x_K^{*i}, \sum_{i=1}^{m} (\delta_i^* \otimes x_L^{*i})) - (w, 0) - \sum_{j=1}^{n} y^{*j} = 0$$

Since $\hat{Y} + (\omega, 0) \subset C$, we have that $y^* + (\omega, 0) \ll Qe$. From equation 3.5, for each $i \in M$, $x_L^{*i} \leq \sum_{t=1}^{m} (\delta_t^* \otimes x_L^{*t})$. From claim II it follows that $\sum_{t=1}^{m} (\delta_t^* \otimes x_L^{*t}) \leq y_L^*$. We conclude that $x_L^{*i} \ll Qe_L$. Since, the preferences are strictly increasing in public goods, we see from claim III that $(p_L \otimes \delta_i) \gg 0$ for all $i \in M$ and $p \in \Delta$. Furthermore, $\delta_i \gg 0$. Since, $\sum_{t=1}^{m} (\delta_t^* \otimes x_L^{*t}) = \sup_{t \in M} \{x_L^{*t}\}$ we have that there is $x_L^* \in \mathbb{R}_+^l$ such that $x_L^{*i} = x_L^*$ for all $i \in M$.

If $p_K^* x_K^{*h} + (p_L^* \otimes \delta_h^*) x_L^{*h} < r_h^*$ for some $h \in M$, then, because $x_L^{*i} \ll Qe_L$ and the preferences are strictly increasing in public goods, there is $x \in C$, such that $p_K^* x_K + (p_L^* \otimes \delta_h^*) x_L < r_h^*$ and $U_h(x) > U_h(x^{*h})$, contradicting claim III.

Thus, we must have that $p_K^* x_K^{*i} + (p_L^* \otimes \delta_i^*) x_L^{*i} = r_i^*$ for all $i \in M$; so that $p^*\gamma = 0$. Since, $p^* \geq 0$ and $\gamma \in -\mathbb{R}_{++}^l \cup \{0\}$, we conclude that $\gamma = 0$.

Claim V: For each $i \in M$, $U_i(x^{*i}) \geq U_i(x)$ for every $x \in X^i$ such that $p_K^* x_K + (p_L^* \otimes \delta_i^*) x_L \leq r_i^*$.

Suppose there are $i \in M$ and $x' \in X^i$ such that $U_i(x^{*i}) < U_i(x')$. From assumption A.5, we have that $x = (0, x'_o, x'_L) \leq x'$ satisfies $U_i(x) = U_i(x') > U_i(x^{*i})$. Take the convex combination $\mu x + (1 - \mu) x^{*i}$ with $\mu \in [0, 1]$. Since $x^{*i} \in \hat{X}^i$ by i) $x^{*i} \ll Qe$ and for a μ^* small enough $\mu^* x + (1 - \mu^*) x^{*i} \in C$. But, by assumption A.7 we have that $U_i(\mu^* x + (1 - \mu^*) x^{*i}) > U_i(x^{*i})$. As a consequence of claim III, $\mu^* x + (1 - \mu^*) x^{*i}$ does not satisfy the budget constraint of agent i. After some computation we obtain that $p_K^* x'_K + (p_L^* \otimes \delta_i^*) x'_L > r_i^*$, and the claim is proved.

Finally, we prove that this Generalized Lindahl equilibrium obtained is in the Core. Suppose there are $S \subset N$ and $((x^i, z)_{i \in S}, y)$ such that

$$(\sum_{i \in S} x^i, z) = (\sum_{i \in S} \omega^i, 0) + y \text{ and} \tag{3.6}$$

$$U_i(x^i, z) \geq U_i(x_K^{*i}, x_L^*) \text{ for all } i \in S \tag{3.7}$$

with at least one strict inequality.

By claim III,

$$p_K^* x^i + (p_L^* \otimes \delta_i^*) z \geq r_i^* \text{ for all } i \in S$$

with at least one strict inequality. Hence,

$$p_K^* \sum_{i \in S} x^i + \sum_{i \in S} (p_L^* \otimes \delta_i^*) z > \sum_{i \in S} r_i^*.$$

Since, $0 \leq \sum_{i \in S} \delta_i \leq 1$ we have that $\sum_{i \in S} (p_L^* \otimes \delta_i^*) \leq p_L^* \otimes 1 = p_L^*$ and

$$p_K^* \sum_{i \in S} x^i + p_L^* z > \sum_{i \in S} r_i^*.$$

Now, by feasibility, equity 3.6, we derive that

$$p^*(\sum_{i \in S} x^i, z) = p^*((\sum_{i \in S} \omega^i, 0) + y) > \sum_{i \in S} r_i^*.$$

Finally, since $p^* y^* = 0$ then $\sum_{i \in S} r_i^* = p^*(\sum_{i \in S} \omega^i, 0)$.

Furthermore, $p^*((\sum_{i \in S} \omega^i, 0) + y) > p^*(\sum_{i \in S} \omega^i, 0)$. Thus, $p^* y > 0$.

Since $p^* y^* = 0$ and by feasibility $y = (a_c, a_o, b)$ satisfies $a_c \geq -\omega_c$, the fact that $p^* y > 0$ contradicts $p^* \in \phi(y^*)$.

□

Although, it was proved by Sharkey (1989) that in this context the Core of an economy with public goods is non-empty by means of the balanced condition, with this theorem we find a particular selection of it in which consumers maximize their utility subject to the budget constraints and the firm maximizes its profits subject to the availability of inputs. It is shown the existence of the social equilibrium with personalized prices that holds in the core of the economy.

4 APPENDIX

Proof of Theorem 2.2

We only provide the changes with respect to the proof of theorem 3.2

Pick $Q \in \mathbb{R}_+$ and let $C = [0, Qe]$. We may choose $\epsilon > 0$, Q and B again such that the following hold

i) $\hat{Y} + (\omega, 0) \subset C$ and $\hat{X}^i \subset C$.

ii) $\{s \in T / \sum_{j=1}^n y^j(s) + (\omega, 0) \in \mathbb{R}_+^{k+l}\} \subset int(B)$.

The correspondence F is now defined on $G = \prod_{i=1}^m C \times B \times \Delta_\epsilon \times \prod_{i=1}^n \Delta \times D$, where $F = \prod_{i=1}^5 F_v$ and

$F_1(x, s, p, (p^j)_{j\in N}, \delta) = \prod_{i=1}^m f_i(p, \delta, p_K\omega^i + \sum_{j=1}^n \theta_{ij} p y^j(s))$

$F_2(x, s, p, (p^j)_{j\in N}, \delta) = \{(\sigma_j)_{j\in N} \in B : \quad \sum_{j=1}^n (p-p^j)(\sigma_j - \sigma_j') \geq 0 \quad \forall (\sigma_j')_{j\in N} \in B\}$

$F_3(x, s, p, (p^j)_{j\in N}, \delta) = \{q \in S_\epsilon : \quad (q-q')(\sum_{i=1}^m (x_K^i, (\delta_i \otimes x_L^i)) - \sum_{j=1}^n y^j(s) - (\omega, 0)) \geq 0 \qquad \forall q' \in S_\epsilon\}$

$F_4(x, s, p, (p^j)_{j\in N}, \delta) = \phi((y^j(s))_{j\in N})$

$F_5(x, s, p, (p^j)_{j\in N}, \delta) = \{\delta \in D : \quad \delta_j \in \arg\max \sum_{i=1}^m \delta_{ij} x_{Lj}^i \qquad \forall j = 1, \ldots, l\}$

By Kakutani's theorem we find a fixed point $(x^*, s^*, p^*, (p^{*j})_{j\in N}, \delta^*)$. Letting

$$\gamma = (\sum_{i=1}^m x_K^{*i}, \sum_{i=1}^m (\delta_i^* \otimes x_L^{*i})) - (w, 0) - \sum_{j=1}^n y^{*j},$$

we have

$$x^{*i} \in f_i(p^*, \delta^*, r_i^*) \text{ for each } i \in M \tag{4.8}$$

$$\sum_{j=1}^n (p^* - p^{*j}) s_j^* \geq \sum_{j=1}^n (p^* - p^{*j}) s_j \quad \forall (s_j)_{j\in N} \in B \tag{4.9}$$

$$p^*\gamma \geq p\gamma \quad \forall p \in S_\epsilon \tag{4.10}$$

$$(p^{*j})_{j\in N} \in \phi((y^{*j})_{j\in N}) \tag{4.11}$$

$$x_L^{*i} \leq \sum_{h=1}^m (\delta_h^* \otimes x_L^{*h}) \text{ for all } i \in M \tag{4.12}$$

The proof of claims (I) and (II) is analogous to theorem 3.2. It is only necessary to take into account that there are n firms. In the proof claim (III) it is obtained that $p^*y^{*j} = 0$ for all $j \in N$ and $p^*_K = 0$ which contradicts assumption A.4.3. The rest is now analogous.

Finally, claims (IV) and (V) follow the same argument as previously but with n firms.

□

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