NUMERICAL REPRESENTATION FOR LOWER QUASI-CONTINUOUS PREFERENCES*

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ABSTRACT

A weaker than usual continuity condition for acyclic preferences is introduced. For preorders this condition turns out to be equivalent to lower continuity, but in general this is not true. By using this condition, a numerical representation which is upper semicontinuous is obtained. This fact guarantees the existence of maxima of such a function, and therefore the existence of maximal elements of the binary relation.

Keywords: Numerical Representation; Maximal Elements; Lower Quasi-Continuous Preferences.

1 Introduction

In order to obtain the existence of numerical representation for preorders (or more general binary relations) in topological spaces, a usual assumption is the lower continuity of the relation. This assumption is also used in order to ensure the existence of maximal elements on compact topological spaces (see Bergstrom (1975), Walker (1977)). Campbell and Walker (1990) introduce the notion of weakly continuous relations and obtain a numerical representation for preorders satisfying such a condition. Moreover they prove the existence of maximal elements for weakly continuous pseudotransitive binary relations. As they show, this result can not be extended to acyclic relations. In a recent work, Tian and Zhou (1995) use the condition of transfer continuity in order to ensure the existence of maximal elements of acyclic binary relations.

In order to obtain the existence of maximal elements of acyclic relations, and also to provide the existence of upper semicontinuous numerical representations, we introduce a condition that we call lower quasi-continuity. This kind of continuity implies the one in Campbell and Walker's work, but it is weaker than the usual lower continuity. The condition used in Tian and Zhou (1995) and the lower quasi-continuity are independent properties as we will see later.

Throughout the paper $\succ$ will represent an asymmetric binary relation defined on a set X and from this the preference-indifference relation $\succeq$, the indifference relation $\sim$ and the transitive closure $\succ\succ$ are defined as usual:
$x \succeq y$ if and only if not $(y \succ x)$
$x \sim y$ if and only if not $(y \succ x)$ and not $(x \succ y)$
$x \succ\succ y$ if and only if $x = x_1 \succ x_2 \succ \cdots \succ x_n = y$, for some $x_1, x_2, \cdots, x_n \in X$
The first one is a complete and reflexive binary relation, while the second one is reflexive and symmetric. We say that relation $\succ$ is

(1) Acyclic if $x_1 \succ x_2 \succ \cdots \succ x_n$ implies $x_1 \succeq x_n$
(2) Quasitransitive if $x \succ y \succ z$ implies $x \succ z$
(3) Pseudotransitive if $x \succ y \succeq z \succ w$ implies $x \succ w$
(4) Preorder if $x \succeq y \succeq z$ implies $x \succeq z$

Given a binary relation $\succ$ defined on a set X and a subset $A \subseteq X$, $x \succ A$ will represent $x \succ a, \forall\, a \in A$.

The binary relation $\succ$ defined on a topological space X is said to be lower continuous if for every $x \in X$, the set $\{z \in X \mid x \succ z\}$ is open.

Definition 1 Given a binary relation $\succ$ defined on a topological space, it is said to be *lower quasi-continuous* if whenever $x \succ y$ there is a neighborhood of y, N(y), which satisfies $x \succ\succ N(y)$, that is, $x \succ\succ z$ for every $z \in N(y)$.

When the binary relation is quasitransitive, lower quasi-continuity coincides with lower continuity, but, as the following example shows, this is not generally true.

Example 1 Let $X = \mathbb{R}$ and let $\succ$ be the binary relation defined as

$$x \succ y \Longleftrightarrow y+1 \geq x > y$$

This is an acyclic relation which is not lower continuous, but is lower quasi-continuous.

A real function $u:X \rightarrow \mathbb{R}$ is called a utility function for the binary relation $\succ$ if $x \succ y$ holds if and only if $u(x) > u(y)$. A utility function exists only when the binary relation is a preorder. For a more general kind of binary relations, only the existence of weak utility functions can be ensured (see for instance, Peleg (1970) and Peris and Subiza (1995)), that is a real valued function u such that $x \succ y$ implies $u(x) > u(y)$. This kind of numerical representation does not provide complete information about the binary relation, since $u(x)>u(y)$ does not guarantee $x \succ y$. Nevertheless it is useful because every element which maximizes this function is a maximal element and so, the existence of maxima of this function ensures the existence of maximal elements.

Rader (1963) uses upper semicontinuous utility functions to represent preorders and thus the existence of the maximum is guaranteed. A function $u:X \rightarrow \mathbb{R}$ defined on a topological space is said to be upper semicontinuous if for every $c \in \mathbb{R}$ the set $\{x \in X \mid u(x) \geq c\}$ is closed. If u is upper semicontinuous then it achieves its maximum on every compact subset. In this work we obtain an extension of the result in Rader to the case of acyclic relations, obtaining the existence of upper semicontinuous weak-utility functions.

2 Existence of Numerical Representation

In order to ensure the existence of continuous numerical representations, two different tools have been used in the literature. One of them consists of imposing some restrictions on the alternative set (usually asking for X to be a second countable topological space). An alternative approach consists of imposing additional conditions on the binary relation (usually asking for the separability of the relation). The first result we are going to obtain proves the existence of an upper semicontinuous weak-utility function when the alternative set is a second countable topological space.

Theorem 1 Let X be a second countable topological space and let $\succ$ be an acyclic and lower quasi-continuous binary relation defined on X. Then there is an upper semicontinuous function $u:X \rightarrow \mathbb{R}$ such that $x \succ y$ implies $u(x)>u(y)$.

Proof. Let $\mathcal{B} = \{B_i\}_{i\in\mathbb{N}}$ be a countable base of the topology. Let us define $f:X \rightarrow \mathbb{R}$ as

$$f(x) = \sum_{x\succ\succ B_k} 2^{-k}$$

and from this function we define

$$u(x) = \sup \{\lim_{n\rightarrow\infty} f(x_n), \text{ for every } \{x_n\} \text{ convergent to } x\}$$

First we prove that function u represents the binary relation. If $x \succ y$ then there is a neighborhood of y such that $x \succ\succ N(y)$, so there exists $k \in \mathbb{N}$ such that $x \succ\succ B_k$. Then,

$$f(x) \geq f(z) + 2^{-k} \text{ for every } z \text{ in } B_k$$

Let $\{y_n\}$ be a sequence convergent to y. Then $y_n \in B_k\ \forall\ n \geq n_0$ for some n_0 and $f(x) \geq f(y_n) + 2^{-k}$. From this we deduce, $f(x) \geq u(y) + 2^{-k}$, so $u(x)>u(y)$.

In order to prove the upper semicontinuity, let us suppose the existence of some $c \in \mathbb{R}$ such that the set $M(c) = \{x \in X \mid u(x) \geq c\}$ is not closed. In this case we can find a sequence $\{x_n\}$ in M(c) with limit $x \notin M(c)$. Then, we have the following:

$$u(x) < c - \varepsilon \text{ for some } \varepsilon > 0$$

$$u(x_n) \geq c\ \forall n \in \mathbb{N}$$

so, for any n we can find a sequence $\{x_n^t\}_{t\in\mathbb{N}}$ convergent to x_n such that

$$\lim_{t\rightarrow\infty} f(x_n^t) > c - \tfrac{\varepsilon}{2}.$$

Let us consider $\mathcal{B}(x) = \{D_n \in \mathcal{B} \mid x \in D_n\}_{n\in\mathbb{N}}$ and let us define

$A_1 = D_1, A_2 = D_1 \cap D_2, ..., A_n = D_1 \cap D_2 \cap \cdots \cap D_n$

Then, for any $n \in \mathbb{N}$ we can find some $k_n, t_n \in \mathbb{N}$ such that:

$f(x_{k_n}^{t_n}) > c - \frac{\varepsilon}{2}$ and

$x_{k_n}^{t_n} \in A_n$

The sequence $\{x_{k_n}^{t_n}\}$ converges to x because of the way it has been constructed. But this is a contradiction, because $\lim_{n\to\infty} f(x_{k_n}^{t_n}) \geq c - \frac{\varepsilon}{2} > u(x)$, which is not possible as a result of the way in which u(x) has been defined. ∎

Remark 1 If X is a metric and separable (that is there exists a dense countable subset of X) topological space, then X is a second countable space and the previous Theorem holds in this case.

The alternative approach in the literature to obtain continuous numerical representations consists of removing the restrictions on the space of alternatives, but asks for the separability of the binary relation, that is the existence of a countable subset $Q \subseteq X$ verifying that whenever $x \succ y$ there exists $d \in Q$ such that $x \succ d \succ y$. The condition we will use is weaker than separability (although for quasitransitive relations both conditions coincide) and we call it weak-separability.

Definition 2 A binary relation is said to be *weak-separable* if there is a countable subset Q of X such that when $x \succ y$ there exists $d \in Q$ such that $x \succ\succ d \succ\succ y$.

Theorem 2 Let X be a topological space and let $\succ$ be an acyclic, weak-separable and lower quasi-continuous binary relation defined on X. Then there is an upper semicontinuous function $u: X \to \mathbb{R}$ such that $x \succ y$ implies $u(x)>u(y)$.

Proof. Let $Q = \{q_i , i \in \mathbb{N}\}$ be the countable set which provides the separability of the binary relation and let $T = \{(a_k, b_k) \in Q \times Q \mid b_k \succ\succ a_k\}$. For each $k \in \mathbb{N}$ we can take a countable set

$T_k \subseteq \{ s \in Q \mid b_k \succ\succ s \succ\succ a_k \}$

such that it is ordered with relation $\succ\succ$ and has neither the first nor the last element due to the weak-separability. Thus a one-to-one mapping g_k from T_k to the rational numbers in (0,1) can be made in such a way that for all s,t $\in T_k$, $s \succ\succ t$ if and only if $g_k(s) > g_k(t)$. Let us define $u_k: X \to \mathbb{R}$ and $u: X \to \mathbb{R}$ as follows:

$$u_k(x) = \begin{cases} 1 & \text{if } \{s \in T_k \mid s \succ\succ x\} = \emptyset \\ \inf\{g_k(s),\ s \in T_k \,;\, s \succ\succ x\} & \text{otherwise} \end{cases}$$

$$u(x) = \sum_{k=1}^{\infty} u_k(x) 2^{-k}.$$

We will see first that u(x) is a weak utility function; let $x,y \in X$ such that $x \succ y$. Then $s \succ\succ x$ implies $s \succ\succ y$, so for any $k \in \mathbb{N}$, $u_k(x) \geq u_k(y)$. Moreover, the weak-separability of the binary relation ensures the existence of k^* and $s \in T_{k^*}$ such that $x \succ\succ s \succ\succ y$, and then $u_{k^*}(x) > u_{k^*}(y)$. Then $u(x) > u(y)$ and this is a weak-utility function.

In order to prove the upper semicontinuity of the function u(x) it is sufficient to prove that for each k, $u_k(x)$ is upper semicontinuous, because of the uniform convergence of the series that defines u(x). To do it, let $c \in \mathbb{R}$ and consider the set $L(c) = \{x \in X \mid u_k(x) < c\}$. Let $x \in L(c)$. If $L(c) \neq X$, there exists $s \in T_k$ such that $s \succ\succ x$, with $g_k(s) < c$. Then, there are $a_1, a_2, ..., a_n \in X$ such that $s=a_1 \succ a_2 ... \succ a_{n-1} \succ a_n = x$. The lower quasi-continuity of the relation gives us the existence of a neighborhood N(x) such that $a_{n-1} \succ\succ N(x)$. Then, for each $z \in N(x)$, $u_k(z) \leq u_k(a_{n-1}) \leq g_k(s) < c$; so $N(x) \subseteq L(c)$ and this is an open set. ∎

3 Existence of Maximal Elements

As we have mentioned, upper semicontinuity guarantees the existence of maximum on compact subsets, and so the existence of maximal elements (it is obvious that every element which maximizes a weak-utility function is a maximal element of the binary relation). Thus, as an immediate consequence of the previous results, we obtain:

Corollary 1 Let X be a second countable topological space and let $\succ$ be an acyclic and lower quasi-continuous binary relation defined on X. Then there exists a maximal element on every compact subset of X.

Corollary 2 Let X be a topological space and let $\succ$ be an acyclic, weak-separable and lower quasi-continuous binary relation defined on X. Then there exists a maximal element on every compact subset of X.

Campbell and Walker (1990) prove the existence of maximal elements on compact subsets by asking for a weaker continuity condition. But they ask for pseudotransitive binary relations in order to guarantee this fact. As they show in an example, weak lower continuity is not sufficient for acyclic relations to have maximal elements. In this sense, the results in Corollaries 1 and 2 are complementary to Campbell and Walker's result.

If we are interested in analyzing directly the existence of maximal elements, the following result provides a generalization of Bergstrom (1975) and Walker's (1977) result.

Theorem 3 Let X be a topological space and let $\succ$ be an acyclic and lower quasi-continuous binary relation defined on X. Then there exists a maximal element on every compact subset of X.

Proof. Suppose that there is not a maximal element on a compact subset $D \subseteq X$. Then, for any $x \in D$ there exists $y \in D$ such that $y \succ x$. In this case,

$$D = \bigcup_{x \in D} \{ y \in D \mid x \succ\succ y \}$$

and the lower quasi-continuity implies that this is an open cover of the compact subset D. Then there exists a finite subcover

$$D = \bigcup_{i=1}^{n} \{ y \in D \mid x_i \succ\succ y \}$$

Consider the finite set $A = \{x_1,...,x_n\}$. As $x_1 \in D$, then there exist $i \in \{1,2,...,n\}$ such that $x_i \succ\succ x_1$. If $i = 1$, we have a contradiction. Call this element x_2 , $x_2 \succ\succ x_1$. By repeating this reasoning, as A is finite, we obtain the existence of a cycle, and therefore a contradiction. ■

Remark 2 Note that the existence of maximal elements is ensured without the restrictions used in Corollaries 1 and 2 (X a second countable topological space or separability of the relation). The advantage of those results is given by the fact that they provide a method to obtain maximal elements: to maximize a real valued function

Tian and Zhou (1995) prove a similar result by using a condition that they call *transfer continuity* which states that whenever $x \succ y$ there exists a point x' and a neighborhood N(y) of y such that $x' \succ N(y)$. This condition was used by Sonnenschein (1971) in order to prove the existence of maximal elements for relations (not necessarily acyclic) satisfying a convexity condition. The following examples show that transfer continuity and lower quasi-continuity are independent properties.

Example 2 Let $X = \mathbb{R}$ and let $\succ$ be the binary relation defined as

$$x \succ y \Longleftrightarrow \begin{cases} x > y & if\ x,y \in (-\infty, 0] \\ x > y & if\ x,y \in [0, +\infty) \end{cases}$$

This relation is acyclic and is lower quasi-continuous but is not transfer continuous.

Example 3 Let $X = [0,1]$ and let $\succ$ be the binary relation defined as

$$x \succ y \Longleftrightarrow x > y \text{ , } x \neq 1 \text{ ; } 1 \succ 0$$

This relation is transfer continuous but is not lower quasi-continuous.

The following definition introduces a kind of continuity which generalizes both lower quasi-continuity and transfer continuity.

Definition 3 Given a binary relation $\succ$ defined on a topological space, it is said to be *transfer lower quasi-continuous* if whenever $x \succ y$ there exists a point x' and a neighborhood N(y) of y such that $x' \succ\succ N(y)$.

Note that Example 2 shows a relation which is transfer lower quasi-continuous but not transfer continuous; Example 3 shows a transfer lower quasi-continuous relation which is not lower quasi-continuous. Next result proves the existence of maximal elements for acyclic binary relations which are transfer lower quasi-continuous on a compact set.

Theorem 4 Let D be a compact topological space and let $\succ$ be an acyclic and transfer lower quasi-continuous binary relation defined on D. Then the set of maximal elements of $\succ$ on D is non-empty.

Proof. Suppose that there is not a maximal element. Then for every element $y \in D$ there exists $x \in D$ such that $x \succ y$. Because the transfer lower quasi-continuity, there is $x(y) \in D$ such that $x(y) \succ\succ N(y)$ for some neighborhood of y. Consider the open cover of D given by

$$D = \bigcup_{y \in D} N(y)$$

As D is compact there exists a finite subcover

$$D = \bigcup_{i=1}^{n} N(y_i)$$

and call $x_i = x(y_i)$, $i = 1, ..., n$. If we consider the finite set $A = \{x_1, ..., x_n\}$ then by following a similar argument as in Theorem 3 we obtain the existence of a cycle, and therefore a contradiction. ■

Remark 4 Note that this result does not generalize Theorem 3 because, like the condition used in Tian and Zhou, transfer lower quasi-continuity is not a "local" property, in the sense that we can not ensure the existence of maximals in closed subsets of D. The following example, taken from Tian and Zhou (1995), shows this fact.

Example 4 (Tian and Zhou, 1995). Let the binary relation $\succ$ defined on [0,3] as

$$x \succ y \Leftrightarrow \begin{cases} x > y & if \quad x,y \in \mathbb{Q} \\ x > y & if \quad x,y \notin \mathbb{Q} \\ x \in \mathbb{Q} & and \quad y \notin \mathbb{Q} \end{cases}$$

where $\mathbb{Q}$ stands for the set of rational numbers. This relation is transfer lower quasi-continuous in [0,3] and then it has a maximal element ($x^* = 3$). Nevertheless, on the subset [0,e] there is not maximal element.

The powerful of Theorem 3 is that it ensures the existence of maximal elements on every non-empty compact subset of the topological space.

REFERENCES

T.C. Bergstrom, Maximal elements of acyclic relations on compact sets, Journal of Economic Theory 10 (1975) 403-404.

D.E. Campbell and M. Walker, Maximal elements of weakly continuous relations, Journal of Economic Theory 50 (1990) 459-464.

B. Peleg, Utility functions for partially ordered topological spaces, Econometrica 38 (1970) 93-96.

J.E. Peris and B. Subiza, A weak utility function for acyclic preferences, Economics Letters 48 (1995) 21-24.

T. Rader, The existence of a utility function to represent preferences, Review of Economic Studies 30 (1963) 229-232.

H. Sonnenschein, Demand theory without transitive preferences, with applications to the theory of competitive equilibrium, in: J.S. Chipman, L. Hurwicz, M.K. Richter and H. Sonnenschein, eds., Preferences, Utility and Demand (Harcourt Brace Jovanovich, New York, 1971).

G. Tian and J. Zhou, Transfer continuities, generalizations of the Weierstrass and maximum theorem: a full characterization, Journal of Mathematical Economics 24 (1995) 281-303.

M. Walker, On the existence of maximal elements, Journal of Economic Theory 16 (1977) 470-474.

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