

## **PIGOUVIAN TAXES: A STRATEGIC APPROACH\***

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# Pigouvian Taxes: A Strategic Approach

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## Abstract

This paper analyzes the problem of designing mechanisms to implement efficient solutions in economies with externalities. We provide two simple mechanisms implementing the Pigouvian Social Choice Correspondence in environments in which coalitions can be formed. Finally, we study economies in which agents are incompletely informed, and provide a mechanism that implements this social choice correspondence in Bayesian equilibrium.

keywords: Pigouvian Taxes, Mechanism Design, Implementation

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## 1. Introduction

This paper presents three mechanisms implementing the Pigouvian Social Choice Correspondence in economies with externalities. In these environments, it is well-known that competitive equilibria fail to be Pareto efficient. A way to solve this problem was pointed out by Pigou (1920). He proposed the establishment of a taxation system. When taxes are introduced in the market and agents are price and tax-takers, Pareto efficient outcomes are reached.

A classical criticism to this approach is derived from the severe informational problems faced by the regulator. The usual way to tackle informational problems is by way of the Theory of implementation. In this context, Varian (1994) presents a two stage “compensation mechanism” which implements Pigouvian equilibria in subgame perfect Nash equilibrium (SPE in the sequel). The compensation mechanism seems to be a nice solution to this problem. Nevertheless, this mechanism presents two main problems. Firstly, in economies for which there is an agent with linear preferences, SPE yielding outcomes which fail to be Pareto efficient exist. (See Examples ?? and ?? below.). Secondly, and more importantly, the solution concept used by Varian, namely Subgame Perfect Nash Equilibrium, is problematic in the sense that it can not be easily extended to environments in which coalitions can be formed, or in which agents have incomplete information. We believe that the extension to these environments is crucial because of their relevance to practical problems. In this paper three mechanisms are presented which implement the Pigouvian SCC and deal with the issues of coalition formation and incomplete information.

The first mechanism implements the Pigouvian SCC in strong equilibrium. This mechanism grasps some ideas from Varian’s *compensation mechanism*. An important difference between Varian’s mechanism and ours is that the output is not decided by a single individual. This is the reason for which allocations failing to be Pareto efficient can be supported by SPE in the compensation mechanism.

The second mechanism doubly implements the Pigouvian SCC in Nash and strong Nash equilibrium. This mechanism is more complicated than the first one, but it has an additional advantage in that the results do not depend either on the possibility of coalition formation or the extent of these coalitions.

The two mechanisms presented above assume that agents have complete information. Nevertheless, there are many cases in which agents' information is not complete. Which is why we design a mechanism to implement the Pigouvian SCC in such a case.

The rest of the paper is organized as follows. Section ?? presents the model. Implementation of Pigouvian SCC in strong Nash equilibrium using a continuous mechanism is reported in Section ?. Section ? presents a mechanism that doubly implements the Pigouvian SCC in Nash and strong equilibrium. The incomplete information case is analyzed in Section ?, where a mechanism implementing the Pigouvian SCC in Bayesian equilibrium is provided.

## 2. The Model

Let us consider the following externality problem involving  $n + 1$  agents. Let  $I = \{0, 1, \dots, n\}$  be the set of agents. Agent 0 consumes  $q$  units of a good generating an external effect on the others. We suppose that each agent's preferences are representable by a utility function,  $u_i$ , which depends on two variables. The first one,  $T_i \in \mathbb{R}$ , will play the role of (a transfer of) numéraire, whereas the second,  $q \in \mathbb{R}_+$ , measures the consumption of the good that generates the externality of agent 0. We assume that each  $u_i$  is quasilinear. Thus, agent  $i$ 's utility function can be expressed as

$$u_i(t_i, q) = T_i + \pi_i(q)$$

where  $\pi_i$  is convex for all  $i$ , strictly increasing for agent 0 and strictly decreasing for the others. An *economy* can be represented by a vector  $e = (u_0, u_1, \dots, u_n)$ . Let  $\mathcal{E}$  denote the set of all admissible economies. Given an economy  $e$ , we say that an *allocation*  $z = (T_0, T_1, \dots, T_n, q)$  is *feasible* for  $e$  if  $q \in \mathbb{R}_+$  and  $\sum_{i \in I} T_i = 0$ . Let  $Z$  denote the set of all feasible allocations. In order to simplify notation,

we can extend agents preferences (and utility functions) over allocations in the following way. We say that agent  $i$  weakly prefers allocation  $z = (T_0, \dots, T_n, q)$  to  $z' = (T'_0, \dots, T'_n, q')$  if  $u_i(T_i, q) \geq u_i(T'_i, q')$ . In this case, we will also write  $u_i(z) \geq u_i(z')$ .

The properties we introduce next are minimal conditions that should be satisfied by any solution in order to be considered satisfactory. The first minimal requirement for an allocation is *Pareto efficiency*. Given an economy  $e$ , we say that a feasible allocation  $z$  is efficient if there is no feasible allocation  $z'$  which is (weakly) preferred to  $z$  by all the individuals, and strictly preferred by some of the agents. That is  $z \in Z$  is Pareto efficient if for all  $z' \in Z$  and  $i \in I$ ,  $u_i(z) > u_i(z')$  implies that there is some  $j$  in  $I$  such that  $u_j(z) < u_j(z')$ . The second condition is that of *individual rationality*. The idea conveyed by this concept is very related to the preservation the individual rights of agents not to be disturbed. In this sense, we think that agents should reach an utility level at least as high as the utility they have when no externality is made and no transfer of numéraire affects them. Thus, an allocation  $z$  is said to be individually rational whenever  $u_i(z) \geq u_i(0, 0)$  for any agent.

We are interested in social choice correspondences satisfying the two minimal conditions mentioned above.

It is well-known that, in competitive equilibrium, the level of output chosen by agent 0 fails to be Pareto efficient and individually rational. A way to solve this problem was pointed out by Pigou (1920). His solution involves intervention by a regulator who imposes a tax system.

**Definition 2.1.** *The Pigouvian Social Choice Correspondence,  $\phi_p : \mathcal{E} \rightarrow \mathbb{R}^n \times \mathbb{R}_+$ , associates to each economy,  $e = (u_0, u_1, \dots, u_n) \in \mathcal{E}$ , a tax system,  $(t_i^*)_{i \in I} \in \mathbb{R}^n$ , and an output  $q^* \in \mathbb{R}_+$  such that*

1.  $q^*$  maximizes  $u_i(T_i, q) = u_i(t_i^* q, q) = t_i^* q + \pi_i(q)$  for all  $i \in I$  and
2.  $\sum_{i \in I} t_i^* = 0$ .

Note that the quantity  $q$  may be interpreted as a public good produced by a firm (agent 0) to be bought by the rest of individuals. Thus, the Lindahl SCC coincides with the Pigouvian SCC in our domain of economies. In such a case, given an economy  $e = (u_0, u_1, \dots, u_n) \in \mathcal{E}$ ,  $t_i$  is agent  $i$ 's individualized price of the public good, and condition ?? represents the usual way of defining the price at which the firm sells the public good.

The Pigouvian SCC has good properties. For instance, it is Pareto efficient and individually rational. However, in many cases the regulator does not have access to the information needed to compute the Pigouvian taxes. This is where implementation theory comes into the picture. The following concepts are standard in the implementation literature.

**Definition 2.2.** A mechanism  $\Gamma$ , is a list of strategy spaces,  $M = (M_i)_{i \in I}$ , and an outcome function,  $f : \times_{i \in I} M_i \rightarrow \mathbb{R}^n \times \mathbb{R}_+$ .

Given  $e = (u_0, u_1, \dots, u_n) \in \mathcal{E}$ , let  $(\Gamma, e)$  denote a normal form game. Let  $\chi$  be an equilibrium concept and  $\chi(\Gamma, e)$  be the set of  $\chi$ -equilibria of the game  $(\Gamma, e)$ .

**Definition 2.3.** The mechanism  $\Gamma$  implements the SCC  $\phi : \mathcal{E} \rightarrow \mathbb{R}^n \times \mathbb{R}_+$  in  $\chi$ -equilibria if  $f(\chi(\Gamma, e)) = \phi(e)$  for all  $e \in \mathcal{E}$  with  $\chi(\Gamma, e) \neq \emptyset$ .

Let us introduce Varian's (1994) compensation mechanism for the two agents case. Firm 0 produces output  $q$ . Payoff for this firm is given by

$$\pi_0(q) = pq - C(q) \quad (2.1)$$

where  $p$  is the competitive price of the output, which is assumed to be exogenous to the model. Firm 0's production imposes an externality on the other agent.

**Definition 2.4.** The Compensation Mechanism. (Varian, 1994)

*Announcement stage.*- Agents 0 and 1 simultaneously announce the magnitude of the appropriate Pigouvian unitary tax; denote the announcement of firm 0 by  $t_0$  and the announcement of agent 1 by  $t_1$ .

*Choice stage.*- The regulator makes side payments to the agents so that they face the following payoff functions:

$$\begin{aligned} \Pi_0 &= pq - C(q) + t_1q - \alpha(t_0 + t_1)^2 \\ \Pi_1 &= t_0q - E(q) \end{aligned}$$

The parameter  $\alpha > 0$  is of arbitrary magnitude.

Thus, both individuals select simultaneously Pigouvian taxes at the first stage and agent 0 selects the output level at the second stage.

Varian (1994) shows that the above mechanism implements in SPE the Pigouvian social choice correspondence when agents' preferences are convex. However, we provide two examples which show that the Varian's results are only valid when preferences are strictly convex.

**Example 2.5.** Firm 0 having constant returns to scale.

Consider the following profit functions

$$\begin{aligned} \pi_0(q) &= pq - cq, \quad p > c > 0 \\ \pi_1(q) &= -\frac{e}{2}q^2 \quad e > 0 \end{aligned}$$

In such a case the Pigouvian equilibrium is  $q^* = \frac{p-c}{e}$ ,  $t_0^* = p - c$ ,  $t_1^* = c - p$ . Nevertheless stating  $\hat{q} = 0$ ,  $\hat{t}_0 = p - c$ ,  $\hat{t}_1 = c - p$  is a subgame perfect equilibrium since the firm has to make zero profits anyway in the second stage.

**Example 2.6.** Linear externality.

Let the functions be

$$\begin{aligned}\pi_0(q) &= pq - C(q) = pq - Aq^c, & c > 1, p, A > 0 \\ \pi_1(q) &= -E(q) = -eq, & e > 0\end{aligned}$$

The Pigouvian taxes are  $t_0^* = -t_1^* = e$ . Then,  $\Pi_1 = t_0^*q - eq = 0$  for all  $q$ . So there are equilibria where firm 1 chooses an arbitrary  $t_1$  (which yields an undesirable outcome in the second stage), since agent 1 makes zero payoff anyway in the second stage.

Thus we see that the compensation mechanism, in the case of linear preferences, does not solve the problem of finding a mechanism that implements the Pigouvian SCC. More seriously, coalition formation and incomplete information can not be considered since clear extension of SPE to cope with these possibilities does not exist. Thus, we are led to search for new mechanisms in which the said characteristics can be easily incorporated.

### 3. Implementing the Pigouvian Correspondence in Strong Equilibrium by Means of a Continuous Mechanism

This section presents a simple and continuous mechanism which implements, in strong Nash equilibrium, the Pigouvian Social Choice Correspondence. Let us introduce the mechanism  $\Gamma^S$ . Let  $M_i = \mathbb{R}^2$  be the strategy space for each  $i \in I$ . A strategy for agent  $i \in I$  is a pair  $m_i = (m_i^1, m_i^2) \in \mathbb{R}^2$  and a profile of strategies is given by  $m = (m_0, m_1, \dots, m_n) \in \mathbb{R}^{2n}$ . Agents are asked to declare their Pigouvian tax,  $m_i^1$ . Each agent's tax will be independent from her own strategy, except when she states a low tax for herself. The second component for each agent's strategy,  $m_i^2$ , is related to the incremental output level. Each agent simultaneously announces a strategy,  $m_i = (m_i^1, m_i^2) \in \mathbb{R}^2$ . The outcome function is

$$f(m_0, m_1, \dots, m_n) = (t_0, \dots, t_n, q) \quad (3.1)$$

where  $q = \max\left\{0, \sum_{i=0}^n m_i^2\right\}$  and, for each  $i$  in  $I$ ,  $t_i = \min\left\{m_i^1, -\sum_{j \neq i} m_j^1\right\}$ .

Then the output level is implemented and each agent  $i \in I$ , receives a compensation (or pays a tax) of  $T_i = t_i q$ . So her payoff is  $t_i q + \pi_i(q)$ . For notational convenience, for a given strategy profile  $\tilde{m}$ , let  $q(\tilde{m}) = \max\left\{0, \sum_{i=0}^n m_i^2\right\}$ ,  $t_i(\tilde{m}) = \min\left\{m_i^1, -\sum_{j \neq i} m_j^1\right\}$ , and  $T_i(\tilde{m}) = q(\tilde{m}) t_i(\tilde{m})$ .

This mechanism captures an aspect of Varian's compensation mechanism, namely each agent's tax is independent of her own strategy.

We next state a result connecting strong and Nash equilibrium for this mechanism.

**Lemma 3.1.**  $\hat{m} = (\hat{m}_0, \hat{m}_1, \dots, \hat{m}_n)$  is a strong equilibrium of the above mechanism if and only if (i)  $\hat{m}$  is a Nash equilibrium and (ii)  $\sum_{i=0}^n \hat{m}_i^1 = 0$

**Proof.** The sufficient condition is obvious. We concentrate on the necessary condition.

Let  $\hat{m}$  be a Nash equilibrium satisfying condition (ii) in Lemma ???. Suppose this is not a strong equilibrium. So, there is a coalition  $S \subseteq N$  and strategies  $\tilde{m}_s \in M_S$  such that  $u_i(T_i(\tilde{m}_s, \hat{m}_{-s}), q(\tilde{m}_s, \hat{m}_{-s})) > u_i(T_i(\hat{m}), q(\hat{m}))$  for each  $i \in S$ . Let us consider the following cases,

(i)  $t_i(\tilde{m}_s, \hat{m}_{-s}) = t_i(\hat{m})$  for some  $i \in S$ . Then,  $u_i(t_i(\tilde{m}_s, \hat{m}_{-s})q, q) = u_i(t_i(\hat{m})q, q)$  for any output level  $q$ , so  $q(\hat{m}) \in \arg \max_q u_i(t_i(\tilde{m}_s, \hat{m}_{-s})q, q)$  if, and only if  $\hat{q} \in \arg \max_q u_i(t_i(\hat{m})q, q)$ . A contradiction.

(ii)  $t_i(\tilde{m}_s, \hat{m}_{-s}) \neq t_i(\hat{m})$  for all  $i \in S$ . Since  $\sum_{j \in I} t_j(\tilde{m}_s, \hat{m}_{-s}) \leq 0 = \sum_{j \in I} t_j(\hat{m})$ , there should be an agent  $i \in S$ , such that  $t_i(\tilde{m}_s, \hat{m}_{-s}) < t_i(\hat{m})$ . Then, it holds that  $u_i(t_i(\hat{m})q(\tilde{m}_s, \hat{m}_{-s}), q(\tilde{m}_s, \hat{m}_{-s})) \geq u_i(t_i(\tilde{m}_s, \hat{m}_{-s})q(\tilde{m}_s, \hat{m}_{-s}), q(\tilde{m}_s, \hat{m}_{-s})) > u_i(t_i(\hat{m})q(\hat{m}), q(\hat{m}))$ . A contradiction. ■

The main result in this section is the following.

**Theorem 3.2.** *The mechanism  $\Gamma^S$  implements in strong Nash equilibrium the Pigouvian SCC.*

**Proof.** Let  $e = (u_0, u_1, \dots, u_n) \in \mathcal{E}$  be an economy. We will first prove that every Pigouvian equilibrium can be supported by a strong Nash equilibrium of the game  $(\Gamma, e)$ .

Let  $(t_0^*, \dots, t_n^*, q^*) \in \mathbb{R}^n \times \mathbb{R}_+$  be a Pigouvian allocation for  $e$ . We show that the strategy profile  $\bar{m}$ , where  $\bar{m}_i = \left(t_i^*, \frac{q^*}{n+1}\right)$  for each  $i \in I$ , is a strong Nash equilibrium for  $(\Gamma^S, e)$  yielding this Pigouvian allocation. Note that no unilateral deviation by an agent can force a lower tax  $t_i^*$  for her. Therefore, each agent  $i \in I$  selects her incremental output level  $m_i^2$  so to maximize her payoff  $t_i^* \left(m_i^2 + \frac{nq^*}{n+1}\right) + \pi_i \left(m_i^2 + \frac{nq^*}{n+1}\right)$ . Since  $(t_0^*, \dots, t_n^*, q^*)$  is a Pigouvian allocation for  $e$ ,  $\bar{m}_i^2$  maximizes the function above. Thus  $\bar{m}$  is a Nash equilibrium of  $(\Gamma^S, e)$ . Since  $\sum_{i \in I} \bar{m}_i^1 = 0$ , by Lemma ??,  $\bar{m}$  is a strong Nash equilibrium of  $(\Gamma^S, e)$ .

On the other hand, let  $\bar{m} \in \mathbb{R}^{2n}$  be a strong Nash equilibrium of  $(\Gamma^S, e)$ . By Lemma ??,  $\bar{m}$  is a Nash equilibrium of  $(\Gamma^S, e)$  and  $\sum_{i \in I} \bar{m}_i^1 = 0$ . Since  $\bar{m}$  is a Nash equilibrium of  $(\Gamma^S, e)$ ,  $q(\bar{m})$  maximizes  $t_i(\bar{m})q + \pi_i(q)$  for each  $i \in I$ . Given that  $\sum_{i \in I} \bar{m}_i^1 = 0$ , each agent  $i \in I$  will behave as a price-taker. (Note that she can not decrease her own tax.). Therefore,  $(t_1(\bar{m}), \dots, t_n(\bar{m}), q(\bar{m}))$  is a Pigouvian allocation for  $e$ . ■

## 4. Double Implementation of the Pigouvian Correspondence

This section introduces a mechanism which doubly implements the Pigouvian Social Choice Correspondence, in Nash and strong Nash equilibrium. This mechanism is related to the one presented by Corchón and Wilkie (1990).

In order to simplify the presentation we introduce a mechanism which is discontinuous. Nevertheless, the mechanism can be made continuous by making the appropriate modifications. (See Corchón-Wilkie, 1990.) Thus the trade-off between the mechanism presented in Section ?? and the one in this section is that of simplicity versus a more robust equilibrium concept.

Let us introduce the mechanism  $\Gamma^H$ . Let  $M_i = \mathbb{R}^2$  be the strategy space for each  $i \in I$ . A strategy for agent  $i \in I$  is a pair  $m_i = (m_i^1, m_i^2) \in \mathbb{R}^2$  and a profile of strategies is given by  $\tilde{m} = (m_0, m_1, \dots, m_n) \in \mathbb{R}^{2n}$ . For each  $i \in I$ ,  $m_i \in \mathbb{R}^2$  will be understood as the tax (or subsidy) she proposes for herself,  $m_i^1$ , and an incremental quantity on the production level,  $m_i^2$ . Each agent simultaneously announces a strategy,  $m_i = (m_i^1, m_i^2) \in \mathbb{R}^2$ . The outcome function is

$$f(\tilde{m}) = (T_0(\tilde{m}), \dots, T_n(\tilde{m}), q(\tilde{m})) \quad (4.1)$$

with  $T_i(\tilde{m}) = m_i^1 \max \left\{ 0, \sum_{i=0}^n m_i^2 \right\}$  if  $\sum_{j \in I} m_j^1 = 0$  and  $-\gamma \left| \sum_{i=0}^n m_i^1 \right|$  otherwise, where  $\gamma$  is a positive real number, it can be chosen to be as small as desired. Whereas that  $q(\tilde{m}) = \max \left\{ 0, \sum_{i=0}^n m_i^2 \right\}$  if  $\sum_{j \in I} m_j^1 = 0$  and  $q(\tilde{m}) = 0$  in any other case.

A natural interpretation can be provided for this mechanism. Each agent simultaneously announces her unitary subsidy,  $m_i^1$ , and how much the output level should be changed,  $m_i^2$ . Then the output level is implemented and she receives a compensation (or pays a tax) of  $T_i(\tilde{m})$ . So her payoff will be  $T_i(\tilde{m}) + \pi_i(q(\tilde{m}))$ . This compensation will depend upon two factors. Firstly, if the announced taxes balance the sum of transfers among agents, each agent's compensation is  $q$  times the tax she announced. Secondly, if such a balance does not hold, no output is produced and every agent is penalized to pay a fee.

We next introduce our main result in this section.

**Theorem 4.1.** *The mechanism  $\Gamma^H$  doubly implements in Nash and strong equilibrium the Pigouvian SCC.*

**Proof.** Let  $e = (u_0, u_1, \dots, u_n) \in \mathcal{E}$  be an economy. We will first prove that every Pigouvian equilibrium can be supported by a Nash equilibrium of the game  $(\Gamma^H, e)$ .

Let  $(t_0^*, \dots, t_n^*, q^*) \in \mathbb{R}^n \times \mathbb{R}_+$  be a Pigouvian allocation for  $e$ . We show that the strategy profile  $\bar{m}$ , where  $\bar{m}_i = (t_i^*, \frac{q^*}{n+1})$  for each  $i \in I$ , is a Nash equilibrium of  $(\Gamma^H, e)$  yielding such a Pigouvian allocation. First, it is clear that such strategies yield the desired Pigouvian allocation. Now we show that they are a Nash equilibrium. Consider an unilateral deviation of player  $i \in I$ . If she does not change  $\bar{m}_i^1$  she can get any  $q' \in \mathbb{R}_+$  at a price  $t_i^*$ . By definition of a Pigouvian allocation she can not improve it by doing  $q' \neq q$ . Furthermore, if she changes  $\bar{m}_i^1$  then  $q = 0$ . Suppose, by way of contradiction, that she improves. But then

$$\pi_i(0) > -\gamma \left| m_i^1 + \sum_{j \neq i} t_j^* \right| + \pi_i(0) > t_i^* q + \pi_i(q)$$

contradicting that  $(t_0^*, \dots, t_n^*, q^*)$  is a Pigouvian allocation for  $e$ .

Let  $\hat{m} = (\hat{m}_0, \hat{m}_1, \dots, \hat{m}_n) \in \mathbb{R}^{2n}$  be a Nash equilibrium of  $(\Gamma^H, e)$ . We show that this yields a Pigouvian allocation for  $e$ . Consider two cases.

Case a)  $\sum_{i \in I} \hat{m}_i^1 \neq 0$ . In such a case, it is easy to see that, for agents other than 0, there is a strategy, namely  $\bar{m}_i = \left( -\sum_{j \neq i} \hat{m}_j^1, -\sum_{j \neq i} \hat{m}_j^2 \right)$  yielding  $q = t_i = 0$ , a contradiction.

Case b)  $\sum_{i \in I} \hat{m}_i^1 = 0$ . But then, given  $\hat{m}_{-i}$ , each agent is a price-taker so the outcome associated with the Nash Equilibrium is a Pigouvian allocation of  $e$ .

The argument above yields the same result for the strong equilibrium case. First, let  $(t_0^*, \dots, t_n^*, q^*) \in \mathbb{R}^n \times \mathbb{R}_+$  be a Pigouvian allocation for  $e$ . It is easy to see that the strategy profile  $\bar{m}$ , where  $\bar{m}_i = (t_i^*, \frac{q^*}{n+1})$  for each  $i \in I$ , is a strong Nash equilibrium of  $(\Gamma^H, e)$  yielding such a Pigouvian allocation. The argument is similar to the one provided for the Nash equilibrium case. On the other hand, let  $\hat{m} = (\hat{m}_0, \hat{m}_1, \dots, \hat{m}_n) \in \mathbb{R}^{2n}$  be a strong Nash equilibrium of  $(\Gamma^H, e)$ . Since  $\hat{m}$  is a strong Nash equilibrium of  $(\Gamma^H, e)$ , it is a Nash equilibrium. Therefore,  $f(\hat{m})$  is a Pigouvian allocation for  $e$ . ■

## 5. Implementation of the Pigouvian SCC when Agents are Incompletely Informed

This section studies the problem of implementing the Pigouvian SCC when agents are incompletely informed. We present a mechanism capturing some ideas from Corchón and Ortuño-Ortín (1995). These authors consider a model in which each agent is completely informed about her own neighbors, but may not be informed about others agents' characteristics. They present a mechanism to implement in Bayesian equilibrium any social choice correspondences satisfying monotonicity and no veto power.

We need some additional notation and definitions for this section. All the relevant information about the preferences of agent  $i$ , called the attribute of  $i$ , is summarized by  $\theta_i$ . Let  $\Theta_i$  be a finite set of possible attributes of agent  $i$  and let  $\Theta = \times_{i \in I} \Theta_i$  be the set of possible states.

We assume that agent  $i$ 's preferences can be represented by a utility function  $u_i : \mathbb{R} \times \mathbb{R}_+ \times \Theta_i \rightarrow \mathbb{R}$ , such that for some  $\pi : \mathbb{R}_+ \times \Theta_i \rightarrow \mathbb{R}$  and all  $(T_i, q) \in \mathbb{R} \times \mathbb{R}_+$ , we have

$$u_i(T_i, q, \theta_i) = T_i + \pi(q, \theta_i) \text{ for each } i \in I.$$

We assume that agents have continuous and convex preferences. We also suppose that for all  $\theta_0 \in \Theta_0$ ,  $\pi(q, \theta_0)$  is increasing in  $q$  and, for all  $i \in I \setminus \{0\}$  and for all



$\theta_i \in \Theta_i$ ,  $\pi(q, \theta_i)$  is decreasing in  $q$ . Thus, an *economy* is a vector  $\theta = (\theta_0, \theta_1, \dots, \theta_n)$ . Let  $\mathcal{E}$  be the set of admissible economies. Let us define the Pigouvian SCC in this framework.

**Definition 5.1.** The *Pigouvian Social Choice Correspondence (SCC)*,  $\phi_P : \mathcal{E} \rightarrow \mathbb{R}^n \times \mathbb{R}_+$ , associates to each  $\theta = (\theta_0, \theta_1, \dots, \theta_n) \in \mathcal{E}$ , a set of taxes,  $\phi_P^I(\theta) = (\phi_P^0(\theta), \phi_P^1(\theta), \dots, \phi_P^n(\theta)) \in \mathbb{R}^n$ , and an output  $\phi_P^{n+1}(\theta) \in \mathbb{R}_+$  such that

1.  $\phi_P^{n+1}(\theta) \in \arg \max_{q \in \mathbb{R}_+} \phi_P^i(\theta)q + \pi_i(q, \theta_i)$  for all  $i \in I$  and
2.  $\sum_{i \in I} \phi_P^i(\theta) = 0$ .

We describe the informational framework. Let  $\psi_i$  denote the information that agent  $i$  has about agents in  $I$ . In the usual terminology,  $\psi_i$  is the type of agent  $i$ . The distinction between types and attributes is made in order to distinguish between information that matters for first best efficiency (attributes) and information needed to play a Bayesian equilibrium (types).  $A^i$  denotes the set of individuals about which agent  $i$  is completely informed. Given the information that agents have, we can define a partition on  $I$ . Two agents will be in the same element of the partition whenever they share the same information about others' characteristics.. We will assume that each element of such a partition will contain at least three agents. Thus, we can construct a function  $G : I \rightarrow 2^I$ , such that, (1) for each  $i \in I$ ,  $G(i)$  contains, at least, three elements, (2) for each pair of agents  $i$  and  $j$  if  $G(i) \neq G(j)$  then  $G(i) \cap G(j) = \emptyset$ , and (3)  $\cup_{i \in I} G(i) = I$ . Notice that each agent  $i$  is completely informed about the characteristics of agents in  $G(i)$ . This might be interpreted by the existence of close links among certain agents. We also suppose that this partition is common knowledge for agents and it is known by the designer. Let us assume that agents have a common prior distribution  $\rho$  on  $\Theta$ , and that  $\rho(\theta)$  is common knowledge for each  $\theta$ . Finally, we assume that  $\rho(\theta) > 0$  for all  $\theta$  in  $\Theta$ , and that agents preferences on risky outcomes can be represented by a von Neumann-Morgenstern utility function. The conditional probability distribution for an agent  $i$  can be defined by the following expression,

$$\rho_i(\psi/\psi_i) = \frac{\rho(\psi)}{\sum_{\Xi_{I \setminus A^i}} \rho(\psi_i, \psi_{I \setminus A^i})}$$

where  $\Xi_{I \setminus A^i} = \times_{j \in I \setminus A^i} \Xi_j$  is the set of information of agents about whom  $i$  is not informed.

We now define the equilibrium concept that we are going to use throughout this section.

**Definition 5.2.** Let  $\Gamma$  be a mechanism, we say that the vector of strategies  $m = (m_0(\psi_0), m_1(\psi_1), \dots, m_n(\psi_n))$  is a *Bayesian Equilibrium* for such a game if, for a given state of the world  $\theta$  and a common prior distribution  $\rho(\cdot)$ , for all agent  $i \in I$ , and for all strategy  $\hat{m}_i \in M_i$ ,

$$\begin{aligned} \sum_{\Xi_{I \setminus A^i}} \rho(\psi/\psi_i) \pi_i \left( f \left( m_{A^i}(\psi_i), m_{I \setminus A^i}(\psi_{I \setminus A^i}) \right), \theta_i \right) \geq \\ \sum_{\Xi_{I \setminus A^i}} \rho(\psi/\psi_i) \pi_i \left( f \left( \hat{m}_i(\psi_i), m_{A^i \setminus \{i\}}(\psi_i), m_{I \setminus A^i}(\psi_{I \setminus A^i}) \right), \theta_i \right) \end{aligned} \quad (5.1)$$

Example ?? shows that the Pigouvian SCC in general cannot be implemented in Bayesian Equilibrium if our assumptions on the informational structure are removed.

**Example 5.3.** Consider a two-agent economy. Let  $\theta_i = \{1, 2\}$  for all  $i = 0, 1$ . Let  $\rho(\theta_0 = 1, \theta_1 = 1) = \rho(\theta_0 = 2, \theta_1 = 2) = 0.2$ , and  $\rho(\theta_0 = 1, \theta_1 = 2) = \rho(\theta_0 = 2, \theta_1 = 1) = 0.3$ . Consider the following profit functions,

$$\pi_0(\theta_0, q) = 3q - \theta_0 q$$

$$\pi_1(\theta_1, q) = -\frac{\theta_1}{2} q^2$$

In such a case the Pigouvian allocations are the following

	$q$	$t_0$	$t_1$	$\pi_0$	$\pi_1$
$(\theta_0 = 1, \theta_1 = 1)$	2	-2	2	0	2
$(\theta_0 = 1, \theta_1 = 2)$	1	-2	2	0	1
$(\theta_0 = 2, \theta_1 = 1)$	1	-1	1	0	$\frac{1}{2}$
$(\theta_0 = 2, \theta_1 = 2)$	$\frac{1}{2}$	-1	1	0	$\frac{1}{4}$

Because of the revelation principle (Harris and Townsend 1981, Myerson 1985) we know that any Social Choice Correspondence which is implementable in Bayesian Equilibrium should satisfy that truth-telling is a Bayesian Equilibrium. Thus, we need to check if Incentive Compatibility is satisfied in our example. Let us consider a direct mechanism that selects the Pigouvian equilibrium for given announcements of the agents. The following table summarizes the expected payoff for agent 0 depending on her own characteristics,  $\theta_0$ , and her statements,  $\theta_0^0$  assuming that the other agent is truthful when announcing her characteristic.

Firm 0	$\theta_0^0 = 1$	$\theta_0^0 = 2$
$\theta_0 = 1$	0	0.7
$\theta_0 = 2$	-1.6	0

Thus firm 0 will always state that it has high costs, i.e.  $\theta_0^0 = 2$ , independently on its type. ■

Now we present the mechanism,  $\Gamma^B$ . Let the strategy space for agent  $i \in I$  be

$$M_i = \Theta_{G(i)} \times \mathbb{R}^2$$

A message for agent  $i \in I$  is given by  $m_i(\theta_{G(i)}) = (\theta_{G(i)}^i, q_i, m_i^1) \in \mathbb{R}^{|G(i)|+2}$ . Thus each agent is asked to report the following information. Firstly, she has to state which is the attribute of each agent in the same element of the partition in which she is. Secondly, it has to announce the output level she wants. And finally, it has to state the tax (resp. subsidy) she has to pay (resp. receive). We next define the outcome function. We will consider three cases.

### Case 1. The Consensus Case

If  $m = (m_0, m_1, \dots, m_n)$  is such that  $\theta_{G(i)}^i = \theta_{G(j)}^j$  for all  $i$  and for all  $j \in G(i)$  then  $f(m_0, m_1, \dots, m_n) = \phi_P(\hat{\theta})$ , the Pigouvian allocation associated to the characteristics unanimously revealed by the agents.

### Case 2. The Dissident Case

There exists a unique agent, say  $i$  such that  $\theta_{G(i)}^i \neq \theta_{G(j)}^j$  for all  $j$  in  $G(i) \setminus \{i\}$ , then

$$f(m) = \begin{cases} \phi_P(\hat{\theta}) & \text{if } \pi_i(\phi_P^i(\hat{\theta}), q_i, \theta_i^j) > \pi_i(\phi_P^i(\hat{\theta}), \phi_P^{n+1}(\hat{\theta}), \theta_i^j) \\ (\phi_P^i(\hat{\theta}), q_i) & \text{otherwise} \end{cases}$$

where  $\theta_i^j$  is the characteristic of agent  $i$  announced by each  $j$  in  $G(i) \setminus \{i\}$ , and  $\hat{\theta}$  is defined in the following way.  $\hat{\theta}_h$  is the characteristic announced by firm  $h$ ,  $h \neq i$ , and  $\hat{\theta}_i = \theta_i^j$ .

Case 3. If Cases ?? and ?? do not apply, then if  $m_i^1 = \max_{j \in I} m_j^1$

$$f(m_0, m_1, \dots, m_n) = (T_0, T_1, \dots, T_n, 0)$$

where  $T_i = m_i^1$  and  $T_j = -\frac{m_i^1}{n}$  for all  $j \neq i$ .<sup>1</sup>

We next introduce our main result in this section.

**Theorem 5.4.** *The mechanism  $\Gamma^B$  implements in Bayesian equilibria the Pigouvian SCC for  $n \geq 3$ .*

**Proof.** Let  $\theta = (\theta_0, \theta_1, \dots, \theta_n)$  be an economy. Consider the truth-telling strategy,  $m_i(\theta_{G(i)}) = (\theta_{G(i)}^i, q_i, m_i^1)$  for all  $i \in I$  where  $q_i$  and  $m_i^1$  are two arbitrary numbers. It is clear that if  $n$  agents follow this strategy, no deviation from the

<sup>1</sup>If there are two or more agents for which the tax they report is the maximal, any tie-break rule is valid in order to define the rule. For instance, we can employ the individual which is lower ranked in the set of agents  $I$ .

remaining agent is going to make any difference in any state since the Pigouvian allocation is the best that any dissident can get. Therefore, it is clear that this strategy yields the Pigouvian allocation and it is a Bayesian equilibrium.

Consider now a profile of strategies that are a Bayesian equilibrium for  $(\Gamma^H, e)$ , say  $m(\theta)$ . First, notice that no dissidence is possible, i.e. it cannot be the Case ???. Indeed if there were a single dissident at state  $\theta$ , say  $i$ , all agents in  $G(i)$  would know it. In this case, any agent  $j$  in  $G(i) \setminus \{i\}$  could get a subsidy as great as it wanted to by forcing herself to be in Case ???. Since agents' preferences are quasilinear, her utility level can be as high as she likes. This contradicts that  $m(\theta)$  is a Bayesian equilibrium. A similar argument can be provided to see that messages announced by agents cannot be such that Case ??? applies. Thus, the only possibility is that agents messages are such that the outcome is in Case ???. Let us suppose that at some announced state  $\tilde{\theta}$ , the outcome of a Bayesian equilibrium is not in  $\phi_P(\theta)$ . But then, there exists some  $q$  such that

$$u_i(\phi_P^i(\tilde{\theta}), q, \theta_i) > u_i(\phi_P^i(\tilde{\theta}), \phi_P^{n+1}(\tilde{\theta}), \theta_i)$$

and because  $\phi_P(\tilde{\theta})$  is a Pigouvian equilibrium for  $\tilde{\theta}$  then

$$u_i(\phi_P^i(\tilde{\theta}), q, \theta_j^i) \leq u_i(\phi_P^i(\tilde{\theta}), \phi_P^{n+1}(\tilde{\theta}), \theta_j^i)$$

Thus, by announcing  $m_i = (\theta_{G(i)}^i, q, m_j^i)$  with  $\theta_{G(i)}^i \neq \theta_{G(i)}^j$  for all  $j \in G(i) \setminus \{i\}$  this firm is better off since for every  $\tilde{\theta} = \theta$  its dissidence does not count and for  $\tilde{\theta} \neq \theta$  its profits are greater. ■

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