# MERGERS FOR MARKET POWER IN A COURNOT SETTING AND MERGER GUIDELINES[*]

**Ramón Faulí** [**]

[**] Ramón Faulí: University of Alicante.

# MERGERS FOR MARKET POWER IN A COURNOT SETTING AND MERGER GUIDELINES

**Ramón Faulí**

## A B S T R A C T

The US Merger Guidelines consider that the anticompetitive effect of a horizontal merger is increasing in the initial market concentration and decreasing in the elasticity of demand. These ideas are studied in a setting where identical firms compete à la Cournot and marginal cost is constant. The former relationship holds if demand is convex, but it may fail to be true if demand is concave. The latter one only holds if the elasticity of demand is increasing in the degree of concavity. This condition is satisfied by linear demands, constant elasticity demands and demands that are log-linear in price.

1. Introduction.

Horizontal mergers initially attracted public attention because they restricted competition. From the Sherman Antitrust Act of 1890 to the Celler-Kefauver Act of 1950, increasingly stricter laws were passed to prevent mergers intended "to lessen competition, or to create a monopoly". A more positive assessment of mergers was proposed by Williamson (1968). He pointed out that mergers might increase welfare if the economies they generate compensate the losses caused by lower competition. Receptive to these new ideas, the 1984 Guidelines revision stated that efficiencies generated by mergers should be considered when making decisions as to whether or not to challenge a merger. Therefore, the antitrust body, before approving a merger, should compare the loss caused by lower competition with the efficiency gains generated through merger.

Unavailability of key information makes the evaluation of both effects difficult (Posner (1976)). In spite of this, successive US Merger Guidelines (the last one being issued in 1992) have been specifically devoted to establishing clear and feasible criteria to compare the anticompetitive effect of different mergers.[1] Feasibility requires that only observable variables be used in the process. This has forced Merger Guidelines to restrict their attention to the (premerger) elasticity of demand and market concentration.[2]

The Merger Guidelines suppose that welfare is inversely related to concentration measured by the Herfindahl-Hirshman Index (HHI). Furthermore, increases in concentration are more welfare-reducing the greater the initial market concentration: "The 1992 Guidelines focus on market shares and concentration data to create a presumption of illegality (if the HHI is above 1800 and it is increased by more than 100 points) or legality (if the HHI is below 1000, or

---

[1] Criteria to measure the efficiency gain are not specified.

[2] "In particular, the larger are the market shares of the participant firms, or the smaller is the industry elasticity of demand, the greater must be the learning effect or scale economies in order for price to fall. It is perhaps encouraging that these are exactlty the factors that Merger Guidelines instruct antitrust officials to consider" (Farrell and Shapiro (1990) p. 114 emphasis added).

between 1000-1800 and the increase is less than 100)." (Gellhorn and Kovacic (1994) p. 396-7). Conditions become tighter as concentration increases.

Considerations about the elasticity of demand appear at the market delineation stage: "The relevant market is delineated by means of an analysis of what set of products (at associated locations) has sufficiently inelastic demand as a group that a hypothetical profit-maximizing monopoly supplier of the set would impose at least a "small but significant and nontransitory increase in price"". (Willig (1991). p. 283 emphasis added). The lower the elasticity of demand the smaller the relevant market the greater the concentration and therefore the more anticompetitive a merger.

In line with other papers (for example Farrell and Shapiro (1990) and Willig (1991)), the present one tests the validity of the previous ideas embodied in the Merger Guidelines in a theoretical framework. This should help to highlight the advantages and the weakness of the present procedures in order to achieve later improvements.

Farrell and Shapiro (1990) consider a setting in which firms compete à la Cournot, the product is homogenous and cost and demand functions are general. Their main point is that with cost asymmetries, welfare may increase with concentration. Mergers will indeed increase welfare if the losses caused by lower competition are outweighed by the cost savings obtained by transferring output from less efficient plants to more efficient ones.

I pay attention to the effect on welfare of the elasticity of demand and the initial market concentration. To focus on demand characteristics I simplify the cost side by assuming that the marginal cost of firms is constant while allowing for more general demand functions than those of Farrell and Shapiro (1990). Furthermore, this assumption on costs allows us to limit our attention to the anticompetitive effect of mergers by assuming absent any efficiency change due to merger.

In this setting, assuming constant-elasticity demands[3], Willig (1991) obtained that the welfare loss is increasing in the initial market concentration. I find that this relationship holds good for convex demands, but it may fail to be true if demand is concave.

The inverse relationship between the elasticity of demand and the welfare loss is not true in general. The anticompetitive loss depends on the curvature[4] (degree of concavity) of demand and not on the slope of demand: the lower the degree of concavity the greater the welfare loss. Then, the elasticity of demand will only help to determine the welfare loss if it enables us to know the degree of concavity. Furthermore, if a high elasticity implies a high degree of concavity, the idea embodied in the Merger Guidelines will be validated. This condition requires that the domain of all possible demands be "quite" homogenous. It is satisfied if demands are either linear, log-linear in price or have a constant elasticity. It is not satisfied, for example, if linear and constant-elasticity demands belong to the domain.

The structure of the paper is as follows. In the following Section, I study the effect of the degree of concavity and market concentration on the welfare loss of a merger of a given number of firms. Although I focus on welfare[5], I add some results on profitability to highlight the opposition between welfare and profitability in this setting: the more a merger reduces welfare the more profitable it will be. In Section III, I find conditions under which the welfare implications derived by the US Merger Guidelines from the elasticity of demand and market concentration hold good in the present model. Final comments end the paper.

---

[3]In fact, Willig (1991) assumes that while the elasticity does not depend on price, it can be affected by the composition of the relevant market. These assumptions allow him to replicate the market delineation stage.

[4] Corchón and González-Maestre (1994) analyse a model where firms can create independent divisions. They get that divisionalization is more profitable the more concave demand is.

[5] For further results on profitability see Faulí-Oller (1996).

## 2. Model and results.

We have N firms competing à la Cournot. They all have the same constant marginal cost ($c > 0$) and sell the same product. The inverse demand function $P(X)$ (price as a function of quantity X) is twice-continuously differentiable and $P'(X) < 0$. Demand at price c is positive but finite.

We consider the possibility of a group of firms merging. Like non-merging firms, the merging entity produces at marginal cost c and behaves à la Cournot.[6] The merger generates synergy gains. If $k+1$ firms merge, they amount to k F, where $F \geq 0$.[7]

In order to calculate the equilibrium with n independent firms, we define $x_i$ as sales of firm i. Given the sales of other firms ($x_{-i}$), the output of firm i that maximizes its profit satisfies:

$$P'(x_i+x_{-i})\, x_i + P(x_i+x_{-i}) - c = 0 \tag{1}$$

I assume that:

$$\frac{P''(X)X}{P'(X)} + n > 0 \qquad \text{(for any X, such that } P(X) \geq c\text{).} \tag{2}$$

Appendix I shows that asymmetric equilibria do not exist and that (2) guarantees the existence and uniqueness of a symmetric equilibrium. Then in equilibrium $(n-1)\, x_i = x_{-i}$ and therefore equilibrium sales (X) satisfy:

$$P'(X)\, \frac{X}{n} + P(X) - c = 0 \tag{3}$$

---

[6] Levin (1990) makes no assumption on the behavior of the merged entity.

[7] In other words, the cost function of a merged entity grouping $k+1$ firms is given by $C(x) = c\, x - k\, F$, where x is output. The origin of F is left unexplained. We can not assume that they are savings in fixed costs. Fixed costs would destroy uniqueness of the equilibrium. No further effort is devoted to F because it plays a marginal role in the present paper: I focus on the anticompetitive effect of mergers assuming F only to allow for socially beneficial mergers.

The previous equation defines implicitly the equilibrium sales as a function of the number of independent firms (X(n)). For computational purposes, it is appropriate to consider n as a continuous variable, although it has only an economic meaning when it is a natural number. This will enable us to differentiate all relevant variables with respect to n. The effect of discrete changes in the number of firms will be calculated as the infinite sum of the effects of marginal changes (Seade (1980) p. 482). Differentiating (3) with respect to n we have:

$$P''X'\frac{X}{n} + \frac{P'X'}{n} - \frac{P'X}{n^2} + P'X' = 0$$

By rearranging we obtain the proportional increase in output due to a marginal increase in the number of firms:

$$\frac{X'}{X} = \frac{1}{n \ (\beta(n)+n+1)}, \tag{4}$$

$$\text{where } \beta(n) = \frac{P''(X(n))X(n)}{P'(X(n))} \ .$$

Gross individual profits (synergy benefits excluded) in equilibrium are given by:

$$\pi(n) = \frac{[P(X(n))-c] \ X(n)}{n} \ .$$

Differentiating with respect to n we have:

$$\pi'(n) = P'X' \frac{X}{n} + X' \frac{P-c}{n} - \frac{(P-c) \ X}{n^2} \ .$$

Rearranging we have the proportional increase in profits due to a marginal increase in the number of firms (n):

$$\frac{\pi'(n)}{\pi(n)} = \frac{X'}{X} \ (\frac{P'X}{(P-c)} + 1) - \frac{1}{n} \ .$$

Using (3) and (4) leads to:

$$-\frac{\pi'(n)}{\pi(n)} = \frac{n-1}{n\ (\beta(n)+n+1)} + \frac{1}{n}$$

By integrating the previous expression we obtain the following,

$$\int_{N-k}^{N} -\frac{\pi'(n)}{\pi(n)}\ dn = \ln[\frac{\pi(N-k)}{\pi(N)}] = \int_{N-k}^{N} (\frac{n-1}{n\ (\beta(n)+n+1)} + \frac{1}{n})\ dn.$$

Then we have that the proportional increase in profits due to a merger of k+1 firms amounts to,

$$\frac{\pi(N-k)}{\pi(N)} = \text{Exp}[\ \int_{N-k}^{N} (\frac{n-1}{n\ (\beta(n)+n+1)} + \frac{1}{n})\ dn] \tag{5}$$

We study below the profitability and the welfare effect of mergers. A merger of k+1 firms is said to be profitable[8] if $\pi(N-k) -(k+1)\ \pi(N) + k\ F \geq 0$. Similarly, a merger of k+1 firms is welfare-increasing if $W(N-k) - W(N) + k\ F \geq 0$, where W(n) stands for gross social welfare with n independent firms.

(4) and (5) will play a decisive role in determining whether a merger is profitable (welfare-increasing) or not. This can be understood once we rewrite the two conditions above in the following way. The condition on profitability can be rewritten as:

$$\pi(N)[\frac{\pi(N-k)}{\pi(N)} - (k+1)] + k\ F \geq 0. \tag{6}$$

The condition on welfare is equivalent to:

---

[8] The definition seems rather obvious, but one should take into account that when the merger process is fully specified as a game (e.g. Kamien and Zang (1990)) the condition is necessary but not sufficient for a merger to be carried out in equilibrium (Gaudet and Salant (1992)).

$$kF - \Pi(N) \int_{N-k}^{N} \frac{\Pi(n)}{\Pi(N)} \frac{X'(n)}{X(n)} \, dn \geq 0, \tag{7}$$

where $\Pi(n)$ stands for gross industry profits.[9]

## 2.1. The effect of the degree of concavity.

To obtain a clear-cut result on the effect of the degree of concavity, I have to study mergers in markets whose demands are of a "comparable" size. Comparability is defined in Definition 1. Changes in the size of the market affect profitability and welfare, because they affect the relative importance of the synergy gains.

The effect of the degree of concavity of demand should be understood through its effect on (4), the proportional increase in output due to a marginal increase in the number of firms (n). Its effect is negative, so that the more concave the demand the slower the increase in competition with n. This has a direct implication on the profitability of mergers, because mergers try to reduce competition as much as possible by reducing n. Then, they will be less successful the greater the degree of concavity. As far as the welfare loss is concerned, as competition is restrained less with concave demands, the welfare loss will also be lower. In other words, the same reason that limits merger profitability explains that they are less harmful from the social point of view. These ideas are formalized in the Proposition below.

Definition 1: Two mergers are comparable if they have the same: a) number of active firms and b) the same ratio between F and (premerger) profits.

---

[9] $W(N-k) - W(N) + kF = kF - \int_{N-k}^{N} W'(n) \, dn = kF - \int_{N-k}^{N} (P(n)-c)X'(n) \, dn =$

$= kF - \int_{N-k}^{N} \Pi(n) \frac{X'(n)}{X(n)} = kF - \Pi(N) \int_{N-k}^{N} \frac{\Pi(n)}{\Pi(N)} \frac{X'(n)}{X(n)} \, dn$

<u>Proposition I</u>: Assume we have two markets (say P and M) whose demands are given respectively by P(X) and M(X). For any profitable (welfare-increasing) merger in market M (P), any comparable merger in market P (M) is also profitable (welfare-increasing) if and only if $\frac{P''(X)X}{P'(X)} \leq \frac{M''(Y)Y}{M'(Y)}$ for any X and Y.

<u>Proof</u>: See Appendix II.

Proposition I also states that the degree of concavity affects the profitability of mergers negatively and affects welfare positively. Two (extreme) examples are presented below to illustrate the issue. The demands with constant degree of concavity take the following form[10]: $P(X) = d - \frac{f}{b+1}X^{b+1}$, where b is the degree of concavity. Some restrictions on the parameters are required. f >0 ensures that demand is downward sloping. $d \neq c$ and sign$\{d-c\}$=sign$\{\frac{f}{b+1}\}$ ensures that demand at price c is positive but finite. In a n-firm industry the equilibrium sales are given by:

$$X(b,n) = [\frac{n(d-c)(b+1)}{f(1+b+n)}]^{[1/(1+b)]}$$

When the degree of concavity tends to infinity, sales tend to a constant that does not depend on the number of firms. Mergers are not able to raise price, because any reduction in the production of insiders is replaced by new production from outsiders. All mergers are welfare-increasing. If F = 0, all mergers, except monopolization, are unprofitable. The following picture illustrates the form of demand in this limit case.

---

[10] If b=-1, the demand takes the form P(X)=h ln X + g, where h<0.
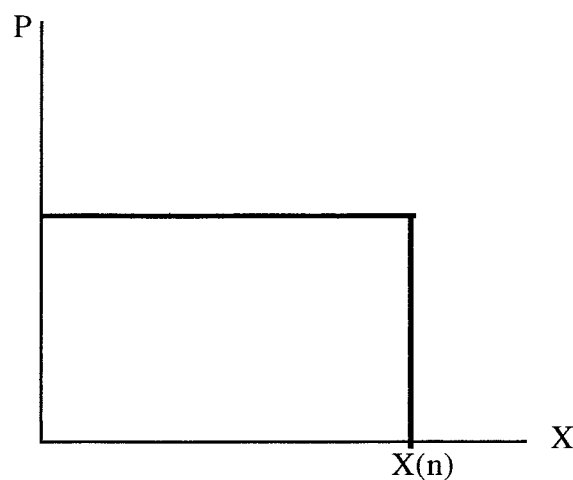
13

P



X(n)                        X

Figure 1. The limit form of demand when the degree of concavity tends to infinity.

To illustrate what happens when demand becomes convex, we take the case where $P(X)=e^{-X}$ and c=0. This demand is convex and the degree of concavity unbounded, because $\beta(X) = -X$ and $P(X) > c$ for any positive output.[11] The profit-maximizing output of firm i is given by the solution of the following equation: $e^{-X}(1-x_i) = 0$. Firms produce one unit of output no matter what the number of firms in the market. This implies that non-participant firms do not increase their production after a merger. Then, all mergers are profitable and welfare-reducing (provided that F does not exceed the individual profits).

## 2.2. The effect of market concentration.

Mergers involving the classical trade-off between competition and efficiency are being considered. Mergers reduce price which increases profitability and reduces welfare. At the same time, they generate synergy gains that increase both profitability and welfare.

---

[11]Although (2) does not hold, second-order conditions are satisfied, because profits are quasi-concave.

The reduction of competition through a merger is limited by the fact that non-participating firms react to it by raising their output. This effect will be less acute the lower the number of firms which free-ride from the output reduction induced by the merger. If we deal with mergers involving a constant number of firms, the greater the initial market concentration, the lower the number of nonparticipants. Therefore, a high level of concentration reinforces the anticompetitive effect of mergers increasing profitability and reducing welfare.

On the other hand, the synergy gains are more important the lower the concentration, because the ratio between F and individual profits is increasing in the number of firms. A low concentration reinforces the efficiency effect of mergers increasing both profitability and welfare. Therefore, to get a clear-cut result as far as profitability is concerned we have to deal with the particular case in which no synergies are obtained. The following Propositions formalize and qualify the intuitions above.

<u>Proposition II</u>: Assume that the degree of concavity of demand is nondecreasing in sales and F=0. If the merger of a given number of firms is profitable, it will also be profitable if the market becomes more concentrated.

<u>Proof</u>: A merger is profitable if $\frac{\pi(N-k)}{\pi(N)}$ - (k+1) $\geq$ 0. Then if the previous expression decreases with N the Proposition is proved. sign $\left\{\frac{\partial(\pi(N-k)/\pi(N))}{\partial N}\right\}$ = sign$\left\{-\frac{\pi'(N)}{\pi(N)} + \frac{\pi'(N-k)}{\pi(N-k)}\right\}$. It is negative because sign$\left\{\frac{\partial(\pi'(n)/\pi(n))}{\partial n}\right\}$=sign$\{n\beta'(n)-\beta-1+(\beta+n+1)^2\}$ is positive given that $\beta'(n) \geq 0$. ●

<u>Proposition III</u>: Assume that the degree of concavity of demand is nondecreasing in sales. If the merger of a given number of firms is welfare-increasing, it will also be welfare-increasing if the market becomes less concentrated.

Proof: A merger is welfare-increasing if $\Delta W = W(N-k) - W(N) + k\ F \geq 0$. Then if the previous expression increases with $N$ the Proposition is proved. Observe that $W'(n)=(P(n)-c)X'(n)=n\pi(n)\dfrac{X'(n)}{X(n)}$. Then,

$$\frac{\partial \Delta W}{\partial N} = W'(N-k) - W'(N) = \frac{\pi(N-k)}{\beta(N-k)+N-k+1} - \frac{\pi(N)}{\beta(N)+N+1} > 0 \text{ given that } \beta(N-k) \leq \beta(N). \quad \bullet$$

Proposition II and III identify market concentration as a determinant of the profitability and social gain of mergers. To obtain the result one must guarantee that changes in the degree of concavity as sales vary, serve to reinforce (and not enter in conflict with) the effects due to changes in concentration. This is the purpose of assuming that the degree of concavity of demand is nondecreasing in sales. Linear and constant elasticity demands satisfy this assumption, whereas demands that are log-linear in price do not. The main implication of Proposition II is that the consummation of a merger makes another merger more profitable. This can be used to explain why mergers occur in waves.

## 3. A look at Antitrust Policy.

US Merger Guidelines use the elasticity of demand and market concentration to assess the anticompetitive effect of mergers. I explain briefly below the effect on welfare both variables are assumed to have. This section aims, mainly, at finding conditions that ensure that, in the present model, the elasticity of demand and market concentration have the effect assumed by the US Merger Guidelines.

### 3.1. The anticompetitive effect of mergers and market concentration.

US Merger Guidelines proxy the anticompetitive impact of a merger by the variation in the Herfindahl-Hirschman Index (HHI), defined as the sum of the squares of the market shares of firms. The postmerger HHI is calculated by assigning to the merged entity the sum of the

premerger shares of the merging parties, and assigning to the other participants their premerger shares.

The following implications are derived from the measurement of concentration. "The 1992 Guidelines focus on market shares and concentration data to create a presumption of illegality (if the HHI is above 1800 and it is increased by more than 100 points) or legality (if the HHI is below 1000, or between 1000-1800 and the increase is less than 100)." (Gellhorn and Kovacic (1994) p. 396-7). These general standards embody the idea that the same increase in the HHI is more welfare-reducing, the greater the market concentration. Proposition IV finds sufficient conditions for this to hold true in our model.

Proposition IV also explores whether or not the result will also hold if mergers are measured by the market share of participating firms.[12] This is the measure used by the European Union.[13]

Proposition IV: Assume that the degree of concavity of demand is nondecreasing in sales and demand is convex. Then mergers that increase the HHI index in the same amount reduce more welfare the greater the market concentration. The same result holds for mergers, short of monopolization, with the same market share of participant firms.

Proof: In a n-firm industry, a merger that increases the HHI in 10000d [market share 100s] reduces independent firms to $R(n) = n + \dfrac{1 - \sqrt{1 + 4\ d\ n^2}}{2}$ [$R(n) = (1-s)\ n + 1$]. The welfare loss due to this merger is given by $\Omega(n) = G(n) - (n-R)\ F$, where $G(n) = W(n) - W(R)$. We have to

---

[12] To get the result, we have to exclude mergers with a market share of 100%, i.e. mergers leading to monopolization independently of the initial concentration. They are obviously more welfare-reducing the greater the initial number of firms.

[13] In opposition to US Merger Guidelines, the implementation of the Merger Regulation of the European Union is not clear on whether a high concentration either alleviates or accentuates the anticompetitive effect of a merger of a given size. A high concentration implies that non-participating firms are large. On the one hand, this is assumed to be good, because it is argued that larger firms will oppose a more vigorous competition to the new merged entity. On the other hand, this is assumed to be bad, because it makes collusion between the new merged entity and the largest remaining producers easier.

show that $\Omega(N_2) - \Omega(N_1) = G(N_2) - G(N_1) - (N_2-N_1)\,F$ is negative if $N_1 < N_2$. It is indeed negative if $G(N_2) - G(N_1) < 0$. A sufficient condition for this is that $G'(n) < 0$ for any $N_1 \le n \le N_2$.

$$G'(n) = \Pi(n)\,\frac{X'(n)}{X(n)} - \Pi(R)\,\frac{X'(R)}{X(R)}\,\frac{\partial R}{\partial n} =$$

$$= \Pi(n)\,\frac{X'(n)}{X(n)}\,[\,1 - \frac{\Pi(R)}{\Pi(n)}\,\frac{X'(R)}{X(R)}\,\frac{X(n)}{X'(n)}\,\frac{\partial R}{\partial n}\,] =$$

$$= \Pi(n)\,\frac{X'(n)}{X(n)}\,[\,1 - \frac{n\ (\beta(n)+n+1)}{R\ (\beta(R)+R+1)}\,\frac{\partial R}{\partial n}\ \text{Exp}[\int_{R}^{n}\frac{x-1}{x\ (\beta(x)+x+1)}\,dx]\,] \le$$

$$\le \Pi(n)\,\frac{X'(n)}{X(n)}\,[\,1 - \frac{n\ (\beta(n)+n+1)}{R\ (\beta(n)+R+1)}\,\frac{\partial R}{\partial n}\ \text{Exp}[\int_{R}^{n}\frac{x-1}{x\ (\beta(x)+x+1)}\,dx]\,] \le$$

$$\le \Pi(n)\,\frac{X'(n)}{X(n)}\,[\,1 - (\frac{n+1}{R+1})^3\,\frac{n}{R}\,\frac{\partial R}{\partial n}\,].$$

The first inequality comes from the fact that $\beta(n) \ge \beta(R)$. The second inequality comes from the fact that the expression in brackets attains its higher value when demand is linear. We check below that the last expression is negative when mergers compared increase the HHI in the same amount or have the same market share.

### HHI:

Define $K(d) = (\frac{n+1}{R+1})^4\,\frac{\partial R}{\partial n}$ .

$$K'(d) = \frac{n\ (n+1)^4}{(R+1)^5\ (1+4\ d\ n^2)^{3/2}}\,[2n-3-6dn^2+12dn^3+\sqrt{1+4\ d\ n^2}\,(-6dn^2+1)] >$$

$$\frac{n\ (n+1)^4}{(R+1)^5\ (1+4\ d\ n^2)^{3/2}}\,[(6dn^2)(\,2\,n-1-\sqrt{1+4\ d\ n^2}\,)] \ge 0 \text{ for } 0 \le d \le \frac{n-1}{n}\,.$$

Observe that as mergers involve at least two firms we have that $\dfrac{2}{N_1^2} \le d \le \dfrac{N_1-1}{N_1}$ and therefore

$\dfrac{2}{n^2} \le d \le \dfrac{n-1}{n}$ for any $N_1 \le n \le N_2$. Then, $(\dfrac{n+1}{R+1})^3 \dfrac{n}{R} \dfrac{\partial R}{\partial n} > K(d) \ge K(0) = 1$.

Market share:

Define $F(R) = (\dfrac{n+1}{R+1})^3 \dfrac{n}{R} (1-s) = (\dfrac{n+1}{R+1})^3 \dfrac{R-1}{R}$. As $2 \le R(N_1) \le N_1-1$, we have $N_1 \ge 3$ and $2 \le R(n) \le n-1$ for $N_1 \le n \le N_2$. $F'(R) = \dfrac{-3R^2+4R+1}{R^2 \, (R+1)^4} < 0$. Then, $F(R) \ge F(n-1) > 1$.  ●

If demand is strictly concave, Proposition IV may not be true. For example, if demand is given by $P(X)=1-X^8$, the welfare loss of a merger changing market structure from a 5-firm industry to a duopoly (HHI delta $= \dfrac{12}{25}$) is greater than the welfare loss of a merger changing market structure from duopoly to monopoly (HHI delta $= \dfrac{1}{2}$). If demand is given by $P(X)=1-X^3$, the welfare loss of a merger changing market structure from a 6-firm industry to a triopoly (market share $= 66\%$) is greater than the welfare loss of a merger changing market structure from triopoly to duopoly (market share $= 66\%$).

To get a result for concave demands as well, we have to compare mergers reducing the number of independent firms in the same proportion. This is the result of Proposition V.

Proposition V: Assume that the degree of concavity of demand is nondecreasing in sales. Then, a proportional reduction in the number of independent firms, due to merger, is more welfare-reducing the greater the market concentration.

Proof: Call 1-p the proportional reduction in the number of independent firms, due to merger. The welfare loss due to this merger is given by $\Omega(n) = G(n) - n\,(1-p)\,F$, where $G(n)=W(n)-W(pn)$. We have to show that $\Omega(N_2) - \Omega(N_1) = G(N_2) - G(N_1) - (N_2-N_1)\,(1-p)\,F$ is negative if

$N_1 < N_2$. It is indeed negative if $G(N_2) - G(N_1) < 0$. A sufficient condition for this is that $G'(n) < 0$ for any $N_1 \leq n \leq N_2$.

$$G'(n) = W'(n)-pW'(pn) = \Pi(n) \frac{1}{n\ (\beta(n)+n+1)} - p\ \Pi(pn) \frac{1}{pn\ (\beta(pn)+pn+1)} \leq$$

$$\leq \frac{\Pi(n) - \Pi(pn)}{n\ (\beta(n)+n+1)} < 0.$$

## 3.2. The anticompetitive effect of mergers and the elasticity of demand.

Considerations about the elasticity of demand appear in the US Merger Guidelines at the market delineation stage. The relevant market is defined as the smaller set of products whose monopolization would imply a "small but significant increase (usually 5%) in price". This set is obtained by means of an iterative procedure in which beginning with the product merging firms produce new (closest substitute) products are added to the set until the above condition is satisfied. In my model, for example, the relevant market will only include the product sold by merging firms if demand is sufficiently inelastic and allows for significant price increases. Otherwise, the relevant market will include other products.

Elasticity, then, plays a key role, because it affects the degree of market concentration. If elasticity is low, the relevant market will be narrow and concentration high. Instead, if elasticity is high, the relevant market will be large and concentration low. A merger will be more likely to be challenged in the former case than in the latter, because concentration is understood to facilitate the exercise of market power. Therefore, the Merger Guidelines embody the idea that the welfare loss of a merger due to the exercise of market power is greater the less elastic the demand, because it implies a narrower relevant market.

In general, this idea is not true in my model, because the welfare loss rather than depending on the elasticity, depends on the degree of concavity of demand. To illustrate this, suppose we have two demands F and G (as depicted in Figure 2) such that sales in equilibrium are given by x. Therefore, the elasticity is the same in both cases. This implies that profits are also the same in both markets. However, for Proposition I, the welfare loss of a merger will be greater with demand G than with F, because the former is strictly convex and the latter strictly concave.

```
P |
  |
  |    G(X)
  |
  |
  |
  |
  |
  |
  |
  |
  |         F(X)
  |_____
       x              X
```
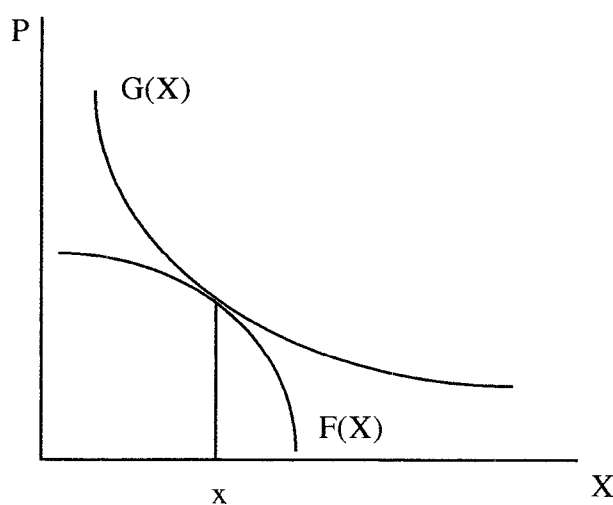
Figure 2.

Relying on the elasticity of demand to rank the welfare loss of mergers as proposed by the Merger Guidelines propose requires that a high elasticity implies a high degree of concavity (Proposition VI). If this is not the case, using a continuity argument, it is possible to find cases where the (premerger) elasticity gives an inaccurate prediction regarding the welfare loss.

The sufficient condition (Proposition VII) includes some additional technical assumptions for the following reason. Suppose we compare the welfare loss of two mergers. The sufficient condition must guarantee that if the premerger elasticity is greater for one merger, it will also be greater in the transition to the postmerger equilibrium.

As in Proposition I, size needs to be controlled. This is the purpose of Definition 2. In this case, I consider that F is constant. Definition 3 characterizes a special domain, where the domain is defined as the set of possible demands. Take two demands in the domain. A linear transformation of one of the demands allows us to equalize the size of both demands without affecting the degree of concavity. "Richness" guarantees that the demand obtained as a linear transformation also belongs to the domain. Definition 4 formally states the prediction on welfare the Merger Guidelines derive from the (premerger) elasticity of demand.

Definition 2: Two mergers are equivalent if they have the same: a) premerger concentration, b) number of participating firms and c) premerger profits.

Definition 3: A domain of demands is rich if P(X) belongs to the domain implies that $\alpha$ P(X), where $\alpha > 0$, also belongs to it.

Definition 4: The elasticity of demand is a good indicator if the welfare loss of equivalent mergers is non-increasing in the (premerger) elasticity of demand.

Proposition VI: (Necessity) Suppose that the domain of demands is rich. If the elasticity of demand is a good indicator then the degree of concavity of demand is nondecreasing in the elasticity of demand.

Proof: See Appendix III.

Proposition VII: (Sufficiency) If a $C^1$ non-decreasing function exists that maps from the elasticity of demand to the degree of concavity for any demand in the domain, then the elasticity is a good indicator.

Proof: See Appendix IV.

If the elasticity of demand and the degree of concavity are denoted by $\varepsilon$ and $\beta$ respectively we have that $\beta=0$ if demand is linear, $\beta=-1-\dfrac{1}{\varepsilon}$ with constant-elasticity demands and $\beta= -\dfrac{1}{\varepsilon}$ if demand is log-linear in price. Therefore, the sufficient condition is satisfied if all demands in the domain conform with one of these types. However, the necessary condition is not satisfied if the domain contains demands of more than one of the previous types.[14]

## 5. Concluding remarks.

Conditions concerning the profitability and the welfare loss of mergers considering general demand functions have been proved in the present paper. The crucial role played by the degree of concavity and market concentration in assessing the anticompetitive effect of mergers has been emphasized. On the other hand, any predictive power to the premerger elasticity of demand has been denied, except when it moves in the same direction as the degree of concavity. Although this condition is very restrictive, it is important to mention that it holds for the most common functional forms of demands: the linear demand, constant elasticity demands and demands that are log-linear in price.

---

[14] Suppose that a linear demand and a constant-elasticity demand (with elasticity f) belong to the domain (similar arguments hold for the remaining combinations of types). Then an output x exists such that the elasticity of the linear demand evaluated at x is lower than f. Then, the necessary condition is not satisfied because $-1-\dfrac{1}{f} < 0$.

## 6. Appendix.

## Appendix I.

I show first that asymmetric equilibria do not exist. Indeed, suppose that they exist and that there are two firms i and j that their outputs are different. Without loss of generality assume that $0 \leq x_i < x_j$. Then,

$$P'(x_i + x_{-i}) \, x_i + P(x_i + x_{-i}) - c \leq 0 \qquad \text{and } P'(x_j + x_{-j}) \, x_j + P(x_j + x_{-j}) - c = 0.$$

If firm i is not active $P(x_i + x_{-i}) - c \leq 0$ which is impossible since demand is strictly downward sloping and firm j is active. Therefore both divisions are active and first order conditions hold with equality. Then , $P'(x_i + x_{-i}) \, x_i = P'(x_j + x_{-j}) \, x_j$ and thus $x_i = x_j$, which is a contradiction.

Then in equilibrium all firms sell the same output. They are active because $P(0) > c$. Therefore, first order conditions are satisfied with equality. Equilibrium sales $(X^*)$ satisfy then:

$$F(X^*) = 0, \tag{8}$$

$$\text{where } F(X) = P(X) - c + \frac{X}{n} \, P'(X).$$

If (8) does not hold, first order conditions are not satisfied for all firms. (8) corresponds to equation (3) in the text. I check below that (8) is also sufficient, by showing that the profit-maximizing output of firm i, given that the others produce $\frac{X^*}{n}$, is $\frac{X^*}{n}$. Define $Y = \frac{n-1}{n} X^* + x_i$ and observe that $F'(X) = P'(X) + \frac{P'(X)}{n} + \frac{P''(X) \, X}{n} = \frac{P'(X)}{n} (n+1+ \frac{P''(X)X}{P'(X)})$ is negative given (2).

If $x_i < \dfrac{X^*}{n}$, the marginal profit of firm i is positive, because $P(Y)-c+x_iP'(Y) > P(Y) - c + \dfrac{Y}{n}$

$P'(Y) = F(Y) > 0$.

The second inequality comes from the fact that $Y < X^*$ and $F'(X) < 0$.

If $x_i > \dfrac{X^*}{n}$, the marginal profit of firm i is negative, because $P(Y) - c + x_i P'(Y) < P(Y) - c + \dfrac{Y}{n}$

$P'(Y) = F(Y) < 0$.

The second inequality comes from the fact that $Y > X^*$ and $F'(X) < 0$.

Therefore, $(P(X^*)-c) \dfrac{X^*}{n} > (P(Y)-c) \, x_i$ for all $x_i \neq \dfrac{X^*}{n}$.

Solution to (8) exists and is unique, because $F(0) > c$, $F(P^{-1}(c)) < c$ and $F'(X) < 0$.

## Appendix II.

For the following proofs the following notation is needed:

Notation.

$$\beta_P(X) = \frac{P''(X)X}{P'(X)}.$$
$$\varepsilon_P(X) = -\frac{P(X)}{P'(X)X}.$$

$X_P(n,c)$ output in equilibrium with n firms when demand is $P(X)$ and cost c.

$\Pi_P(n,c)$ industry profits in equilibrium with n firms when demand is $P(X)$ and cost c.

$\pi_P(n,c)$ firm profits in equilibrium with n firms when demand is $P(X)$ and cost c.

$W_P(n,c)$ gross social welfare in equilibrium with n firms when demand is $P(X)$ and cost c.

$\beta_P(n,c) = \beta_P(X_P(n,c))$.

$\varepsilon_P(n,c) = \varepsilon_P(X_P(n,c))$.

## Proof of Proposition I:

(<-) Define $\lambda$ as the ratio between F and the premerger individual profits. Given (6) if a merger

of k+1 firms is profitable in market M we have $0 \leq \text{Exp}[\int_{N-k}^{N} (\dfrac{n-1}{n\ (\beta_M(n,c_M)+n+1)} + \dfrac{1}{n})\ dn] - k$

$-1 + k\lambda \leq \text{Exp}[\int_{N-k}^{N} (\dfrac{n-1}{n\ (\beta_P(n,c_P)+n+1)} + \dfrac{1}{n})\ dn] -k -1 +k\ \lambda$. This implies that the merger is

also profitable in market P. The inequality follows from $\beta_P(n,c_P) \leq \beta_M(n,c_M)$. Given (7) if a

merger of k+1 firms is welfare-increasing in market P we have:

$$0 \leq k\lambda - N \int_{N-k}^{N} \text{Exp}[\int_{n}^{N} (\dfrac{x-1}{x\ (\beta_P(x,c_P)+x+1)})\ dx]\ (\dfrac{1}{n\ (\beta_P(n,c_P)+n+1)})dn \leq$$

$$k\lambda - N \int_{N-k}^{N} \text{Exp}[\int_{n}^{N} (\dfrac{x-1}{x\ (\beta_M(x,c_M)+x+1)})\ dx]\ (\dfrac{1}{n\ (\beta_M(n,c_M)+n+1)})\ dn. \text{ This implies that the}$$

merger is also welfare-increasing in market M.

(->) Suppose contradictorily that outputs $x_P$ and $x_M$ exist such that $\beta_P(x_P) > \beta_M(x_M)$. As these

functions are continuous exists $\delta > 0$ s.t. for $x \in (x_P-\delta, x_P]$ and $y \in (x_M-\delta, x_M]$ we have that

$\beta_P(x) > \beta_M(y)$. Define $c_P = P(x_P)$ and $c_M = M(x_M)$ .

As $\lim_{n->\infty} X_j(n,c_j) = x_j$, then exists N s.t. for any $n > N$ we have that $X_j(n,c_j)-x_j < \delta$ and

therefore $\quad \beta_P(n,c_P) > \beta_M(n,c_M)$ $\qquad\qquad\qquad\qquad$ (9)

## Welfare.

Define $F_j$ as the synergy gain in market j. Suppose synergy gains satisfy:

$F_P = W_P(N+6,c_P) - W_P(N+5,c_P)$. $\qquad\qquad\qquad$ (10)

$\dfrac{F_M}{\pi_M(N+6,c_M)} = \dfrac{F_P}{\pi_P(N+6,c_P)} = \lambda$. $\qquad\qquad\qquad$ (11)

Then when there are N+6 firms in each market, both markets are comparable for (11). We have that a two-firm merger is welfare-increasing in market P for (10). Furthermore, a two-firm merger is not welfare-increasing in market M, because

$$\frac{W_M(N+5,c_M)-W_M(N+6,c_M)}{\pi(N+6)} + \lambda < \frac{W_P(N+5,c_P)-W_P(N+6,c_P)}{\pi(N+6)} + \lambda = 0.$$

The inequality comes from (9). The equality from (10).

Profitability.

Suppose that synergy gains satisfy:

$$F_M = 2\,\pi_M(N+6,c_M) - \pi_M(N+5,c_M). \tag{12}$$

$$\frac{F_P}{\pi_P(N+6,c_P)} = \frac{F_M}{\pi_M(N+6,c_M)} = \lambda. \tag{13}$$

Then when there are N+6 firms in each market, both markets are comparable for (13). We have that a two-firm merger is profitable in market M for (11). Furthermore, a two-firm merger is not profitable in market P because $\frac{\pi_P(N+5,c_P)}{\pi_P(N+6,c_P)} - 2 + \lambda < \frac{\pi_M(N+5,c_M)}{\pi_M(N+6,c_M)} - 2 + \lambda = 0.$ The inequality comes from (9). The equality from (12).

Observe that if N is large enough (2) is satisfied and (12) is positive, because $\lim_{n\to\infty} \frac{\pi(n-1)}{\pi(n)} = 1.$

**Appendix III.**

Proof of Proposition VI.

Suppose contradictorily that demands P(X) and M(X) belonging to the domain and outputs $x_P$ and $x_M$ exist such that $\varepsilon_P(x_P) < \varepsilon_M(x_M)$ and $\beta_P(x_P) > \beta_M(x_M)$. As $\varepsilon_j(X)$ and $\beta_j(X)$ (j=P,M) are continuous, $\delta > 0$ exists s.t. for $x \in (x_P-\delta, x_P]$ and $y \in (x_M-\delta, x_M]$ we have that $\varepsilon_P(x) < \varepsilon_M(y)$ and for $z \in (x_P-\delta, x_P]$ and $u \in (x_M-\delta, x_M]$ we have that $\beta_P(x) > \beta_M(y)$. Define $c_j = P(x_j)$.

As $\lim_{n\to\infty} X_j(n,c_j) = x_j$, then N exists s.t. for any n > N we have that $X_j(n,c_j)-x_j < \delta$. Then,

$$\beta_P(n,c_P) > \beta_M(n,c_M) \tag{14}$$

$$\varepsilon_P(n,c_P) < \varepsilon_M(n,c_M) \tag{15}$$

Take $\alpha = \dfrac{\Pi_P(N+3,c_p)}{\Pi_M(N+3,c_M)}$ and define $K(X)=\alpha\ M(X)$. $K(X)$ belongs to the domain, because it is

rich. Then, we have:

$X_K(n,\alpha c_M)=X_M(n,c_M)$.

$$\beta_K(n,\alpha c_M)=\beta_M(n,c_M) \tag{16}$$

$$\varepsilon_K(n,\alpha c_M)=\varepsilon_M(n,c_M) \tag{17}$$

$$\Pi_K(N+3,\alpha c_M)=\Pi_P(N+3,c_M) \tag{18}$$

$W_K(N+3,\alpha c_M)-W_K(N+2,\alpha c_M) > W_P(N+3,c_P)-W_P(N+2,c_P)$, because (14), (16) and (18). As

$\varepsilon_K(N+3,\alpha c_M) > \varepsilon_P(N+3,c_P)$ for (15) and (17), the elasticity of demand is not a good indicator.


**Appendix IV.**


Proof of Proposition VII.


Take two demands in the domain, $P(X)$ and $M(X)$, and costs $c_P$ and $c_M$ such that
$\Pi_P(N,c_P)=\Pi_M(N,c_M)$ and $\varepsilon_M(N,c_M) > \varepsilon_P(N,c_P)$. We have to show that:


$$W_P(N,c_P)-W_P(N-k,c_P) \geq W_M(N,c_M) - W_M(N-k,c_M), \text{ for } 1 \leq k \leq N-1. \tag{19}$$


By assumption a $C^1$ non-decreasing function $g$ exists satisfying that $\beta_j(x)=g(\varepsilon_j(x))$ $(j=P,M)$.

Manipulating we have that:


$$\frac{\partial\varepsilon_j(n,c_j)}{\partial n} = \frac{\partial\varepsilon_j(X)}{\partial X}\frac{\partial X_j(n,c_j)}{\partial n} =$$
$$\frac{\partial\varepsilon_j(X)}{\partial X}\frac{X_j(n,c_j)}{\varepsilon_j(n,c_j)}\frac{\varepsilon_j(n,c_j)}{X_j(n,c_j)}\frac{\partial X_j(n,c_j)}{\partial n} = -\frac{1+\varepsilon_j(n,c_j)+g(\varepsilon_j(n,c_j))\ \varepsilon_j(n,c_j)}{n(1+n+g(\varepsilon_j(n,c_j)))} \tag{20}$$


(20) defines a differential equation of the type:

$$\overset{\cdot}{x} = f(t,x).$$

Given an initial condition, it has a unique and $C^1$ solution if f is $C^1$. This is the case because n $\geq$ 1, (2) and g is $C^1$. Then $\varepsilon_M(n,c_M) > \varepsilon_P(n,c_P)$ for n $\in [N-k,N]$. Therefore, $\beta_M(n,c_M)=g(\varepsilon_M(n,c_M)) \geq \beta_P(n,c_M)=g(\varepsilon_P(n,c_P))$ for n $\in [N-k,N]$. This implies (19).

7. <u>References</u>.

Corchón, L.C. and González-Maestre, M. (1994) "On the competitive effects of divisionalization". WP-AD 94-10. A discusión. University of Alicante.

Farrell and Shapiro (1990) "Horizontal Mergers: an Equilibrium Analysis". American Economic Review. March. 107-126.

Faulí-Oller, R. (1996) "On merger profitability in a Cournot setting." Mimeo.

Gaudet, G., and S. W. Salant (1992) "Mergers of producers of perfect complements competing in price." Economics Letters 39. 359-364.

Gellhorn, E. and W. E. Kovacic (1994). *Antitrust Law and Economics in a Nutshell*. West Publishing Co. St. Paul, Minn.

Kamien M., and I. Zang (1990) "The limits of monopolization through acquisition." Quarterly Journal of Economics. May. 465-499.

Levin, Dan. "Horizontal mergers: the 50-Percent Benchmark". American Economic Review Vol. 80 No. 5.

Posner R.A. (1976) <u>Antitrust Law: An Economic Perspective</u>. Chicago: University of Chicago Press.

Salant S.W., S. Switzer , and R.J. Reynolds (1983) "Losses from horizontal merger: the effects of an exogenous change in industry structure on Cournot-Nash equilibrium". Quarterly Journal of Economics. May. 185-199.

Seade, J. (1980) "On the effects of entry". Econometrica, Vol 84, No.2 March. 479-489.

Williamson, O. (1968) "Economies as an antitrust defence: the welfare tradeoffs." American Economic Review. Vol. 58. 18-36.

Willig, R.D. (1991) "Merger analysis, industrial organization theory, and mergers guidelines". Brookings Papers in Economic Activity: Microeconomics.