MULTIPLE ADVERSE SELECTION *

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ABSTRACT

We study an adverse selection model, with a principal and several agents, where contracting is under asymmetric information. The number of agents is finite and types are "continuous" and independent. We analyze two settings. In the first one, the performance functions of mechanisms may depend on all the reported types. In the second one, each performance function depends only on the respective announced type.

Under the standard hypotheses in the basic one-agent adverse selection model and the independence assumption, there is not loss of generality if the principal considers only mechanisms for which every agent reports his true type as a dominant strategy. We consider also the relaxation of the monotonicity hypothesis about the agents' welfare and we will prove that the former "equivalence" between the Bayesian implementation and the dominant strategy one stands firm in some cases.

We examine the properties of the optimal mechanism, supposing that the principal's "virtual income" depends on the agents' performances only through the aggregate total performance (which is natural in the context of regulation of a good produced by an oligopoly), and also, assuming the frame of regulation of a monopolist with several independent divisions (or the one of a group of firms), each one producing a different good.

Unlike the standard properties of the optimal mechanisms in the basic one-agent adverse selection model, in our model the optimal mechanism may ask very efficient agents for an individual performance higher than the one of complete information.

We show also that if agents are symmetrical, the principal may prefer ex ante to hire more than one agent.

Keywords: Adverse Selection; Independent Types; Optimal Mechanisms.

1.- INTRODUCTION

This paper aims at analyzing the adverse selection relationship of a principal with several agents. An example is the Japanese automobile firm: Japanese management made much less extensive use of competitive bidding that the U.S. management [see Milgrom & Roberts (1992), chapter 16]. We can also think about the internal structure of a firm whose head office is related to several divisions. Our approach may also represent the regulation of an oligopoly producing a homogeneous good or the one of a monopoly with several divisions (or the one of a group of firms), each one producing a different good.

We study an adverse selection model, with a principal and several agents, where the "type" of each agent is only known by himself before contracting. The number of agents is finite and types are independent realizations of absolutely continuous random variables with strictly positive density functions.

The utility function of each agent is quasilinear and separable. It is the addition of three terms: his remuneration, a personalized function clustering externalities which are originated by agent's actions, and a disutility (or utility) depending only on his type and on his individual action (performance). The principal's utility function is the difference between a profit, depending on agent's actions and types, and the sum of payments multiplied by a parameter representing principal's preferences on the agents' welfare.

We consider two settings. In the first one, individual performance functions depend on every reported type and in the second one, each individual performance function is only based on the type announced by the respective agent. We are interested in the properties of optimal performances and in the principal's ex ante preferences on the agents' number.

Model and results.

Our first setting represents situations where an economic agent (the principal) proposes personalized take-it-or-leave-it "prices" and "quantities" to another agents, so that each individual "menu" of contracts depends on every announced type. Think of a producer, of a variable quality good, being ignorant of the consumers' preferences upon the quality, or a firm purchasing an input, which is unfamiliar with the suppliers' productivity. Such menu of contracts may represent also, for example, several classes of managerial compensations in some enterprises [see the chapter 13 of Milgrom & Roberts (1992)].

We will make assumptions warranting that there is not loss of generality if the principal considers only mechanisms for which reporting the true type is a dominant strategy for each agent. The proof of this "equivalence", between the Bayesian implementation and the dominant strategy one, is realized by showing that the two of the respective optimization programs are equivalent to the "virtual" program consisting in the maximization of a distortion of the principal's expected utility of complete information, taking into account the social cost of revelation of the agents' hidden information.

We prove also that this equivalence stands firm, in some cases, when each agent's welfare is not monotonous in his type. Nevertheless, in this case, there is two classes of distortions in the principal's expected utility and the optimal mechanism may ask different types for the same performance (bunching).

For our first setting, we examine the properties of the optimal mechanism considering three cases. In both two first cases we suppose that the principal's "virtual income" [1] depends on the agents' performances only through the aggregate total performance (this is natural, for example, in the frame of the regulation of a homogeneous good).

In the first case [Linear Case], we assume that each agent's utility depends linearly on his type [2] (in the frame of the regulation of a homogeneous good, this case represents that every firm has a constant marginal production cost). We prove that, under some conditions, the principal prefers to offer direct dominant strategy mechanisms such that only the most virtually efficient agents (generically only one) produce. Moreover, if agents are symmetrical, the principal may prefer ex ante to hire more than one agent because the expected efficiency of the virtually best agent increases with the agents' number.

For this Linear Case, if agents are symmetrical, the properties of optimal performances are similar to those of the basic one-agent model: Although they are discontinuous, the aggregate total performance is monotonous and each individual performance is monotonous in the respective agent's type. Moreover each individual performance is lower than the one of complete information. Nevertheless, if the agents' ex ante distributions of types are different, the optimal mechanism may ask an agent, with a null performance under complete information, for a positive performance: virtual efficiencies induce an "order" over agents which may be different to the one induced by efficiencies.

In the second case [Convex Case], agents are symmetrical and each agent's type enters into the respective utility function affecting its third term which is a strictly convex (or concave if it represents a profit) function in his performance (in the frame of the regulation of an

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[1] It is the principal's virtual utility plus the total agents' virtual cost [following the terminology of Myerson (1981)].

[2] His type multiplies his action in his utility function.

homogeneous good, this case represents that each firm has a strictly increasing marginal production cost). Although the aggregate total performance is lower than the one of complete information, if the set of contractual types is heteroge neous enough, the optimal mechanism can ask the most efficient agents for an individual performance higher than the one of complete information. The intuition is that the principal "saves" informational rent of an agent inducing the truthful revelation of others agent's types.

Considering an agents' quadratic disutility function we prove that, for the Convex Case, the principal may also prefer ex ante to hire more than one agent.

In the third case, we examine the regulation of a monopolist with several independent divisions (or the one of a group of firms), each one producing a different good. Considering a cuadratic surplus and assuming that each type is the constant marginal cost of each division (or firm), we prove that, when goods are substitutes, the optimal regulatory mechanism may ask the most efficient divisions (or firms) for an individual production higher than the one of complete information. Nevertheless, when goods are complements, every individual production is lower than the one of complete information.

In our second setting, feasible mechanisms have each individual performance function based only on the announced type by the respective agent. This is justified if two circumstances coincide: on the one hand, if only contracts depending on the agents' (verifiable) performances are feasible and on the other hand [like in Demski & Sappington (1984)], if each agent's performance depends only on both an individual state (type) and a hidden action, in such a way that the state is privately observed by him-self before taking the action.

We show firstly that if the principal can only propose contracts depending on observed performances, the implementation by contracts is equivalent to the one by direct mechanisms for which each individual performance function is based only on the announced type of the respective agent.

Now, the equivalence between the Bayesian implementation and the dominant strategy one is obtained under less assumptions and the optimal mechanism entails that the most efficient agents may be asked for individual performances higher than the correspondent ones under complete information.

Differences with the previous research.

It is known that the basic adverse selection model, with only one agent and a continuum of types can be interpreted like the problem of a principal in front of a continuum of agents with independent types [see Guesnerie & Laffont (1984)]. Our model, on the contrary, assumes a finite agents' number.

There are adverse selection models, with several agents, where types are correlated [see, for example, Demski & Sappington (1984) and Cremer & McLean (1988)]. The main conclusion is that the principal is able to exploit the information owned by each agent about rival types (correlation) and he implements optimally the complete information solution. On the contrary, we suppose type-independence and this implies that all agent will obtain an informational rent and that the complete information mechanism cannot be the solution of incomplete information.

Note that in our model there is not an initial auction like, for example, in Laffont & Tirole (1987) who analyze the problem where several firms may carry out an indivisible public project having a large value for consumers. Under asymmetrical information they characterize the optimal Bayesian auction and prove that it can be implemented by a dominant strategy auction. In our model, the principal contracts several agents without utilizing a mechanism to select one agent. As mentioned above, an example of this class of relationship is the Japanese automobile firm. We can also think about the internal structure of a firm or about the regulation of an oligopoly.

Mookherjee & Reichelstein's (1992) analyze a general agency model, with a principal and several agents with independent types, and they prove that, the above equivalence between the Bayesian implementation and the dominant strategy one holds under some conditions. Nevertheless, their model can not be applied, for example, in the setting of the regulation of a private good, because the utility which they assume for their agents does not take into account the agent's externalities.

Our analysis proves that the equivalence between the Bayesian implementation and the dominant strategy one can be extended to another frames as regulation.

We consider also the relaxation of the monotonicity hypothesis upon the agents' welfare [implicitly supposed by Mookherjee & Reichelstein (1992)] and we will prove that the equivalence stands firm in some cases.

Lewis & Sappington (1989) analyze the regulation of a monopolist with a production costs function which is not monotonous in type. They prove that the monopolist may prefer understate or overstate his type and that the optimal mechanism may ask for a performance with bunching.

We consider also such a relaxation, with a cost function more general than the one supposed by Lewis & Sappington (1989). We prove that the equivalence, between the Bayesian implementation and the dominant strategy one, stands firm under hypothesis which are natural in the frame of regulation of the industrial pollution of a group of firms.

Structure of the paper.

The paper is organized as follows: Section 1 sets up the general model. Section 2 analyzes the equivalence between the Bayesian implementation and the dominant strategy one in our first setting. Section 3 explores the relaxation of the monotonicity hypothesis upon the agents' welfare. Section 4 analyzes the Linear Case, the Convex case and the regulation case. Finally, Section 5 explores our second setting.

2.- GENERAL FORMULATION

We consider a principal hiring n agents.
The agent i's utility function (i=1,..,n) is:

$$s_i + V_i(x,\theta_i), \; \theta_i \in [\underline{\theta}_i \, , \bar{\theta}_i] = \Theta_i$$

where $x=(x_1,\ldots,x_n) \in \mathbb{R}^n_+$ is the vector of agents' verifiable performances (actions), the parameter θ_i is the agent i's type and s indicates his remuneration.

We suppose the following separability assumption. We will consider two versions to cover more cases.

<u>Separable Utility</u> (SU)

$V_i(x,\theta_i) = R_i(x) - v_i(x_i,\theta_i) \;\; \forall \; i=1,\ldots,n.$ (SU-)

$V_i(x,\theta_i) = R_i(x) + v_i(x_i,\theta_i) \;\; \forall \; i=1,\ldots,n$ (SU+)

The principal's utility function is:

$$B(x,\theta) - \lambda \sum s_i$$

where $\theta=(\theta_1,\ldots,\theta_n) \in \Theta := \Pi\Theta_i$ is the vector of contractual types and $\lambda \geq 0$ represents the principal's preferences on the agents' welfare. The parameter λ may also be the shadow cost of publics funds in the regulation setting.

In order to fix ideas, let us consider several specifications of the model.

i) If the principal was a producer of a variable quality good, we should have $\lambda=1$, $R_i \equiv 0$, $B(x,\theta) = - C(x)$, where $C(\cdot)$ would be the cost function. Type θ_i , in SU+, would measure the for good of quality x_i.

ii) If the principal was a firm purchasing an input, we should have $\lambda=1$, $R_i \equiv 0$, $B(x,\theta) = [P(\Sigma x_i)\text{-}c] \; \Sigma x_i$, where $P(\cdot)$ would be the inverse demand function and c would be the marginal cost. Type θ_i, in SU-, would be the supplier i's cost parameter and s_i would be the price for the quantity x_i bought to supplier i.

iii) In the case of the regulation of a private good, the regulator's utility function would be the sum of the consumers' net surplus plus the firms' total profit weighed with 1-λ, where $\lambda \in [0,1]$:

$$S(\Sigma x_i) - P(\Sigma x_i) \Sigma x_k - \Sigma s_k + (1-\lambda) \Sigma [s_k + P(\Sigma x_i) x_k - v_k(x_k,\theta_k)].$$

The function $v_k(\cdot,\cdot)$ would be the firm k's production cost, the consumers' gross surplus would be represented by $S(Q) = \int_0^Q P(q)dq$ and $P(\cdot)$ would denote the inverse demand function.

Moreover $R_k(\mathbf{x}) = P(\Sigma x_i)x_k$, and therefore: $B(\mathbf{x},\boldsymbol{\theta}) = S(\Sigma x_i) - \lambda P(\Sigma x_i) \Sigma x_k - (1-\lambda) \Sigma v_k(x_k,\theta_k)$.

iv) For the regulation of a monopoly with several independent divisions (or of a group of firms), each one producing a different private good, the objective function of government would be:

$$S(\mathbf{x}) - \Sigma P_k (\mathbf{x}) x_k - \Sigma s_k + (1-\lambda) \Sigma [s_k + P_k (\mathbf{x}) x_k - v_k (x_k,\theta_k)]$$

where $S(\cdot)$ would be the consumers' gross surplus, $P_k(\cdot)=\partial_{x_k} S(\cdot)$ would be the price of the good k and $v(\cdot,\cdot)$ would be the firm k's production cost. According to our formulation, we should have

$$B(\mathbf{x},\boldsymbol{\theta}) = S(\mathbf{x}) - \lambda \Sigma P_k (\mathbf{x}) x_k - (1-\lambda) \Sigma v_k(x_k,\theta_k), \quad R_k(\mathbf{x}) = P_k(\mathbf{x}) x_k.$$

We assume that types are independent realizations of absolutely continuous random variables with density and distribution functions denoted respectively by $f_i(\theta_i)$ and $F_i(\theta_i)$, $\theta_i \in \Theta_i$, with $f_i(\cdot) > 0$, $i=1,\dots,n$. Each agent, before contracting, observes privately his type realization. For each agent i, let $E^i\{\cdot\}$ denote the expectation with regard to the random vector $\theta_{-i} := (\theta_1,\dots,\theta_{i-1},\theta_{i+1},\dots,\theta_n)$. From the revelation principle [see, for instance, Myerson (1982)] we know that the principal will offer a direct revelation mechanism that induces truthful revelation of the agents' types. Such mechanisms are based on the vector of announced types $\boldsymbol{\theta} = (\theta_1,\dots,\theta_n) \in \Theta := \Pi\Theta_i$ and they will be denoted:

$$\mathbf{t} : \Theta \to \mathbb{R}^n , \mathbf{x} : \Theta \to \mathbb{R}^n_+ .$$

We assume that the transfer functions $\mathbf{t}(\cdot)$ and the performance functions $\mathbf{x}(\cdot)$ are bounded and continuously differentiable functions almost everywhere. Note that they can be discontinuous [3]. For the second setting we will assume moreover that the component $x_i(\cdot)$ $i=1,\ldots,n$ of all feasible vector of performances $\mathbf{x}(\cdot)$ depends only on $\boldsymbol{\theta}$ in Θ.

The optimal mechanism has to solve the following program BP:

Bayesian program (BP)

$$\max_{\mathbf{x}(\cdot),\mathbf{t}(\cdot)} E\{ B(\mathbf{x}(\theta),\theta) - \lambda \Sigma t_i(\theta) \}$$

s.t.

$$\theta_i \in \arg\max_{\hat{\theta}_i} E^i\{ t_i(\theta_{-i},\hat{\theta}_i) + V_i(\mathbf{x}(\theta_{-i},\hat{\theta}_i),\theta_i) \} \qquad \forall\theta_i,\ \forall i \tag{1}$$

$$E^i\{ t_i(\theta) + V_i(\mathbf{x}(\theta),\theta_i) \} \geq u_i \qquad \forall\theta_i,\ \forall i$$

The problem would be *simplified* if we could replace the self-selection Bayesian constraints (1) by the correspondent ones of the dominant strategy implementation. In that case, the principal would solve the program:

Dominant strategy program (DP)

$$\max_{\mathbf{x}(\cdot),\mathbf{t}(\cdot)} E\{ B(\mathbf{x}(\theta),\theta) - \lambda \Sigma t_i(\theta) \}$$

s.t.

$$\theta_i \in \arg\max_{\hat{\theta}_i} \{ t_i(\theta_{-i},\hat{\theta}_i) + V_i(\mathbf{x}(\theta_{-i},\hat{\theta}_i),\theta_i) \} \quad \forall\theta_i,\ \forall\theta_{-i},\ \forall i \tag{2}$$

$$E^i\{ t_i(\theta) + V_i(\mathbf{x}(\theta),\theta_i) \} \geq u_i \qquad \forall\theta_i,\ \forall i$$

Constraints (2) point out that the mechanism induces truthful revelation of types like a dominant strategy for agents. An additional advantage of the dominant strategy implementation is that it may mitigate the problem of multiple equilibria.

Similarly, let BP' and DP' denote the programs for the second setting.

[3] In the case of only one agent, under weak assumptions. the must be continuous at the optimum [see Guesnerie & Laffont (1984)].

2.- THE EQUIVALENCE OF IMPLEMENTATIONS IN THE FIRST SETTING

In this section we will extend the Mookherjee & Reichelstein's (1992) analysis showing that the programs BP and DP are equivalent.

They consider the case $V_i(y,\theta_i) = - v_i(h_i(y),\theta_i)$, where $y \in Y$ represents the general decision to be implemented and $h_i(y) \in \mathbb{R}\ \forall i$, with a principal's utility equal to $B(y)-\Sigma t_i$. On the contrary, we assume the utility function of each agent being given by two personalized functions (see assumption SU). The first one, $R_i(\cdot)$, gathers together the agents' externalities and the second one, $v_i(\cdot,\cdot)$, represents a personalized cost (or profit) depending on type and on individual action. Moreover, in our model the equivalence would stand if we supposed a bit more general setting with:

$$V_i(y,\theta_i) = R_i(y) - v_i(h_i(y),\theta_i) \text{ or } V_i(y,\theta_i) = R_i(y) + v_i(h_i(y),\theta_i).$$

We will prove the equivalence through the virtual program (VP). We consider two versions corresponding to assumptions SU- and SU+.

Virtual program (VP)

$$\max_{\mathbf{x}(\cdot)} \quad E[\ \Omega(\mathbf{x}(\theta),\theta)\ ]$$

where $\Omega(\cdot,\cdot)$ is defined, under SU-, as

$$\Omega^-(x,\theta) = B(x,\theta) + \lambda \sum V_i(x,\theta_i) - \lambda \sum [\frac{F_i(\theta_i)}{f_i(\theta_i)} \partial_{\theta_i} v_i(x_i,\theta_i)] - \lambda \sum u_i$$

and, under SU+, as

$$\Omega^+(x,\theta) = B(x,\theta) + \lambda \sum V_i(x,\theta_i) - \lambda \sum [\frac{1-F_i(\theta_i)}{f_i(\theta_i)} \partial_{\theta_i} v_i(x_i,\theta_i)] - \lambda \sum u_i$$

Let us give an explanation of the virtual program. Firstly, note that, under complete information and when individual performances can depend on the whole vector of types (first setting), the principal solves, for each θ, the following program:

Complete information program (CIP)

$$\max_{\mathbf{x}} \{\ B(\mathbf{x},\theta) + \lambda\ \Sigma\ V_i(\mathbf{x},\theta) - \lambda\ \Sigma\ u_i\ \}$$

Therefore, the virtual program is a distortion of the one which he would solve if the information was complete. Under SU-, the term

$$-\lambda\sum\left[\frac{F_i(\theta_i)}{f_i(\theta_i)}\partial_{\theta_i}v_i(x_i,\theta_i)\right]$$

is added. For see that this term represents the social cost of revelation of the hidden information, assume that both the disutility and the marginal disutility of each agent increase with his type. As it is better for the principal that the most efficient agents being asked for a performance grater than the one for other less efficient agents, each agent has a natural incentive for overstate his type. Therefore, in order that an agent i of type θ_i-$d\theta_i$ does not report θ_i, he has to be compensated with $\partial_{\theta_i}v_i(x_i,\theta_i)d\theta_i$. The principal weighs the previous compensation with $F_i(\theta_i)$ which is the "amount" of the agent i's types that can claim to be of type θ_i.

Let $\Gamma(\mathcal{P})$ denote the optimal value of the program $\mathcal{P}$. In this section we assume the existence of solutions of every program.

Firstly, we prove that if a mechanism solves BP, it has to verify, as in the basic one-agent adverse selection model, self-selection local incentive constraints and this implies that the principal's optimal expected utility is lower than the optimal value of VP, which takes in account the social cost of the hidden information.

Proposition 1 Under SU we have $\Gamma(BP) \leq \Gamma(VP)$.

Proof: Suppose SU-. The line of argument is similar to the one of the case n=1. Let $[t(\cdot),x(\cdot)]$ a solution of the program BP. The functions

$$u_i(\theta_{-i},\theta_i,\tau) := t_i(\theta_{-i},\tau) + V_i(x(\theta_{-i},\tau),\tau) \qquad (3)$$

are continuously differentiable almost everywhere. From (1) we have:

$$E^i[u_i(\theta_{-i},\theta_i)] := E^i[u_i(\theta_{-i},\theta_i,\theta_i)] \geq E^i[u_i(\theta_{-i},\theta_i,\tau)] \quad \forall\theta_i,\ \forall\tau,\ \forall i$$

and, under assumption SU-, we get:

$$\partial_{\theta_i}E^i[u_i(\theta_{-i},\theta_i)] = -E^i[\partial_{\theta_i}v_i(x_i(\theta_{-i},\theta_i)] \quad a.e.\ on\ \Theta_i,\ \ \forall i$$

Therefore, integrating we have that

$$E^i[u_i(\theta_{-i},\theta_i)] = E^i[\int_{\underline{\theta}_i}^{\overline{\theta}_i} \partial_{\theta_i} v_i(x_i(\theta_{-i},\tau)d\tau] + \hat{u}_i \tag{4}$$

where $\hat{u}_i = E^i[u_i(\theta_{-i},\overline{\theta}_i)] \geq u_i \ \forall i$. By (3) and (4) the following equality holds:

$$E[t_i(\theta_{-i},\theta_i)] = \int_{\underline{\theta}_i}^{\overline{\theta}_i} E^i[-V_i(x(\theta_{-i},\theta_i),\theta_i) + \int_{\underline{\theta}_i}^{\overline{\theta}_i} \partial_{\theta_i} v_i(x_i(\theta_{-i},\tau),\tau)d\tau] f_i(\theta_i)d\theta_i + \hat{u}_i \tag{5}$$

Integrating by parts the second term in (5), and substituting it into (5) we obtain:

$$E[t_i(\theta)] = E[-V_i(x(\theta),\theta_i) + \frac{F_i(\theta_i)}{f_i(\theta_i)} \partial_{\theta_i} v_i(x_i(\theta),\theta_i)] + \hat{u}_i$$

Then, if $[\mathbf{t}(\cdot),\mathbf{x}(\cdot)]$ is a solution of BP we have that

$$\Gamma(BP) = E\{B(\mathbf{x}(\theta),\theta) - \lambda \Sigma t_i(\theta)\} \leq E[\Omega^-(\mathbf{x}(\theta),\theta)] \leq \Gamma(VP-).$$

Under assumption SU+, the equality

$$\int_{\underline{\theta}_i}^{\overline{\theta}_i} E^i[\int_{\underline{\theta}_i}^{\theta_i} \partial_{\theta_i} v_i(x_i(\theta_{-i},\tau),\tau)d\tau] f_i(\theta_i)d\theta_i = E[\frac{1-F_i(\theta_i)}{f_i(\theta_i)} \partial_{\theta_i} v_i(x_i(\theta_{-i},\theta_i),\theta_i)]$$

leads us to $\Gamma(BP) \leq \Gamma(VP+)$. □

Secondly, we will prove that every solution of VP is dominant strategy implementable, under some conditions that we specify below:

Monotone hazard rate (MHR)

F (·)/f (·) is increasing ∀i. (MHR-)

[1-F (·)]/f (·) is decreasing ∀i. (MHR+)

Constant sign (CS)

$\partial_{x,\theta_i} v_i(\cdot,\cdot) > 0$ *and* $\partial_{x,\theta_i} v_i(\cdot,\cdot)$ *is increasing in* θ_i, $\forall i$ (*CS*-)

$\partial_{x,\theta_i} v_i(\cdot,\cdot) > 0$ *and* $\partial_{x,\theta_i} v_i(\cdot,\cdot)$ *is decreasing in* θ_i, $\forall i$ (*CS*+)

Monotone agents' welfare (MAW)

$\partial_{\theta_i} v_i(\cdot,\cdot) \geq 0 \quad \forall i$

Let us comment the above assumptions which are standard in the basic one-agent adverse selection model.

Assumption MHR- is satisfied by most usual distributions and it implies the existence of a sort of decreasing returns for the probability that there are "improvements" on the basic technology (measured by $\bar{\theta}$-θ when the parameter θ represents the productivity) [see the interpretation of assumption in chapter 1 of Laffont & Tirole (1993)]. Note that assumptions MHR- and MHR+ are equivalent when density functions are symmetrical.

Assumptions MHR and CS, for only one agent, guarantee that the principal's program can be relaxed removing self-selection constraints. [see, for instance, Fudenberg & Tirole (1991)]. On the one hand, assumption CS implies the equivalence between constraints and the monotonicity of the performance function and, on the other hand, adding assumption MHR we get that every solution of the "relaxed program" is monotone. If MHR fails, the solution of relaxed program may be not monotone appearing bunching: the optimal performance is constant on an interval of positive measure. This makes the analysis difficult. For our model, with several agents, assumptions MHR and CS imply (assuming a last condition bellow) that each optimal individual performance of the program VP is monotone in the respective agent's type. Therefore every solution of VP is dominant strategy implementable. Knowing if the equivalence holds when MHR fails, is an open problem.

Assumption MAW [implicitly supposed in Mookherjee & Reichelstein (1992)] facilitates greatly the analysis. It implies that each agent's expected utility is monotone in his type and thus, the set of individual rationality constraints can be substituted, without loss of generality, for only one constraint. When there is only one agent, if MAW fails, he can have an incentive to understate his type for some of its realizations, and overstate it for others [see Lewis & Sappington (1989)]. This leads to bunching despite assumption MHR. In Section 3 we show that assumption MAW may be leave out, for some cases, holding up the equivalence between BP and DP.

The last assumption which we consider is:

Independent-on-types virtual income (ITVI)

The function $W(\mathbf{x},\theta)$ does not depend on θ

where, under SU-, we define $W(\cdot,\cdot)$ as

$$W^-(\mathbf{x},\theta) = B(\mathbf{x},\theta) + \Sigma\, [\, \lambda\, R_i(\mathbf{x}) + (1-\lambda)\, v_i(x_i,\theta_i)\,].$$

and under SU+, we define it as

$$W^+(\mathbf{x},\theta) = B(\mathbf{x},\theta) + \Sigma\, [\, \lambda\, R_i(\mathbf{x}) + (\lambda-1)\, v_i(x_i,\theta_i)\,].$$

With regard to assumption ITVI, the utility function of program VP- can be rewritten

$$\Omega^-(\mathbf{x},\theta) = W(\mathbf{x},\theta) - \Sigma\, \gamma_i(x_i,\theta_i) - \lambda\, \Sigma\, u_i$$

where the function

$$\gamma_i(x,\theta) := v_i(x,\theta) + \lambda \frac{F_i(\theta)}{f_i(\theta)} \partial_{\theta_i} v_i(x,\theta) \qquad x \geq 0, \quad \theta \in \Theta_i$$

is, following the terminology of Myerson (1981), the agent i's virtual cost. According to this terminology, $W(\cdot,\cdot)$ represents the principal's "virtual income". Note that with $\lambda=1$, the value $W(\cdot,\cdot)$ is the principal's income when he retains the agents' externalities that will be sent back through transfers.

Assumption ITVI suffices to apply a revealed preference argument showing that each component of any solution $\mathbf{x}(\cdot)$ of program VP is monotone in the respective type.

Note that for the examples of the producer of a variable quality good and the firm purchasing input, we have $R_i \equiv 0$, $\lambda=1$ and therefore ITVI holds. For the example of regulation it is also satisfied because $W \equiv S$.

Proposition 2 Under SU, MHR, CS, ITVI, every solution of program VP is dominant strategy implementable. If moreover MAW holds, we have $\Gamma(VP) \leq \Gamma(DP)$.

Proof: Consider firstly assumption SU-. Let $\mathbf{x}(\cdot)$ be a solution of program VP. Then it has to solve the pointwise maximization of the function $\Omega^-(\mathbf{x},\theta)$. Let see that this implies $x_i(\theta_{-i},\theta_i)$ decreasing in θ_i, for each i. Consider i and $\theta_i < \hat{\theta}_i$. Adding up inequalities:

$$\Omega^-(\mathbf{x}(\theta_{-i},\theta_i),\theta_{-i},\theta_i) \geq \Omega^-(\mathbf{x}(\theta_{-i},\hat{\theta}_i),\theta_{-i},\theta_i),\ \Omega^-(\mathbf{x}(\theta_{-i},\hat{\theta}_i),\theta_{-i},\hat{\theta}_i) \geq \Omega^-(\mathbf{x}(\theta_{-i},\theta_i),\theta_{-i},\hat{\theta}_i)$$

and under ITVI-, we get

$$\gamma_i(x_i(\theta_{-i},\theta_i),\hat{\theta}_i) - \gamma_i(x_i(\theta_{-i},\theta_i),\theta_i) \geq \gamma_i(x_i(\theta_{-i},\hat{\theta}_i),\hat{\theta}_i) - \gamma_i(x_i(\theta_{-i},\hat{\theta}_i),\theta_i) \tag{6}$$

where

$$\gamma_i(x,\theta) := v_i(x,\theta) + \lambda \frac{F_i(\theta)}{f_i(\theta)} \partial_{\theta_i} v_i(x,\theta) \quad x \geq 0, \quad \theta \in \Theta_i$$

Under assumptions MHR- and CS- the function $\partial_x \gamma_i(\cdot,\cdot)$ is strictly increasing in $\theta \in \Theta_i$, and therefore, we have $\partial_x[\gamma_i(\cdot,\hat{\theta}_i) - \gamma_i(\cdot,\theta_i)] > 0$. The inequality (6) implies, then, $x_i(\theta_{-i},\hat{\theta}_i) \leq x_i(\theta_{-i},\theta_i)$.

Because the Spence-Mirrlees' condition holds (it is the first condition in CS-) and the function $x_i(\theta_{-i},\theta_i)$ is decreasing in θ_i for each θ_{-i}, we can apply the habitual line of argument for the one-agent adverse selection models to prove that $x_i(\theta_{-i},\cdot)$ is implementable by a transfer function $t_i(\theta_{-i},\cdot)$. Define

$$t_i(\theta) = -V_i(x(\theta),\theta_i) + \int_{\theta_i}^{\overline{\theta}_i} \partial_{\theta_i} v_i(x_i(\theta_{-i},\tau)\tau) d\tau + \hat{u}_i$$

where $\hat{u}_i$ is arbitrary. From CS- we deduce

$$t_i(\theta_{-i},\theta_i) + V_i(x(\theta_{-i},\theta_i) - [t_i(\theta_{-i},\hat{\theta}_i) + V_i(x(\theta_{-i},\hat{\theta}_i)] = \int_{\theta_i}^{\hat{\theta}_i} [\partial_{\theta_i} v_i(x_i(\theta_{-i},\tau),\tau) - \partial_{\theta_i} v_i(x_i(\theta_{-i},\hat{\theta}_i),\tau)] d\tau \geq 0.$$

So, the mechanism $[\mathbf{t}(\cdot),\mathbf{x}(\cdot)]$ satisfies the constraints (2) and therefore, $\mathbf{x}(\cdot)$ is dominant strategy implementable.

If moreover MAW holds, choosing $\hat{u}_i = u_i \ \forall i$, the previous mechanism $[\mathbf{t}(\cdot),\mathbf{x}(\cdot)]$ verifies the individual rationality constraints (it is feasible for DP) and further we have

$$\Gamma(\text{VP}) = E[\Omega^{-}(\mathbf{x}(\theta),\theta)] = E\{B(\mathbf{x}(\theta),\theta) - \lambda \Sigma t_i(\theta)\} \leq \Gamma(\text{DP}).$$

If we assume SU+, making use of a similar argument, we obtain that $\mathbf{x}(\cdot)$ must satisfy:

$$\gamma_i(x_i(\theta_{-i},\theta_i),\hat{\theta}_i) - \gamma_i(x_i(\theta_{-i},\theta_i),\theta_i) \leq \gamma_i(x_i(\theta_{-i},\hat{\theta}_i),\hat{\theta}_i) - \gamma_i(x_i(\theta_{-i},\hat{\theta}_i),\theta_i)$$

where now

$$\gamma_i(x,\theta) := v_i(x,\theta) - \lambda \frac{1-F_i(\theta)}{f_i(\theta)} \partial_{\theta_i} v_i(x,\theta) \quad x \geq 0, \quad \theta \in \Theta_i$$

verifies $\partial_x[\gamma_i(\cdot,\theta_i)-\gamma_i(\cdot,\hat{\theta}_i)] < 0$ by MHR+ and CS+. Therefore, in this case, $x_i(\theta_{-i},\theta_i)$ is increasing in θ_i for each θ_{-i} and $\mathbf{x}(\cdot)$ is dominant strategy implementable using

$$t_i(\theta) = -V_i(x(\theta),\theta_i) + \int_{\underline{\theta}_i}^{\theta_i} \partial_{\theta_i} v_i(x_i(\theta_{-i},\tau)\tau)d\tau + \hat{u}_i \quad \square$$

Because the dominant strategy implementation implies the Bayesian one, previous propositions prove immediately that all the three programs BP, DP and VP are equivalent.

Proposition 3 Under assumptions SU, MHR, CS, ITVI, MAW, we have $\Gamma(BP) = \Gamma(DP) = \Gamma(VP)$, that is, every solution $[\mathbf{t}(\cdot),\mathbf{x}(\cdot)]$ of program BP is dominant strategy implementable with transfer functions $\hat{\mathbf{t}}(\cdot)$ satisfying

$$E^i[\ \hat{t}_i(\theta_{-i},\theta_i) - t_i(\theta_{-i},\theta_i)\ ] = 0 \ \forall\theta_i,\ \forall i.$$

Let us conclude this section with the following remark. When $\lambda=0$ programs VP and CIP coincide and the complete information solution is the solution of incomplete information, under assumptions of Proposition 3. This is a property of the basic one-agent adverse selection model [see Guesnerie & Laffont (1984) and Caillaud, Guesnerie, Rey & Tirole (1988)].

3.- RELAXATION OF MONOTONE AGENTS' WELFARE ASSUMPTION IN THE FIRST SETTING

We have indicated previously that, *in the basic one-agent adverse selection model*, if MAW fails, the agent can prefer to understate or overstate his type [see Lewis & Sappington (1989)]. This implies, in spite of assumption MHR, that the optimal performance has "bunching".

In this section we will see that it is possible to relax assumption MAW, in some cases, maintaining the equivalence between BP and DP. Unlike Lewis & Sappington's (1989) analysis, we consider several agents with more general disutility functions. We will assume:

<u>Type-separable disutility</u> (TSD)

For every i, $v_i(x,\theta) = \theta\ \varphi_i(x) + \Psi_i(\theta)$, $x \geq 0$, $\theta \in \Theta_i$, where $\varphi_i(\cdot)$, $\Psi_i(\cdot)$ are continuously differentiable.

Assumption TSD, which generalizes a constant marginal disutility, implies a necessary and sufficient condition for Bayesian implementation. But such condition does *not* involve that each *individual* performance is monotone in the respective type.

Proposition 4 Under SU- and TSD, the transfer functions $\mathbf{t}(\cdot)$ Bayesian implement $\mathbf{x}(\cdot)$ if and only if the following conditions are satisfied:

(a) There are values $\hat{u}_i$, $i=1,\ldots,n$, such that, for each i, the following equality holds almost everywhere in Θ_i:

$$E^i[t_i(\theta_{-i},\theta_i) + V_i(x(\theta_{-i},\theta_i),\theta_i)] = \int_{\theta_i}^{\bar{\theta}_i} E^i\varphi_i[x_i(\theta_{-i},\tau)]d\tau + \Psi_i(\bar{\theta}_i) - \Psi_i(\theta_i) + \hat{u}_i$$

(b) The function $E^i\varphi_i[x_i(\theta_{-i},\cdot)]$ *is decreasing (in θ_i)* $\forall$i.

Proof: Given $\mathbf{t}(\cdot)$ implementing $\mathbf{x}(\cdot)$, applying the argument in the proof of Proposition 1, we obtain the equalities in (a). On the other hand, with a revealed preference argument as in the proof of Proposition 2, we get: $(\hat{\theta}_i-\theta_i)E^i\varphi_i[x_i(\theta_{-i},\theta_i)] = (\hat{\theta}_i-\theta_i)E^i\varphi_i[x_i(\theta_{-i},\hat{\theta}_i)]$, $\forall\hat{\theta}_i$, $\forall\theta_i$, $\forall$i, and then, the function $E^i\varphi_i[x_i(\theta_{-i},\cdot)]$ is decreasing for each i.

Finally, if $\mathbf{t}(\cdot)$, $\mathbf{x}(\cdot)$, satisfy conditions (a) and (b) we have $\forall\hat{\theta}_i$, $\forall\theta_i$, $\forall i$,

$$E^i[t_i(\theta_{-i},\theta_i)+V_i(x(\theta_{-i},\theta_i))]-E^i[t_i(\theta_{-i},\hat{\theta}_i)+V_i(x(\theta_{-i},\hat{\theta}_i))]=\int_{\theta_i}^{\hat{\theta}_i}[E^i\varphi_i[x_i(\theta_{-i},\tau)]-E^i\varphi_i(x_i(\theta_{-i},\hat{\theta}_i))]d\tau \geq 0$$

and therefore, $\mathbf{t}(\cdot)$ implements $\mathbf{x}(\cdot)$. □

Proposition 4 characterizes, in a special case (TSD), the class of Bayesian implementable performance functions, which contains the one of dominant strategy implementable performance functions. Note that, under the conditions of Proposition 3, every solution of PB belongs to the latest class. Under SU- and TSD, Proposition 4 implies that the program BP is equivalent to the following one:

Equivalent Bayesian problem (EBP)

$$\max_{\mathbf{x}(\cdot),\hat{u}} \ T[\mathbf{x}(\cdot)] - \lambda\ \Sigma\ \Psi_i(\theta_i) - \lambda\ \hat{u}_i \quad \text{s.t.}$$

$E^i\varphi_i[x_i(\theta_{-i},\cdot)]$ decreasing $\forall i$

$$\int_{\theta_i}^{\bar{\theta}_i} E^i\varphi_i[x_i(\theta_{-i},\tau)]d\tau+\Psi_i(\bar{\theta}_i)-\Psi_i(\theta_i)+\hat{u}_i \geq 0 \quad \forall\theta_i, \forall i$$

where the functional $T[\cdot]$ is defined by

$$T[x(\cdot)]:=E[B(x(\theta),\theta)+\lambda\sum R_i(x(\theta))]-\lambda\sum\int_{\underline{\theta}_i}^{\bar{\theta}_i}[\![\,\theta_i E^i\varphi_i[x_i(\theta_{-i},\theta_i)]+\int_{\theta_i}^{\bar{\theta}_i}E^i\varphi_i[x_i(\theta_{-i},\tau)]d\tau\,]\!]f_i(\theta_i)d\theta_i$$

Note that if for all i the inequalities $\varphi_i \geq 0$ and $\Psi_i' \geq 0$ hold, assumption TSD implies MAW and the analysis of Section 2 is valid. In order to examine the problem when assumption MAW fails, we will assume, like in Lewis & Sappington (1989), that each "fixed cost" Ψ_i is decreasing and concave in the "marginal cost" θ_i. Thus, we can warrant that, for the optimal performance, each individual rationality constraint is binding at only one value of the respective type.

Lemma 1 Assume SU-, TSD, with $\Psi_i \in C^2$, $\Psi_i' < 0$, $\Psi_i'' < 0$, $\underline{\theta}_i > 0$, $\forall i$. Then, for every BP-optimal performance function, each individual rationality function is binding at only one type. If such types are $\tau_i \in \Theta_i$, $i=1,\ldots,n$, the principal's expected utility is equal to

$$E\{\ W(\mathbf{x}(\theta),\theta) - \Sigma\ \beta_i(\theta_i;\tau_i)\ \varphi_i[x_i(\theta)]\ \} - (1-\lambda)\ \Sigma\ E\psi_i - \lambda\ \Sigma\ \Psi_i(\tau_i) - \lambda\ \Sigma\ u_i$$

where $\beta_i(\cdot;\cdot)$ is defined by:

$$\beta_i(\theta;\bar{\theta}_i):=\theta+\lambda\frac{F_i(\theta)}{f_i(\theta)},\quad \theta\in\Theta_i;\qquad \beta_i(\theta;\underline{\theta}_i):=\theta-\lambda\frac{1-F_i(\theta)}{f_i(\theta)},\quad \theta\in\Theta_i;$$

and for $\tau_i \in \text{int}(\Theta_i)$, $\beta_i(\theta;\tau_i):=\beta_i(\theta;\bar{\theta}_i)$ if $\underline{\theta} \leq \theta < \tau_i$, $\beta_i(\theta;\tau_i):=\beta_i(\theta;\underline{\theta}_i)$ if $\tau_i < \theta \leq \bar{\theta}_i$.

Proof: Let $\mathbf{x}(\cdot)$ be an optimal performance of program EBP. The individual rationality constraints can be rewritten $U_i(\theta_i) \geq u_i$, $\forall\theta_i$, $\forall i$, where

$$U(\theta_i):=\int_{\theta_i}^{\bar{\theta}_i} E^i\varphi_i[x_i(\theta_{-i},\tau)]d\tau+\Psi_i(\bar{\theta}_i)-\Psi_i(\theta_i)+\hat{u}_i,\quad \theta_i\in\Theta_i.$$

Each function $U_i(\cdot)$ is continuous and differentiable, with a derivative equal to:

$$U_i'(\theta_i) = - E^i\varphi_i[x_i(\theta_{-i},\theta_i)] - \Psi_i'(\theta_i)$$

which is of class C^1 almost everywhere [$U_i'(\cdot)$ may be discontinuous because each $x_i(\cdot)$ is C^1 almost everywhere]. Since $E^i\varphi_i[x_i(\theta_{-i},\cdot)]$ is decreasing and $\Psi_i'(\cdot)$ is strictly decreasing, the function $U_i'(\cdot)$ will be strictly increasing and C^1 almost everywhere. Therefore, $U_i(\cdot)$ is continuous, C^1 almost everywhere and strictly convex. As $\mathbf{x}(\cdot)$ is optimal, we obtain that there is only one $\tau_i \in \Theta_i$ such that $U_i(\theta_i) \geq U_i(\tau_i) = u_i$, $\forall\ \theta_i \in \Theta_i$. Thus, we have that

$$\hat{u}_i=u_i-\int_{\tau_i}^{\bar{\theta}_i} E^i\varphi_i[x_i(\theta_{-i},\tau)]d\tau-\Psi_i(\bar{\theta}_i)+\Psi_i(\theta_i),\quad \forall i.$$

Replacing these values into the expression of the principal's expected utility for the program EBP, and after integrating by parts, we get the expression we are looking for. □

To understand Lemma 1 suppose TSD. As under MAW, an agent has incentives to overstate his type, but now a second class of incentives appears: a few efficient agent may prefer understate his type, if in that manner the principal believes that this agent has a higher fixed cost (note that $\Psi_i' < 0$). Therefore, each agent's rent decreases with his type, when type is low, and it increases when type is high. The principal will distort in two fashions his complete information expected utility: he will add (see Lemma 1), for the agent i, the terms

$$-\lambda \frac{F_i(\theta_i)}{f_i(\theta_i)} \partial_{\theta_i} v_i(x_i,\theta_i) \text{ if } \theta_i < \tau_i, \qquad \lambda \frac{1-F_i(\theta_i)}{f_i(\theta_i)} \partial_{\theta_i} v_i(x_i,\theta_i) \text{ if } \theta_i > \tau_i$$

Note that if $\tau_i = \bar{\theta}_i$ $\forall$i holds, only the first class of incentives prevails and the principal's expected utility in Lemma 1 coincides with the virtual function $\Omega(\cdot,\cdot)$, under TSD, in the previous section.

In the rest of this section we will see that, under the considered assumptions, when virtual income is additively separable, concave and increasing in performances and independent of types, the programs BP and DP are equivalents. We show also that each optimal individual performance depends only on the respective type decreasingly.

We consider, then, the following assumption, which is natural if the principal is the owner of a firm with several divisions (agents), each one with the cost function in TSD or if the principal is a regulator of the industrial pollution of several firms (here x would be an verifiable indicator of contamination with $x = +\infty$ representing null contamination).

<u>Independent-on-types and separable virtual income</u> (ITSVI)

$W(x,\theta) = \Sigma\, W_i(x_i)$, where $W_i \in C^2$, $W_i' > 0$, $W_i'' \leq 0$, $W_i(0) > -\infty$, $\forall$i.

Under ITSVI, we can easily verify that the program EBP is equivalent to the independent programs EBP_i, $i=1,\dots,n$, bellow.

<u>Equivalent Bayesian program for agent i</u> (EBP_i)

$$\max_{x_i(\cdot),\hat{u}_i} T_i[x_i(\cdot)] - (1-\lambda)\, E\Psi_i - \lambda\, \Psi_i(\bar{\theta}_i) - \lambda\, \hat{u}_i$$

s.t.

$E^i\varphi_i[x_i(\theta_{-i},\cdot)]$ decreasing

$$\int_{\theta_i}^{\bar{\theta}_i} E^i\varphi_i[x_i(\theta_{-i},\tau)]d\tau + \Psi_i(\bar{\theta}_i) - \Psi_i(\theta_i) + \hat{u}_i \geq u_i \quad \forall\theta_i$$

where

$$T_i[x_i(\cdot)] := \int_{\underline{\theta}_i}^{\bar{\theta}_i} [\![\, E^iW_i[x_i(\theta_{-i},\theta_i)] - \lambda\theta_i E^i\varphi_i[x_i(\theta_{-i},\theta_i)] - \lambda \int_{\theta_i}^{\bar{\theta}_i} E^i\varphi_i[x_i(\theta_{-i},\tau)]d\tau \,]\!]\, f(\theta_i)d\theta_i.$$

The arguments in the proof of Lemma 1 can be applied for each program EBP_i: at the optimum of EBP_i, the agent i's rent is zero only for a certain type τ_i. We will see later (Proposition 5) that, under some conditions, every solution of EBP_i is also a solution of some of the programs $[P(\tau_i)]$, $\tau_i \in \Theta_i$, bellow (precisely the correspondent one of the unique type with null rent).

$$\max_{x_i(\cdot)} E\{ W[x_i(\theta)] - \beta_i(\theta_i;\tau_i)\, \varphi_i[x_i(\theta)] \}$$
$$\text{s.t.}$$
$$E^i\varphi_i[x_i(\theta_{-i},\theta_i)] \geq -\psi_i'(\tau_i),\ \forall\ \theta_i \in [\underline{\theta}_i, \tau_i[$$

$$E^i\varphi_i[x_i(\theta_{-i},\theta_i)] \leq -\psi_i'(\tau_i),\ \forall\ \theta_i \in]\tau_i, \overline{\theta}_i]$$

Note that, if $\tau_i = \overline{\theta}_i$, we consider only the first constraint and if $\tau_i = \underline{\theta}_i$ only the second one.

Previously, we establish hypotheses for which any one of the problems $[P(\tau_i)]$ has only a solution. Moreover, this solution will be independent on θ_{-i} and decreasing. In Appendix 1, we prove the following lemma.

Lemma 2 Suppose

i) $\varphi_i \in C^2$, $\varphi_i \geq 0$, $\varphi_i'(0) \geq 0$, $\varphi_i'(x) > 0$ if $x > 0$, $\varphi_i'' \geq 0$

ii) $\Psi_i \in C^2$, $\Psi_i' < 0$, $\Psi_i'' < 0$

iii) $W_i \in C^2$, $W_i' > 0$, $W_i'' \leq 0$, $W_i'(0) > [\overline{\theta}_i + 1/f_i(\overline{\theta}_i)]\, \varphi_i'(0)$

iv) $\exists\, x^*_i > 0$ / $W_i(x^*_i) - [\overline{\theta}_i + \lambda/f_i(\overline{\theta}_i)]\, \varphi_i(x^*_i) > W_i(0) - \underline{\theta}_i\, \varphi_i(0)$

$\varphi_i(x^*_i) < -\Psi_i'(\underline{\theta}_i)$

v) $W_i'(x)/\varphi_i'(x)$ for $x \in]0, +\infty[$ is strictly decreasing and it goes to a real number strictly lesser than $\underline{\theta}_i - \lambda/f_i(\underline{\theta}_i)$ when x goes to $+\infty$.

vi) $\underline{\theta}_i\, f_i(\underline{\theta}_i) > \lambda$

and moreover MHR+ and MHR-.

Then, each program $[P(\tau_i)]$ has only one solution:

$$x_i(\theta;\tau_i) = \max[\, \overline{\phi}_i(\theta_i), \rho_i(\tau_i)\,] \text{ if } \theta_i \in [\underline{\theta}_i, \tau_i[, \quad x_i(\theta;\tau_i) = \min[\, \underline{\phi}_i(\theta_i), \rho_i(\tau_i)\,] \text{ if } \theta_i \in]\tau_i, \overline{\theta}_i]$$

where $\rho_i(\cdot) := \varphi_i^{-1}[-\Psi_i'(\cdot)]$, and the functions $\overline{\phi}_i(\cdot)$ and $\underline{\phi}_i(\cdot)$ are the only ones satisfying

$$W_i'[\overline{\phi}_i(\theta)] = \beta_i(\theta;\overline{\theta}_i)\, \varphi_i'[\overline{\phi}_i(\theta)],\ \forall\ \theta \in \Theta_i \quad \text{and} \quad W_i'[\underline{\phi}_i(\theta)] = \beta_i(\theta;\underline{\theta}_i)\, \varphi_i'[\underline{\phi}_i(\theta)],\ \forall\ \theta \in \Theta_i.$$

Therefore, $\bar{\phi}_i(\cdot)$ and $\underline{\phi}_i(\cdot)$ are C^1 and strictly decreasing functions and they verify $\bar{\phi}_i(\cdot) < \underline{\phi}_i(\cdot)$. This implies that the solution of $[P(\tau_i)]$ depends only on θ_i, is continuous except at most at τ_i, is decreasing and it may be constant only on a subinterval around τ_i and on the rest it is continuously differentiable.

In Lemma 2, conditions i) and ii) imply that, under SU- and TSD, the disutilities v $(\cdot,\cdot)$ $\forall$i satisfy SC- but they do not verify MAW. Condition iv) allows to assure that each solution of $[P(\tau_i)]$ is strictly positive. The existence and uniqueness are obtained from iii) and v).

By means of Lemmas 1 and 2 we can prove that, in some cases, the assumption MAW may be relaxed preserving $\Gamma(BP)=\Gamma(DP)$.

Proposition 5 Under SU-, TSD, ITSVI and the assumptions of Lemma 2, every optimal performance of program BP is dominant strategy implementable and $\Gamma(BP)=\Gamma(DP)$.

Proof: Let $\mathbf{x}(\cdot)$ be an optimal performance of EBP. From ITSVI each $x_i(\cdot)$ is a solution of EBP_i and, by Lemma 1, there are $\tau_i \in \Theta_i$, i=1,...,n, such that each $x_i(\cdot)$ is feasible for the program $[P(\tau_i)]$ with an objective value equal to

$$E\{ W_i[x_i(\theta)] - \beta_i(\theta_i;\tau_i)\, \varphi_i[x_i(\theta)] \} - (1-\lambda)\, E\Psi_i - \lambda\, \Psi_i(\tau_i) - u_i.$$

Suppose that $x_i(\cdot)$ is not a solution of $[P(\tau_i)]$. From Lemma 2, the only solution $x_i(\cdot;\tau_i)$ of $[P(\tau_i)]$ has to satisfy:

$$E\{W_i[x_i(\theta)]-\beta_i(\theta_i;\tau_i)\varphi_i[x_i(\theta)]\} < E\{W_i[x_i(\theta;\tau_i)]-\beta_i(\theta_i;\tau_i)\varphi_i[x_i(\theta;\tau_i)]\}$$

We know that $x_i(\cdot;\tau_i)$ is a function independent of types θ_{-i}, decreasing and continuous, and piecewise continuously differentiable. Therefore, the function

$$U_i(\theta_i) := \int_{\theta_i}^{\bar{\theta}_i} E^i \varphi_i[x_i(\theta_{-i},\tau;\tau_i)]\,d\tau + \Psi_i(\bar{\theta}_i) - \Psi_i(\theta_i)$$

is continuous, piecewise continuous differentiable, and strictly convex. Moreover, we have:

$$U_i'(\tau_i-) = -\varphi_i(\max[\bar{\phi}_i(\tau_i), \rho_i(\tau_i)]) - \Psi_i'(\tau_i) \leq -\varphi_i(\rho_i(\tau_i)) - \Psi_i'(\tau_i) = 0$$

$$U'(\tau_i+) = -\varphi_i(\min[\underline{\phi}_i(\tau_i), \rho_i(\tau_i)]) - \Psi_i'(\tau_i) \geq -\varphi_i(\rho_i(\tau_i)) - \Psi_i'(\tau_i) = 0.$$

Then, we get $U_i(\theta_i) \geq U_i(\tau_i)$, $\forall \theta_i \in \Theta_i$. Considering $\hat{u}_i = u_i - U_i(\tau_i)$ we obtain $[x_i(\cdot;\tau_i),\hat{u}_i]$ which is feasible for the program EBP_i, with an objective function value equal to

$$T_i[x_i(\cdot;\tau_i)] - (1-\lambda)E\Psi_i - \lambda\Psi_i(\bar{\theta}_i) - \lambda\hat{u}_i =$$

$$E\{ W_i[x_i(\theta;\tau_i)] - \beta_i(\theta_i;\tau_i)\varphi_i[x_i(\theta;\tau_i)] \} - (1-\lambda)E\Psi_i - \lambda\Psi_i(\tau_i) - u_i$$

which is strictly greater than the one of $x_i(\cdot)$, contradicting that we have supposed $x_i(\cdot)$ being a solution of EBP_i.

Necessarily, the function $x_i(\cdot)$ is a solution of $[P(\tau_i)]$ and it is, then, independent on types θ_{-i}, decreasing and dominant strategy implementable. □

The intuition of the above results is the following. Hypotheses of separability and type-independence allow to separate the agents' incentives: at the optimum, each agent is asked for a performance depending only on the type reported by him-self. The principal trades-off the socially optimal performance $[\bar{\phi}_i(\cdot)]$ when an agent has only the first class of incentives (he overstates his type), with the socially optimal one $[\underline{\phi}_i(\cdot)]$ when he has only the second class (he understate his type). As the Spence-Mirrlees condition still holds (an inefficient agent has to be more compensated than an efficient one, for an unitary increase in performance), that trade-off always finishes with a decreasing performance function, which is, therefore, dominant strategy implementable (it may have a flat piece where none of the two class of incentives prevails).

For example, in the frame of regulation of industrial pollution, where x_i would be a verifiable indicator of the firm i's environment cleanness ($x_i=0$ indicates the maximal contamination and $x_i=+\infty$ denotes the minimal one), the hypothesis ITSVI is justifiable. In this context, under TSD, and if each fixed cost of the technology to reduce pollution is decreasing in the marginal cost, the above results imply that there is not loss of generality if the policy of pollution regulation considers only the dominant strategy implementation. Moreover, the socially optimal pollution of each firm will depend only of his individual report and it will be constant in a subinterval of types.

Note that, under the conditions of Proposition 5, the optimal performance of programs BP or DP is obtained solving each EBP_i, whose solution can be calculated maximizing on $\tau_i \in \Theta_i$ the function:

$$\int_{\underline{\theta}_i}^{\tau_i} [\![W_i[\bar{x}_i(\theta;\tau_i)] - \beta_i(\theta;\bar{\theta}_j)\varphi_j[\bar{x}_j(\theta;\tau_j)]]\!] f_i(\theta)d\theta + \int_{\tau_i}^{\bar{\theta}_i} [\![W_i[\underline{x}_j(\theta;\tau_i)] - \beta_i(\theta;\underline{\theta}_i)\varphi_i[\underline{x}_i(\theta;\tau_i)]]\!] f_i(\theta)d\theta - \lambda\Psi_i(\tau_i)$$

where $\bar{x}_i(\cdot;\tau_i) := \max[\bar{\phi}_i(\cdot),\rho_i(\tau_i)]$, $\underline{x}_i(\cdot;\tau_i) := \min[\underline{\phi}_i(\cdot),\rho_i(\tau_i)]$.

It can be proved that, at the optimal value τ_i, the correspondent individual performance function is continuous.

4.- THE PROPERTIES OF OPTIMAL PERFORMANCES IN THE FIRST SETTING

In this section we will apply Proposition 3 in order to analyze the properties of optimal performances. We will examine three cases. In the first two cases, we will assume that the principal's virtual income depends on performances only by means of the aggregate total performance. We will suppose:

Virtual income depending on total performance (VIDTP)

$W(\mathbf{x},\boldsymbol{\theta}) = W(\Sigma\, x_i)$ where $W(\cdot)$ is C^2 and satisfies $W'(\cdot) = 0$, $W''(\cdot) < 0$.[4]

This assumption is verified, for example, in the frame of the regulation of a private good, because, in that case, it is easy to show that the virtual income is equal to $S(\Sigma\, x_i)$, where $S(\cdot)$ denotes the consumers' gross surplus.

The third case that we will examine corresponds to the regulation of a monopolist with several independent divisions (or the one of a group of firms), each one producing a different private good. In this case, we have $W(\mathbf{x},\boldsymbol{\theta}) \equiv S(\mathbf{x})$, where $S(\mathbf{x})$ represents the consumers' gross surplus of the vector of productions **x**. Therefore, condition ITVI holds.

4.1.- HOMOGENEOUS PERFORMANCES AND LINEAR DEPENDENCE

In this subsection we assume VIDTP and

Linear dependence (LD)

$v_i(x,\theta) = \theta\, x + \Psi_i(\theta)$, $x \geq 0$, $\Psi_i(\cdot) \geq 0$, $\Psi_i'(\cdot) \geq 0$, $\theta \in \Theta_i$, $\forall i$.

We can think of regulation of a private good produced by an oligopoly in which every firm has a hidden constant marginal cost and a fixed cost increasing in his marginal cost.

Under VIDTP, LD and complete information, it is optimal for the principal that only the agents with the lowest marginal cost have a positive performance. The following proposition prove that this conclusion can be extended to the case of incomplete information. If previous conditions hold, the principal may prefer to require a performance which is only positive for the most virtually efficient agents (generically only one).

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[4]Note that $W(\cdot,\cdot)$ may depend on λ. We have suppresed the variable simplifying the notation.

Proposition 6 Given assumptions *SU-*, MHR-, LD, VIDTP, suppose that moreover the condition $W'(+\infty) \leq \alpha(\cdot) \leq W'(0+)$ holds, with

$$\alpha(\theta) := \min[\phi_k(\theta_k) / k=1,\ldots,n], \qquad \phi_i(\theta_i) := \theta_i + \lambda \frac{F_i(\theta_i)}{f_i(\theta_i)},\ \forall i$$

Then, the optimal performance function $x^*(\cdot)$ exists, is unique and satisfies:

(a) $x^*_i(\theta) = 0$ if $\phi_i(\theta_i) > \alpha(\theta)$,

(b) if $x^*_i(\theta) > 0$ then $\phi_i(\theta_i) = \alpha(\theta)$,

(c) $\Sigma\, x^*_i(\cdot) = D(\alpha(\cdot))$ where $D := (W')^{-1}$

(d) The principal's optimal expected utility is:

$$W(D(\overline{\alpha})) - \overline{\alpha} D(\overline{\alpha}) + \int_{\underline{\alpha}}^{\overline{\alpha}} D(\alpha) G(\alpha) d\alpha - \sum E\zeta_i - \lambda \sum u_i$$

where $\zeta_i(\cdot) \equiv (1-\lambda)\Psi_i(\cdot) + \lambda\Psi_i(\overline{\theta_i})$ $\forall i$, $G(\cdot)$ is the distribution function of $\alpha(\cdot)$ induced by distributions $\{F_i(\cdot) / i=1,\ldots,n\}$ and $[\underline{\alpha},\overline{\alpha}]$ is its support.

Proof: According to definition of $\alpha(\cdot)$, we have $W(\Sigma\, x_i) - \Sigma\, \phi_i(\theta_i) x_i \leq W(\Sigma\, x_i) - \alpha(\theta)\, \Sigma\, x_i$, and the function $z \rightarrow W(z) - \alpha(\theta) z$ has an absolute maximum at $z = D(\alpha(\theta))$ by VIDTP. Therefore, there is $x^*(\theta) \in \mathbb{R}^n_+$ maximizing $\Omega(x,\theta)$ on $\mathbb{R}^n_+$ for each value of θ (and λ). Moreover we have:

$$\partial_{x_i} \Omega(x,\theta) = W'(\sum x_k) - \phi_i(\theta_i). \tag{7}$$

Consider $I(\theta) := \{i / x^*_i(\theta) > 0\}$. From (7), there is $\xi(\theta)$ such that $\forall i \in I(\theta)$ $\phi_i(\theta_i) = \xi(\theta)$. In addition, the function $\xi(\cdot)$ must verify the equality $W'(\Sigma x^*_k(\cdot)) = \xi(\cdot)$ because $i \in I(\theta)$ implies

$$\partial_{x_i} \Omega(x^*(\theta),\theta) = 0.$$

If there was k such that $\phi_k(\theta_k) < \xi(\theta)$, we should have $k \notin I(\theta)$. But then, the relations $x^*_k(\theta) = 0$ and $\partial_{x_k} \Omega(x^*(\theta),\theta) = \xi(\theta) - \phi_k(\theta_k) > 0$ would hold contradicting that $x^*(\cdot)$ is optimal. Therefore, we have $\xi(\theta) = \alpha(\theta)$ and the properties (a), (b), (c) hold.

Denoting the distribution function of $\alpha(\cdot)$ as $G(\cdot)$, the principal's expected utility is:

$$E[\, W(D(\alpha(\theta))) - \alpha(\theta) D(\alpha(\theta))\,] - \sum E\zeta_i - \lambda \sum u_i = \int_{\underline{\alpha}}^{\overline{\alpha}} [\, W(D(\alpha)) - \alpha D(\alpha)\,] dG(\alpha) - \sum \zeta_i - \lambda \sum u_i$$

and if we integrate it by parts, we obtain (d). □

Proposition 6 has a straightforward interpretation. Under the above assumptions, from Proposition 3, there is not loss of generality if the principal only considers mechanisms for which each agent reports his true type as a dominant strategy. Therefore, the principal has to take into account essentially the agents' efficiencies corrected with the social cost of hidden information, i.e. the *agents' virtual marginal costs* $\phi_i(\cdot)$ $\forall i$ defined in Proposition 6. So, $\alpha(\cdot)$ represents the smallest virtual marginal cost and the principal will elicit a positive performance only from the agents with such a virtual marginal cost $\alpha(\cdot)$.

In the context of regulation of a private good, the conclusion (c) of Proposition 6 indicates that, the socially optimal price is equal to the smallest virtual marginal cost. The condition $W'(+\infty) \leq \alpha(\cdot) \leq W'(0+)$ makes sure that the equality between the principal's virtual marginal income and the least virtual marginal cost is feasible.

Remark 1. Note that, under the conditions of Proposition 6, the optimal performance $x^*_i(\theta)$ of an agent i with a virtual marginal cost equal to $\alpha(\theta)$ may be defined of several ways if there are several agents with such a minimal virtual marginal cost. In this case, for any selection of values $x^*_k(\theta)$; $k \in N(\theta) := \text{argmin}[\phi_j(\theta_j) \,/\, j=1,\ldots,n]$ satisfying the equality (c) of Proposition 6, the agent i's optimal performance $x^*_i(\theta_{-i},\theta_i)$ is decreasing in θ_i because we have:

$$x^*_i(\theta_{-i},\theta_i) = D(\phi_i(\theta_i)) \text{ if } \theta_i < \phi_i^{-1}[\alpha_{-i}(\theta_{-i})], \qquad x^*_i(\theta_{-i},\theta_i) = 0 \text{ if } \theta_i > \phi_i^{-1}[\alpha_{-i}(\theta_{-i})]$$

with $\alpha_{-i}(\theta_{-i}) := \min[\, \phi_j(\theta_j) \,/\, j=1,\ldots,n, j\neq i \,]$, and $D'(\cdot) = 1 \,/\, W''(D(\cdot)) < 0$.

In the linear case examined in Proposition 6 the optimal performance function $x^*(\cdot)$, although it is discontinuous, satisfies some of the properties correspondent to the basic one-agent adverse selection model [which appears, for instance, in Baron (1989)]. The aggregate total performance is decreasing (in any type) and each agent is asked for an optimal performance decreasing in his type.

In order to compare the optimal performance of incomplete information with the one of complete information, note that the complete information solution, which will be called $x^{CI}(\cdot)$, verifies the properties (a), (b), (c) of Proposition 6 for the functions $\phi_i(\cdot)$ and $\alpha(\cdot)$ obtained with $\lambda=0$ (recall that, then, programs VP and CIP coincide). Thus, we have:

$$x^{CI}_i(\theta_{-i},\theta_i) = D(\theta_i) \text{ if } \theta_i < \alpha^{CI}_{-i}(\theta_{-i}), \qquad x^*_i(\theta_{-i},\theta_i) = 0 \text{ if } \theta_i > \alpha^{CI}_{-i}(\theta_{-i})$$

where $\alpha^{CI}_{-i}(\theta_{-i}) := \min[\theta_j \,/\, j=1,\ldots,n, j\neq i]$, and also $\Sigma\, x^{CI}_i(\theta) = D(\alpha^{CI}(\theta))$ with $\alpha^{CI}(\theta) := \min[\theta_j \,/\, j=1,\ldots,n]$.

Like in the one-agent basic adverse selection model, the aggregate total performance of complete information is grater than the one of incomplete information. Nevertheless, at the individual level, this inequality may be reversed. We can show easily that the only case in which it is reversed corresponds to a vector $\boldsymbol{\theta}$ such that, for some agent i, we have

$$\alpha^{CI}_{-i}(\theta_{-i}) < \theta_i \leq \phi_i(\theta_i) < \alpha_{-i}(\theta_{-i})$$

because, in this case, $x^{CI}_i(\boldsymbol{\theta}) = 0 < D(\phi_i(\theta_i)) = x^*_i(\boldsymbol{\theta})$.

Note that the above inequalities cannot hold if the distributions of types are equal. Therefore, we can conclude that, under the assumptions of Proposition 6 and if types are identically distributed, each agent is required to perform under the level of complete information (in order to reduce the informational rent of the most efficient agent's type, like in the basic one-agent adverse selection model).

Nevertheless, when the distributions of types are different, it may happen that an agent, which does not perform under complete information, is asked for a positive performance under incomplete information. The explanation is simple. The supposed assumptions imply that only the agents with the smallest marginal cost or the smallest virtual marginal cost can be required to perform under, respectively, complete information and incomplete information. But the "orders" over agents induced by marginal costs and virtual marginal costs may be different if the distributions of types are distinct.

To analyze the relationship between the principal's ex ante optimal expected utility and the agents' number, under VIDTP and LD, we consider the symmetrical case in which the distributions of types, the agents' fixed costs and reservation utilities coincide:

Symmetrical agents (SA)

$f_i(\cdot) \equiv f(\cdot),\ \ F_i(\cdot) \equiv F(\cdot),\ \ \Psi_i(\cdot) \equiv \Psi(\cdot),\ \ u_i = u,\ \ \forall i.$

In this symmetrical case, an augmentation of the agents' number yields an improvement (decrease) of α according to the first order stochastic dominance. Under the assumptions of Proposition 6, the principal may prefer ex ante to hire several agents.

Corollary 1 Under the assumptions SU-, LD, MHR-, VIDTP, SA and supposing $W'(+\infty) \leq \alpha(\boldsymbol{\theta}) \leq W'(0+)\ \ \forall\boldsymbol{\theta}$, we have that:

(a) The principal's expected utility hiring n agents is:

$$U(n)=W(D(\bar{\alpha}))-\bar{\alpha}D(\bar{\alpha})+\int_{\underline{\alpha}}^{\bar{\alpha}}D(\alpha)[1-[1-F(\phi^{-1}(\alpha))]^{n}]d\alpha-n(E\zeta+\lambda u)$$

(b) If $E\zeta+\lambda u \geq \bar{u}:=\int_{\underline{\alpha}}^{\bar{\alpha}}D(\alpha)[1-F(\phi^{-1}(\alpha))]\log[[1-F(\phi(\alpha))]^{-1}]d\alpha>0$ the principal prefers to hire only one agent. But if $E\zeta+\lambda u < \bar{u}$, he prefers ex ante to hire more than one agent, i. e. $U'(1) > 0$.

Proof: From AS, MHR-, the values of $\alpha(\theta):=\min[\ \phi_k(\theta_k)\ /\ k=1,\ldots,n\ ]$, as a random variable, are placed between $\underline{\alpha}:=\underline{\theta}+\lambda F(\underline{\theta})/f(\underline{\theta})$ and $\bar{\alpha}:=\bar{\theta}+\lambda F(\bar{\theta})/f(\bar{\theta})$ with a distribution function equal to

$$G_n(\alpha):=1-[1-F(\phi^{-1}(\alpha))]^n,\ \text{where } \phi(y):=y+\lambda F(y)/f(y);\ \underline{\theta}\leq y\leq\bar{\theta}.$$

From Proposition 6, the optimal expected utility of a principal hiring n agents is the one in (a). Then, we have that $\partial_n U(n) = V(n) - (E\zeta+\lambda u)$, where

$$V(n):=\int_{\underline{\alpha}}^{\bar{\alpha}}D(\alpha)[1-F(\phi(\alpha))]^{n}\log[[1-F(\phi^{-1}(\alpha))]^{-1}]d\alpha>0,$$

and we can verify that $\partial_{nn}W < 0$, $V(1) = \bar{u}$, $V(+\infty) = 0$. Therefore, (b) holds. □

Let us interpret the above result in the frame of regulation of a private good produced by an oligopoly. If production technologies have a constant marginal cost and a fixed cost, which are increasing functions of a hidden parameter, and if such a parameter is independent and identically distributed between the regulated firms, despite the adverse selection problem, an ex ante limit in the regulated firms' number, of an optimal regulatory policy, will be decreasing in the firm's expected fixed cost.

Remark 2. Changing SU-, MHR- by SU+, MHR+, we should obtain similar results to Proposition 6 and Corollary 1. In that case, we should have $\alpha(\theta):=\max[\ \phi_k(\theta_k)\ /\ k=1,\ldots,n\ ]$ where

$$\phi_i(\theta_i):=\theta_i-\lambda\frac{1-F_i(\theta_i)}{f_i(\theta_i)}\ \forall i$$

we should demand a positive performance to the agents with $\phi_i(\theta_i) = \alpha(\theta)$, and $\Sigma\, x^*_i(\cdot) = D(-\alpha(\cdot))$ would hold. The condition for the existence of the optimal performance would be $W'(+\infty) \leq -\alpha(\cdot) \leq W'(0+)$.

Specifying the model, we can study the behavior of the optimal size of agency (optimal agents' number). As an illustration, consider a firm purchasing input to several suppliers, with a linear demand function, constant returns to scale and a uniform distribution of types. Then we have the following properties (proved in Appendix 2).

Example.- Under the assumptions of Corollary 1, suppose the symmetrical case corresponding to:

$$f(\tau)=1/\beta;\ \ 0<\tau<\beta,\ \ R_i\equiv 0,\ \ B(x,\theta)=[P(\Sigma\, x_i)-c]\ \Sigma\, x_i,$$

$$\lambda=1,\ \ c>0,\ \ P(x)=a-x,\ \ a>c>2\beta,$$

Then, we have that:

(a) The firm's expected profit hiring n suppliers is:

$$U := \frac{(a-c)^2}{4} + \frac{\beta}{n+1}\left[\frac{2\beta}{n+2} - a + c\right] - n(\psi(\overline{\theta}) + u)$$

(b) The profit critical value is $\overline{u}=\beta[9(a-c)-10\beta]/36$.

(c) The optimal number of suppliers n^* is increasing in the profitability a and decreasing in the constant production cost c and in the suppliers' reservation utility u. If $n^* \geq 2$, n^* increases with the uncertainty β, but nevertheless, the firm's optimal expected profit decreases with β.

4.2.- HOMOGENEOUS PERFORMANCE AND CONVEX DEPENDENCE

In the linear case in Subsection 4.1 (Proposition 6), when agents are symmetrical, the optimal performance function $x^*(\cdot)$, although it is discontinuous, satisfies the properties corresponding to the basic one-agent adverse selection model. This conclusion changes when the disutility of each agent is strictly convex in his performance. Although the aggregate total performance is lesser than the one of complete information, the inequality may be individually reversed for the most efficient agents when the set of contractual types is heterogeneous enough.

Proposition 7.- Suppose SU-, MHR-, AS, CS-, VIDTP, MAW, $\lambda > 0$ and

$$v_i(\cdot,\cdot) \equiv v(\cdot,\cdot) \in C^3 \ \forall i, \ \partial_{xx}v(\cdot,\cdot) > 0, \ \partial_{\theta xx}v(\cdot,\cdot) \geq 0.$$

Let $\mathbf{x}^*(\cdot)$ and $\mathbf{x}^{CI}(\cdot)$ be respectively the optimal performance functions of incomplete and complete information and assume that they are strictly positive.[5]

Then the following assertions hold:

(a) For each i, $x^*(\cdot)$, $x^{CI}(\cdot)$ are C^I, strictly decreasing in θ_i and strictly increasing in θ_k for $k \neq i$.

(b) $\theta_i < \theta_j$ implies $x^*_i(\boldsymbol{\theta}) > x^*_j(\boldsymbol{\theta})$, $x^{CI}_i(\boldsymbol{\theta}) > x^{CI}_j(\boldsymbol{\theta})$.

(c) $\Sigma\, x^*_i(\cdot)$ and $\Sigma\, x^{CI}_i(\cdot)$ are strictly decreasing in each θ_k, $\forall k$.

(d) $\Sigma\, x^{CI}_i(\cdot) \geq \Sigma\, x^*_i(\cdot)$, with strict inequality at $\boldsymbol{\theta}$

such that $\theta_k > \underline{\theta}$ for some k, and $\Sigma\, x^{CI}_i(\underline{\boldsymbol{\theta}}) = \Sigma\, x^*_i(\underline{\boldsymbol{\theta}})$ where $\underline{\boldsymbol{\theta}} = (\underline{\theta},\ldots,\underline{\theta})$.

(e) $x^{CI}_i(\boldsymbol{\theta}) < x^*_i(\boldsymbol{\theta})$, if $\boldsymbol{\theta}$ verifies $\theta_i = \underline{\theta}$, $\theta_k > \underline{\theta}$ for some $k \neq i$.

Proof: From Proposition 3, $[\mathbf{t}(\cdot),\mathbf{x}(\cdot)]$ is an optimal mechanism of program BP if and only if the function $\mathbf{x}(\cdot)$is a solution of program VP and then, by VIDTP, $\mathbf{x}^*(\cdot)$ has to pointwise maximize the function $\Omega(\mathbf{x},\boldsymbol{\theta}) := W(\Sigma\, x_i) - \Sigma\, \gamma(x_i,\theta_i)$, where

$$\gamma(x,\theta) := v(x,\theta) + \lambda \frac{F(\theta)}{f(\theta)} \partial_\theta v(x,\theta) + \lambda u.$$

Because $\mathbf{x}^*(\cdot)$ is interior, we have:

$$W'(\Sigma\, x^*_k(\boldsymbol{\theta})) = \partial_x\gamma(x^*_i(\boldsymbol{\theta}),\theta_i) \quad \forall \boldsymbol{\theta},\ \forall i. \tag{8}$$

Let $\xi^*(\cdot)$ be such that $\xi^*(\boldsymbol{\theta}) = W'(\Sigma\, x^*_k(\boldsymbol{\theta}))\ \forall\boldsymbol{\theta}$ and denote $D \equiv (W')^{-1}$.
The function $\partial_x\gamma(\cdot,\theta)$ is C^1 for each θ and moreover $\partial_{xx}\gamma(\cdot,\cdot) > 0$ from the assumptions on $v(\cdot,\cdot)$. Applying the inverse function theorem, there is $\rho(\cdot,\cdot)$ of class C^1 such that $\rho(\cdot,\theta) \equiv [\partial_x\gamma(\cdot,\theta)]^{-1}$ and it satisfies $\partial_z\rho(z,\theta) > 0$, $\partial_\theta\rho(z,\theta) < 0$. Because, by (8), we have:

$$x^*_i(\boldsymbol{\theta}) = \rho(\xi^*(\boldsymbol{\theta}),\theta_i), \quad \forall\boldsymbol{\theta}, \quad \forall i, \tag{9}$$

the value $\xi^*(\boldsymbol{\theta})$ is necessarily a solution of $0 = H(z,\boldsymbol{\theta})$ for each $\boldsymbol{\theta}$, where

[5] They exist and are continuous if, for instance, $v(x,\theta) = \theta\varphi(x)$, $\varphi \in C^2$, $\varphi'' > 0$, $\varphi'(0) = 0$, $\varphi'(+\infty) = +\infty$, $\underline{\theta} > 0$, $W'(0) > 0$.

$$H(z,\theta) := \Sigma\, \rho(z,\theta_i) - D(z) \tag{10}$$

is a C^1 function and verifies $\partial_z H(\cdot,\cdot) > 0$. From the implicit function theorem, $\xi^*(\cdot)$ is of class C^1.

The optimal performance function x $(\cdot)$ of complete information has to be a solution of program CIP and using a similar argument, it must verify:

$$x^{CI}_i(\theta) = \rho^{CI}(\xi^{CI}(\theta),\theta_i), \quad \forall\theta,\ \forall i, \tag{11}$$

letting $\rho^{CI}(\cdot,\theta)$ be the inverse function of $\partial_x v(\cdot,\theta)$ for each θ, and letting $\xi^{CI}(\theta)$ be a solution of $0 = H^{CI}(z,\theta)$, where

$$H^{CI}(z,\theta) := \Sigma\, \rho^{CI}(z,\theta_i) - D(z). \tag{12}$$

The function $\rho^{CI}(\cdot,\cdot)$ is also of class C^1 and satisfies $\partial_z\rho^{CI}(z,\theta) > 0$, $\partial_\theta\rho^{CI}(z,\theta) < 0$. Therefore $\xi^{CI}(\cdot)$ is continuously differentiable.

The performances $x^*_i(\cdot)$ and $x^{CI}_i(\cdot)$ are decreasing in θ_i accordingly to Proposition 2 [$x^*(\cdot)$ and $x^{CI}(\cdot)$ are dominant strategy implementable]. We will show later that the relations

$$\partial_{\theta_k}\xi^*(\theta) > 0,\ \partial_{\theta_k}\xi^{CI}(\theta) > 0$$

hold and then, will have, from (9) and (11), the inequalities $\partial_{\theta_k}x_i^*(\theta) > 0$, $\partial_{\theta_k}x_i^{CI}(\theta) > 0$ for $k \neq i$

because $\partial_\theta\, \rho < 0$ holds. Therefore we will obtain part (a).

As the relations $\partial_\theta\rho(\cdot,\cdot) < 0$ and $\partial_\theta\rho^{CI}(\cdot,\cdot) < 0$ hold, for θ such that $\theta_i < \theta_j$ we have $x^*_i(\theta) = \rho(\xi^*(\theta),\theta_i) > \rho(\xi^*(\theta),\theta_j) = x^*_j(\theta)$ and, similarly, we get $x^{CI}_i(\theta) > x^{CI}_j(\theta)$. Then, we have part (b).

From the properties of ρ, ρ^{CI}, D, and differentiating the expressions $H(\xi^*(\theta),\theta) = 0$, $H^{CI}(\xi^{CI}(\theta),\theta) = 0$, we obtain $\partial_{\theta_k}x_i^*(\theta) > 0$, $\partial_{\theta_k}x_i^{CI}(\theta) > 0$. This implies that part (c) holds because

$D' < 0$.

By the definition of $\gamma(\cdot,\cdot)$ and assumption CS-, the inequality $\partial_x\gamma(\cdot,\theta) \geq \partial_x v(\cdot,\theta)$ holds, and moreover strictly if $\theta > \underline{\theta}$. Therefore, we have $\rho(\cdot,\theta) \leq \rho^{CI}(\cdot,\theta)$ with strict inequality if $\theta > \underline{\theta}$. Then, form (10) and (12) we get $H^{CI}(z,\theta) \geq H(z,\theta)$ $\forall z$ $\forall\theta$ and besides strictly if $\theta_k > \underline{\theta}$ for some k. This implies $\xi^{CI}(\theta) \leq \xi^*(\theta)$ [strictly if there is k such that $\theta_k > \underline{\theta}$], because $H(\cdot,\theta)$ and $H^{CI}(\cdot,\theta)$ are strictly increasing, and (d) holds. It is clear that $\Sigma\, x^{CI}_i(\underline{\theta}) = \Sigma\, x^*_i(\underline{\theta})$, because $\gamma(\cdot,\underline{\theta}) \equiv v(\cdot,\underline{\theta})$.

Finally, let θ be such that $\theta_i = \underline{\theta}$, $\theta_k > \underline{\theta}$ for some k $\neq$ i. Then we have

$$x^{CI}_i(\theta) = \rho^{CI}(\xi^{CI}(\theta),\underline{\theta}) = \rho(\xi^{CI}(\theta),\underline{\theta}) < \rho(\xi^*(\theta),\underline{\theta}) = x^*_i(\theta),$$

because $\xi^{CI}(\theta) < \xi^*(\theta)$, and the part (e) holds. □

Under the conditions of Proposition 7, if the set of contractual types is completely homogeneous, i. e., θ is such that $\theta_i = \theta \in]\underline{\theta}, \overline{\theta}]$ $\forall$i, we will have $x^{CI}_i(\theta) = D(\xi^{CI}(\theta))/n > D(\xi^*(\theta))/n = x^*_i(\theta)$ $\forall$i. By continuity, if the set of types was not heterogeneous enough, we should have $x^{CI}_i(\theta) \geq x^*_i(\theta)$ $\forall$i. However, Proposition 7(e) indicates that the optimal mechanism may ask very efficient agents for an individual performance greater than the one of complete information, if the set of contractual types is heterogeneous enough.

The intuition, under SU-, is the following. Similarly to the basic one-agent adverse selection model, each agent in our model prefers to overstate his type when the principal offers the optimal mechanism of complete information. In order to see this, note that the agent i's utility of type θ_i, announcing $\tau_i > \theta_i$, is:

$$t^{CI}_i(\theta_{-i},\tau_i) + V_i(x^{CI}(\theta_{-i},\tau_i),\theta_i) = v(x^{CI}_i(\theta_{-i},\tau_i),\tau_i) - v(x^{CI}_i(\theta_{-i},\tau_i),\theta_i) + u$$

which is grater than u when $\tau_i > \theta_i$, because condition MAW holds.

Therefore, in order to decrease the agent i's informational rent:

$$R_i(\theta,x(\cdot)) = \int_{\theta_i}^{\overline{\theta}} \partial_\theta v(x_i(\theta_{-i},\tau),\tau)d\tau,$$

the principal has to reduce the required performance, because we suppose the Spence-Mirrlees' condition $\partial_{x\theta}v > 0$.

But, on the other hand, we have $\partial_{\theta_k} x_i^{CI}(\theta) > 0$ (if the agent k's type increases, the agent i's efficiency rises relating to the one of the agent k and the mechanism asks the agent i for more performance). This implies that the agent i's informational rent $R_i(\theta_{-k},\tau_k,x^{CI}(\cdot))$ decreases when the agent k is prevented from announcing the type $\tau_k > \theta_k$. Then, the principal "saves" informational rent of an agent inducing the truthful revelation of types of others agents.

As this diminution in rent increases with the agent i's efficiency, i.e.,

$$\partial_{\theta_i}[\![R_i(\theta_{-k},\tau_k,x^{CI}(\cdot)) - R_i(\theta_{-k},\theta_k,x^{CI}(\cdot))]\!] < 0, \quad \tau_k > \theta_k,$$

the principal may prefer to increase the required performance, in relation to the one of complete information, for very efficient agents if the set of contractual types is heterogeneous enough.

Remark 3. Changing SU-, MHR- by SU+, MHR+ in Proposition 7, we obtain an analogous result. For this we should suppose $v(\cdot,\cdot)$ satisfying

$$\partial_{xx}v(\cdot,\cdot) < 0, \quad \partial_{xx}v(\cdot,\cdot) - \lambda\frac{1-F_i(\cdot)}{f_i(\cdot)}\partial_{\theta xx}v(\cdot,\cdot) < 0.$$ [6]

The functions $x^*_i(\cdot)$ and $x^{CI}_i(\cdot)$ would be s. increasing in θ_i and s. decreasing in θ_k with $k \neq i$. Now $\theta_i < \theta_j$ would imply $x^*_i(\theta) < x^*_j(\theta)$, $x^{CI}_i(\theta) < x^{CI}_j(\theta)$ and the aggregate total performance would be s. increasing in each type. We should have also $\Sigma\, x^{CI}_i(\cdot) \geq \Sigma\, x^*_i(\cdot)$, but now strictly at θ such that $\theta_k < \bar{\theta}$ for some k, and $\Sigma\, x^{CI}_i(\theta) = \Sigma\, x^*_i(\theta)$ if $\theta = (\bar{\theta},\ldots,\bar{\theta})$. Moreover, given i, $x^{CI}_i(\theta) < x^*_i(\theta)$ would hold if θ satisfies $\theta_i = \bar{\theta}$, $\theta_k < \bar{\theta}$ for some $k \neq i$.

In this Convex Case the principal may also prefer ex ante to hire several agents.

Corollary 2 Under the assumptions of Proposition 7, suppose the case:

$$v(x,\theta) = \theta x^2/2 + \Psi(\theta), \quad \underline{\theta} > 0, \quad \Psi(\cdot) \geq 0, \quad \Psi'(\cdot) \geq 0, \quad W'(0) > 0.$$

Then, the optimal performance functions $x^*(\cdot)$ and $x^{CI}(\cdot)$, of incomplete and complete information respectively, exist and are interior.

If the the value $(1-\lambda)E\Psi + \lambda\Psi(\bar{\theta}) + \lambda u$ is low enough, the principal prefers ex ante to hire more than one agent.

Proof: From Proposition 3 and assuming the considered case, if the optimal performance function of incomplete information $x^*(\cdot)$ exists, it will pointwise maximize the function:

$$\Omega(x,\theta) = W(\Sigma\, x_i) - \Sigma\, \phi(\theta_i)x_i^2/2 - n(E\zeta + \lambda u) \tag{13}$$

where $\phi(\theta) = \theta + \lambda\frac{F(\theta)}{f(\theta)}$, $\zeta(\cdot) := (1-\lambda)\Psi(\cdot) + \lambda\Psi(\bar{\theta})$. As $\phi(\cdot) > 0$ and we assume $W'(0) > 0$,

[6] This holds if, for instance, $v(x,\theta) = \theta\varphi(x)$, $\varphi \in C^2$, $\varphi'' < 0$, $\underline{\theta} > 0$, with $\phi(\theta) := \theta - \lambda(1-F(\theta))/f(\theta) \geq 0$, $\forall\theta$. Note that $\gamma(x,\theta) = \phi(\theta)\varphi(x)$ in this case. For an uniform distribution of types, we have $\phi(\cdot) > 0$ if $\underline{\theta}/\bar{\theta} > \lambda/(1+\lambda)$.

$W''(\cdot) < 0$, every maximum of $\Omega(\cdot,\theta)$, on $\mathbb{R}^n_+$, has to be interior. Therefore, if $x^*(\cdot)$ exists, it will be interior.

Using a similar argument for $\Omega^{CI}(x,\theta) = W(\Sigma x_i) - \Sigma \theta_i x_i^2/2 - n(E\Psi+\lambda u)$, we have that if the optimal performance function of complete information $x^{CI}(\cdot)$ exists, it will be interior.

By Proposition 7, if $x^*(\cdot)$ and $x^{CI}(\cdot)$ exist, the values $\xi^*(\theta)=W'(\Sigma x^*_k(\theta))$ and $\xi^{CI}(\theta)=W'(\Sigma x^{CI}_k(\theta))$ will be solutions respectively of $H(z,\theta)=0$ and $H^{CI}(z,\theta)=0$, for each θ, with $H(z,\theta)=zs(\theta)-D(z)$, $H^{CI}(z,\theta)=zs^{CI}(\theta)-D(z)$ where

$$s(\theta):=\sum\frac{1}{\phi_i(\theta_i)},\quad s^{CI}(\theta):=\sum\frac{1}{\theta_i}. \tag{14}$$

Since $s(\cdot) > 0$, we have that $H(W'(0),\theta) = W'(0)s(\theta) > 0$. On the other hand, if $W'(+\infty)\leq 0$ the inequality $H(0,\theta) = -D(0) < 0$ holds; and if $W'(+\infty) > 0$ we get $H(W'(+\infty),\theta) = -\infty$. This implies that for any θ there is a unique $z > 0$ such that $H(z,\theta) = 0$.

Given $\xi^*(\cdot) > 0$ such that $H(\xi^*(\theta),\theta)=0$ for each θ, the function $x^*(\cdot)$ defined by $x^*_i(\theta) = \xi^*(\theta)/\phi(\theta_i)$ $\forall i$ is the only pointwise maximum of $\Omega(\cdot,\cdot)$ and it is, then, the optimal performance function of incomplete information.

A similar argument shows that $x^{CI}(\cdot)$ exists, is unique and interior.

Note that from the implicit function theorem, we have $\xi^*(\theta)=\xi(s(\theta))$ $\forall\theta$, where $\xi(\cdot) > 0$ is a function of class C^1 on $s>0$ satisfying:

$$s\,\xi(s) = D(\xi(s)) \quad \forall s>0. \tag{15}$$

Moreover, we obtain $\xi'(s) = \xi(s)/[D'(\xi(s))-s] < 0$ deriving (15).

Substituting $x^*(\cdot)$ into (13) and by (14) and (15), the principal's expected utility can be rewritten as

$$E[\Omega(x^*(\theta),\theta)] = E[J(s(\theta))] - n(E\zeta+\lambda u) \tag{16}$$

where $$J(s):= W[D(\xi(s))] - \xi(s)\,D(\xi(s))/2.$$

If we derive $J(\cdot)$, we can use (15) to obtain: $J'(s) = [\xi(s)]^2/2 > 0$.

Let F(s;n) be the distribution function of $s(\cdot)$ induced by the random vector $\boldsymbol{\theta}$ corresponding to n agents. Consider n' > n. We have:

$$F(s;n') = P[\![\sum_{i=1}^{n} \frac{1}{\phi(\theta_i)} + \sum_{j=n+1}^{n'} \frac{1}{\phi(\theta_j)} \leq s]\!] \leq P[\![\sum_{i=1}^{n} \frac{1}{\phi(\theta_i)} \leq s]\!] = F(s;n)$$

since $\phi(\cdot) > 0$. On the other hand, we have that $1/\phi(\cdot) \in [1/\phi(\bar{\theta}), 1/\underline{\theta}]$ and, therefore, for each $s \in\ ]n/\phi(\bar{\theta}), n'/\phi(\bar{\theta})[\ \cup\ ]n/\underline{\theta}, n'/\underline{\theta}[$ the inequality F(s;n') < F(s;n) holds.

The latter points out that an increase of agents' number n produces an improvement of F($\cdot$;$\cdot$) according to the first order stochastic dominance.

As J'($\cdot$) > 0, the function E[J($s(\boldsymbol{\theta})$)] has to be strictly increasing in the agents' number. Finally, from (16), we get that if $E\zeta+\lambda u$ is lower enough, the principal prefers ex ante to hire more than one agent. □

The intuition is similar to the one of linear case. Now, the principal's expected utility depends increasingly on $s(\boldsymbol{\theta})$, a value which measures the total marginal utility that, for the principal, represents contracting n agents of types $\boldsymbol{\theta}$, because $\phi(\theta)$ is the virtual marginal disutility of an agent with type θ. The distribution of s($\cdot$) increases with the agents' number n, according to the first order stochastic dominance, and therefore, the principal's expected utility increases with n if the reservation utility is low.

Let us interpret the above results in the frame of the firm which purchases an input to several exclusive suppliers, each one of them possessing a strictly increasing marginal production cost which depends on his hidden type according to Proposition 7. In this case, as $\lambda=1$ and $R_i \equiv 0\ \forall i$ we have $W=B=[P(\Sigma x_i)-c]\Sigma x_i$, if we assume that the firm transforms an unity of input to an unity of output with a constant marginal cost equal to c. On the one hand, if types are independents, the firm may ex ante prefer to contract with several suppliers because the ex ante distribution of the "total" virtual marginal costs of engaged suppliers improves with the number of them. On the other hand, when the firm has several suppliers, unlike the case with only one, the firm may require a very efficient supplier to produce a quantity of input greater than the one of complete information. The reason is that a decrease in a supplier's input quantity, to reduce his informational rent, makes an increase of the others supplier's quantities more desirable for the firm and, therefore, the firm has to make a trade-off between the several possible reductions of supplier's quantities with regard to the ones of complete information.

4.3.- HETEROGENEOUS PERFORMANCE: THE CASE OF REGULATION

In this subsection we will apply the results of Section 2 in order to analyze the properties of the optimal regulatory mechanism in the bellow setting of regulation of several private goods. Consider the regulation of a monopoly with several independent divisions (or the one of a group of firms), each one producing a different good. Let x_i be the quantity of good i produced by the division (or firm) $i=1,\dots,n$.

The production cost of the division i is $C_i(x_i,\theta_i)$ and we assume the hypotheses of Proposition 3 implying the equivalence between the Bayesian implementation and the dominant strategy implementation. Therefore, we will suppose that $F_i(\cdot)/f_i(\cdot)$ is increasing, $\partial_{x_i\theta_i}C_i > 0$, $\partial_{x_i\theta_i}C_i$ is increasing in θ_i and $\partial_{\theta_i}C_i \geq 0$, for any i.

Let $S(x)$ denote the consumers' gross surplus. It depends on the vector $x=(x_1,\dots,x_n)$ of productions of goods. Therefore the (linear) price is $P_i(x)=\partial_{x_i}S(x)$. The regulator's objective function is

$$S(x) - \Sigma P_i(x)x_i - \Sigma s_i + (1-\lambda) \Sigma [s_i + P_i(x)x_i - C_i(x_i,\theta_i)]$$

and we can verify that, according to our formulation, we have:

$$B(x,\theta) = S(x) - \lambda \Sigma P_i(x)x_i - (1-\lambda) \Sigma C_i(x_i,\theta_i)$$

with a virtual income equal to $W(x,\theta) = S(x)$.

In order to obtain intuitions about optimal productions, we assume the following quadratic frame:

$$S(x) = a\,\Sigma x_i + \frac{(d-1)}{2}\,\Sigma x_i^2 - \frac{d}{2}[\Sigma x_i]^2 + b$$

where $d \in]-1/(n-1), 1]$, $a>0$, and

$$C_i(x_i,\theta_i) = \theta_i x_i + \Psi_i(\theta_i),\ \ \theta_i \in \Theta_i,\ \ x_i \geq 0,\ \ \underline{\theta} > 0,\ \ \Psi_i(\cdot) \geq 0,\ \ \Psi_i'(\cdot) \geq 0\ \forall i.$$

Thus, the price of good i is

$$P_i(x) = a - x_i - d \sum_{j=1, j\neq i}^{n} x_j.$$

Note that, for $d<0$ goods are complements and for $d>0$ goods are substitutes.

Now, conclusions change according to the class of goods. When goods are complements, any optimal production is lower than the one of complete information. But when goods are substitutes, this inequality may be reversed for the most efficient divisions (or firms) if there are another ones which are not so efficient.

Proposition 8 If $a>0$ is high enough, in the above quadratic case of regulation, the optimal prices and productions of incomplete information $x^*(\cdot)$, $P^*(\cdot)$ and the ones of complete information $x^{CI}(\cdot)$, $P^{CI}(\cdot)$ verify the following properties:

(a) $\partial_{\theta_i} x_i^* < 0, \quad \partial_{\theta_i} x_i^{CI} < 0, \quad \forall i.$

(b) If goods are substitutes ($d>0$): $\partial_{\theta_j} x_i^* > 0, \quad \partial_{\theta_j} x_i^{CI} > 0, \quad \forall j \neq i.$

If goods are complements ($d<0$): $\partial_{\theta_j} x_i^* < 0, \quad \partial_{\theta_j} x_i^{CI} < 0, \quad \forall j \neq i.$

(c) When goods are substitutes ($d>0$), given θ such that $\theta_i = \underline{\theta}_i$, $\theta_j > \underline{\theta}_j$ for some $j \neq i$, we have that $x^{CI}_i(\theta) < x^*_i(\theta)$.
When goods are complements ($d<0$), we have $x^{CI}_i(\theta) \geq x^*_i(\theta)$ $\forall\theta$, with strict inequality if $\theta \neq (\underline{\theta}_1, \underline{\theta}_2, \ldots, \underline{\theta}_n)$.

(d) $P^{CI}_i(\theta) = \theta_i$, $P^*_i(\theta) = \theta_i + \lambda F_i(\theta_i)/f_i(\theta_i)$, $\forall i$.

Proof: Hypotheses imply that, applying Proposition 3, the vector of optimal productions $x^*(\cdot)$ of incomplete information is obtained by pointwise maximizing the function

$$\Omega(x,\theta) = S(x) - \Sigma\, \phi_i(\theta_i)x_i - \Sigma\, E\zeta_i, \quad \text{where } \zeta_i(\cdot) := (1-\lambda)\Psi_i(\cdot) + \lambda\Psi_i(\overline{\theta}_i) \quad \forall i$$

and the virtual marginal production cost of division (or firm) i is $\phi_i(\theta_i) := \theta_i + \lambda F_i(\theta_i)/f_i(\theta_i)$.

As $\partial_{x_i x_i}\Omega = -1$, $\partial_{x_i x_j}\Omega = -d$, $j \neq i$, the matrix M of second-derivatives of Ω with respect the variables $(x_1, \ldots, x_n)$ satisfies $x\, M\, x = (d-1)\, [\, \Sigma\, x_i^2 - [d/(d-1)]x^2\,] = (d-1)\, \Sigma\, (x_i - \alpha x)^2 < 0, \ \forall x$

where $x := \Sigma x_i$ and $\alpha = \frac{1}{n}[\![\, 1 + [1 + \frac{nd}{(1-d)}]^{1/2}\,]\!] > 1$ because $nd/(1-d) > -1$

when $-1/(n-1) < d < 1$.

Therefore, the matrix M is negative definite and any x verifying $P_i(x) = \phi_i(\theta_i)$ $\forall i$ is a absolute maximum of the function $\Omega(\cdot, \theta)$.

Adding the expressions $a - x_i - d \sum_{j=1, j\neq i}^{n} x_j = \phi_i(\theta_i)\ \forall i,$ we obtain

$$\sum x_i^*(\theta) = \frac{na - \sum \phi_j(\theta_j)}{1+(n-1)d}$$

$$x_i^*(\theta) = \frac{1}{1+(n-1)d} [\![a + [\frac{d}{1-d}] \sum_{j=, j\neq i}^{n} \phi_j(\theta_j) - [\frac{1+(n-2)d}{1-d}] \phi_i(\theta_i)]\!],\ \forall i.$$

As the complete information solution verifies the above expressions if $\phi_i(\theta_i)$ is changed by θ_i for all i, the assertions (a) and (b) hold.

The assertion (c) is satisfied also, because, for all i and all $\boldsymbol{\theta}$,

$$x_i^*(\theta) - x_i^{CI}(\theta) = \frac{\lambda}{1+(n-1)d} [\![[\frac{d}{1-d}] \sum_{j=1, j\neq i}^{n} \frac{F_j(\theta_j)}{f_j(\theta_j)} - [\frac{1+(n-2)d}{1-d}] \frac{F_i(\theta_i)}{f_i(\theta_i)}]\!].$$

Finally, it is clear that the assertion (d) holds. □

The intuition of this proposition complements the one of Proposition 7. Like above, to reduce the informational rent of a division (or firm) the regulator (principal), in principle, has to ask it for a production lower than the one of complete information.

When goods are complements, if the quantity of a good goes down, the marginal social value of another goods decreases also. Therefore, the regulator distorts productions away from first-best allocations because informational rents and marginal social values go "in the same direction".

When goods are substitutes, informational rents and marginal social values go in opposite directions: if the production of a good decreases, the marginal social value of the other goods increases. The optimal distortion, with regard to the complete information solution, has to make a trade-off among the reductions in rent of divisions. Like in the former subsection, the regulator may prefer to increase the elicited production, with regard to the one of complete information, from a very efficient division.

5.- SECOND SETTING: INDIVIDUAL PERFORMANCES BASED ON INDIVIDUAL TYPES

This section analyzes our second setting, where feasible mechanisms have each individual performance based only on the type reported by the respective agent. This class of mechanisms is justifiable under two conditions: contracts depending on performances are the only feasible, and every agent takes a hidden action, after privately observing his type, which determines univocally his performance.

There are plenty of economic situations in which the first condition holds: the remuneration of some agents (workers, employees, sellers) is based directly on performances or another indicators [see the chapters 12 and 13 of Milgrom & Roberts (1992)] (not on mechanisms). To give a more precise sense to the second condition, which is also assumed in the model with correlated types of Demski & Sappington (1984), suppose that the agent i's (verifiable) performance is $x_i = X_i(a_i, \theta_i)$, where a_i denotes the hidden action (for instance, an effort) which the agent takes after observing his type θ_i and accepting his contract. Assume that the agent i's utility function is $U_i(t_i, \mathbf{x}, a_i) = t_i + R_i(\mathbf{x}) - \Psi_i(a_i)$, where $\Psi_i(\cdot)$ denotes the disutility of his action. Given $a_i = \varphi_i(x_i, \theta_i)$ where $\varphi_i(\cdot, \theta_i)$ is the inverse function of $X_i(\cdot, \theta_i)$, we can rewrite the agent's utility function as $t_i + R_i(\mathbf{x}) - \Psi_i(\varphi_i(x_i, \theta_i))$ and we obtain a similar expression to the ones in Section 2.

Let us remark that if the feasible contracts depends only on performances $\mathbf{x} = (x_1, \ldots, x_n)$, every strategy $x_i(\cdot)$ of agent i depends only on his type θ_i.

In spite of this constraint, supposing the above two conditions, we prove that the principal can utilize, in an equivalent manner, mechanisms $[\mathbf{t}(\cdot), \mathbf{x}(\cdot)]$ such that each individual performance function $x_i(\cdot)$ depends only on the agent i's announced type θ_i. We will denote $x_{-i}(\theta_{-i})$ the performance vector:

$$(x_1(\theta_1), \ldots, x_{i-1}(\theta_{i-1}), x_{i+1}(\theta_{i+1}), \ldots, x_n(\theta_n)).$$

Proposition 9

(a) Let $S_i(\cdot)$, $x_i(\cdot)$, $i = 1, \ldots, n$, be satisfying

$$x_i(\theta_i) \in \operatorname{argmax}\{\ E^i[\ S_i(x_{-i}(\theta_{-i}), x_i) + V_i\ (x_{-i}(\theta_{-i}), x_i, \theta_i)\ ]\ /\ x_i \geq 0\ \}\quad \forall \theta_i,\ \forall i \qquad (17)$$

Then, the functions $t_i(\cdot)$, $i=1,\dots,n$, which are defined $t_i(\theta_i) = E^i[\, S_i(x_{-i}(\theta_{-i}),x_i(\theta_i))\,]$, $i=1,\dots,n$, verify

$$\theta_i \in \operatorname{argmax}\{t_i(\tau_i) + E^i[\, V_i(x_{-i}(\theta_{-i}),x_i(\tau_i),\theta\,)\,] \;/\; \tau_i \in \Theta_i\,\} \quad \forall\theta_i,\ \forall i \qquad (18)$$

(b) Given $t_i(\cdot)$, $x_i(\cdot)$, $i=1,\dots,n$, satisfying the expressions (18) there are contracts $S_i(\cdot)$, $i= 1,\dots n$, verifying (17) and such that:

$$E^i[\, S_i(x_{-i}(\theta_{-i}),x_i(\theta_i))\,] = t_i(\theta_i), \quad \forall\theta_i,\ \forall i.$$

Proof: (a) Let $S_i(\cdot)$, $x_i(\cdot)$, $i=1,\dots,n$, be verifying (17). Given functions $t_i(\cdot)$ defined above, as each $x_k(\cdot)$ depends only on θ_k for each k and the mathematical expectation is a linear operator, it is easy to show that if we apply (17) for $x_i=x_i(\theta_i)$, we obtain (18).

(b) Let $t_i(\cdot)$, $x_i(\cdot)$, $i=1,\dots,n$, satisfying (18). From (18) we have that $x_i(\theta_i)=x_i(\theta_i')$ implies $t_i(\theta_i)= t_i(\theta_i')$. Therefore, the following contracts are well defined and they satisfy (17):

$$S_i(x_i) = t_i(\theta_i) \ \text{ if } \exists\ \theta_i \,/\, x_i=x_i(\theta_i), \quad S_i(x_i) = m_i \ \text{ if } x_i \neq x_i(\theta_i)$$

where

$$m_i \le \inf\{\, t_i(\theta_i) + E^i[\, V_i(x_{-i}(\theta_{-i}),x_i(\theta_i),\theta_i) - V_i(x_{-i}(\theta_{-i}),x_i\,,\theta_i) \,/\, \theta_i \in \Theta_i,\ x_i \ge 0\,\}. \ \square$$

5.1.- THE EQUIVALENCE OF IMPLEMENTATIONS IN THE SECOND SETTING

We have just shown that contract implementation is equivalent to the one realized by mechanisms verifying (18). On the other hand, it is obvious that there is not loss of generality if, in the Bayesian program BP', we change self-selection constraints for the ones defined in (18). The equivalence between programs BP' and DP' holds now, under the assumption

Spence-Mirrlees' condition (SM)

$\partial_{x\theta_i} v_i(\cdot,\cdot) > 0 \ \forall i$

Proposition 10 Under SU, SM, we have $\Gamma(BP') = \Gamma(DP')$.

Proof: Given a feasible mechanism $[\mathbf{t}(\cdot),\mathbf{x}(\cdot)]$ for the program DP', $[\tilde{\mathbf{t}}(\cdot),\mathbf{x}(\cdot)]$ is a feasible mechanism for BP', with $\tilde{t}_i(\cdot) := E^i[t_i(\theta_{-i},\cdot)]$, $i=1,\dots,n$, because the functions $\tilde{\mathbf{t}}(\cdot)$ satisfy (18).

Moreover, it is evident that the equalities $E[t_i(\cdot)]=E[\tilde{t}_i(\cdot)]$ $\forall i$ hold. Then, we have $\Gamma(DP') \leq \Gamma(BP')$.

Given a feasible mechanism $[t(\cdot),x(\cdot)]$ for the program BP', using a similar argument like the one in the proof of Proposition 2 (for the function Ω), applied to the constraints (1) [or (18)], we obtain for each τ_i, θ_i and i:

$$E^i[\ V_i(x_{-i}(\theta_{-i}),x_i(\theta_i),\theta_i) - V_i(x_{-i}(\theta_{-i}),x_i(\theta_i),\tau_i)\ ] \geq E^i[\ V_i(x_{-i}(\theta_{-i}),x_i(\tau_i),\theta_i) - V_i(x_{-i}(\theta_{-i}),x_i(\tau_i),\tau_i)\ ].$$

Because the function $x_k(\cdot)$ depends only on θ_k for all k, by SU- we have:

$$v_i(x_i(\theta_i),\tau_i) - v_i(x_i(\theta_i),\theta_i) \geq v_i(x_i(\tau_i),\tau_i) - v_i(x_i(\tau_i),\theta_i).$$

Therefore, condition SM implies that $x_i(\cdot)$ is decreasing $\forall i$.

On the other hand, using a similar argument to the one in Proposition 1, we have $\forall\theta_i$, $\forall i$:

$$t_i(\theta_i) = E^i[-V_i(x_{-i}(\theta_{-i}),x_i(\theta_i),\theta_i) + \int_{\theta_i}^{\bar{\theta}_i} \partial_{\theta_i} v_i(x_i(\tau),\tau)d\tau] + \hat{u}_i.$$

But as $x_i(\cdot)$ is decreasing and we assume SM, the transfer functions

$$\tilde{t}(\theta_{-i},\theta_i) = -V_i(x_{-i}(\theta_{-i}),x_i(\theta_i),\theta_i) + \int_{\theta_i}^{\bar{\theta}_i} \partial_{\theta_i} v_i(x_i(\tau),\tau)d\tau + \hat{u}_i,\ \ i=1,\dots,n$$

dominant strategy implement $x(\cdot)$ and they satisfy $E[\tilde{t}_i(\cdot)] = E[t_i(\cdot)]$ for every i. We have, then, $\Gamma(BP') \leq \Gamma(DP')$.

Under SU+, a similar argument shows that each $x_i(\cdot)$ is increasing and dominant strategy implementable. □

5.2.- PROPERTIES OF THE OPTIMAL PERFORMANCES IN THE SECOND SETTING

Consider, as in Section 2, the virtual program which will be called VP'. Such a program is similar to VP except that feasible functions $x(\cdot)$ satisfy that every $x_k(\cdot)$ depends only on θ_k. Then, the program VP' is not equivalent generally to the pointwise maximization of the function $\Omega(x,\theta)$. Nevertheless, under the assumptions of Proposition 3, the equivalence among the three programs BP', DP' and VP' holds.

Proposition 11 Under the assumptions SU, MHR, CS, MAW, ITVI, we have

(a) $\Gamma(BP') = \Gamma(DP') = \Gamma(VP')$.

(b) Supposing that the function $\Omega(\cdot,\cdot)$ is C^1 and concave in **x**, the performance function $\mathbf{x}(\cdot)$ is optimal if and only if the following conditions hold:

$$E^i[\partial_{x_i}\Omega(x_{-i}(\theta_{-i}),x_i(\cdot),\theta_{-i},\cdot)] \leq 0, \ a.e. \ \forall i$$

$$E^i[\partial_{x_i}\Omega(x_{-i}(\theta_{-i}),x_i(\cdot),\theta_{-i},\cdot)]x_i(\cdot) = 0, \ a.e. \ \forall i$$

Proof: (a) A similar argument to the one in Proposition 1, shows that $\Gamma(BP') = \Gamma(DP') \leq \Gamma(VP')$.

Under SU-, given a solution $\mathbf{x}(\cdot)$ of VP', we have for all i and for all θ_i

$$x_i(\theta_i) \in argmax \, [\![\, E^i[\,W(x_{-i}(\theta_{-i}),x_i)) - \sum_{k=1,k\neq i}^{n} \gamma_k(x_k(\theta_k),\theta_k)\,] - \gamma_i(x_i,\theta_i) \ / \ x_i \geq 0 \,]\!]$$

where the functions $\gamma_k(\cdot,\cdot)$ are defined in Proposition 2. Utilizing an argument similar to its proof, we have that $x_i(\cdot)$ is decreasing and, therefore, dominant strategy implementable. Since we assume MAW, we get $\Gamma(VP') \leq \Gamma(DP')$.

If we suppose SU+ the line of argument is analogous.

(b) By the part (a), $\mathbf{x}(\cdot)$ is an optimal performance function if and only if it is a solution of program VP'. Supposing the assumptions in the part (b), the functional $T(\cdot)$, defined by $T(\mathbf{x}(\cdot)) := E[\Omega(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})]$, is Fréchet differentiable and concave on the convex cone $\mathcal{P}$ of positive functions in the normed space of piecewise continuously differentiable functions $\mathbf{x}(\cdot)$, satisfying that each $x_k(\cdot)$ depends only on θ_k.

Therefore [see section 8.7 of Luenberger (1969)], $\mathbf{x}(\cdot)$ maximizes $T(\cdot)$ on $\mathcal{P}$ if and only if:

$$\delta T(\mathbf{x}(\cdot);\mathbf{h}(\cdot)) \leq 0, \ \forall \, \mathbf{h} \in \mathcal{P}, \ \ \delta T(\mathbf{x}(\cdot);\mathbf{x}(\cdot)) = 0,$$

where $\delta T(\mathbf{x}(\cdot);\cdot)$ represents the Fréchet differential of the functional $T(\cdot)$ at the vector $\mathbf{x}(\cdot)$.

Because

$$\delta T(x(\cdot);h(\cdot)) = E[\, \sum \partial_{x_i}\Omega(x(\theta),\theta)h_i(\theta_i)\,]$$

and moreover the density functions of types are strictly positive and $\mathcal{P}$ is formed by positive functions, we can deduce easily the conditions of part (b). □

Under complete information, we assume also, in this second setting, that the feasible "equivalent" mechanisms have each individual performance function depending only on the respective type. Therefore, under complete information, the principal will solve the program:

$$(\text{CIP'}) \max_{\mathbf{x}(\cdot)} \quad E[\, B(\mathbf{x},\theta) + \lambda\, \Sigma\, V_i(\mathbf{x},\theta_i) - \lambda\, \Sigma\, u_i \,].$$

Assuming that the principal's virtual income depends on performances only through the aggregate total performance, the optimal performance demanded to the most efficient agents may be greater than the one of complete information.

Corollary 3 Given the assumptions SU-, MHR-, LD, VIDTP, AS, suppose the case:

$$W(z) = a(1-e^{-bz}), \text{ with } a>0,\ b>0,\ ab > \phi^n(\bar{\theta})/(E\phi)^{n-1},\ \underline{\theta}>0$$

where $\phi(\theta) := \theta + \lambda\, F(\theta) / f(\theta)$, $\lambda > 0$. Then,

(a) The optimal performance functions of incomplete and complete information satisfy respectively:

$$x_i^*(\theta_i) = \frac{1}{b}\log[\frac{E\phi}{\phi(\theta_i)\mu_i}],\quad x_i^{CI}(\theta_i) = \frac{1}{b}\log[\frac{E\theta}{\theta_i\mu_i^{CI}}],\quad i=1,\ldots,n$$

where $0 < \mu_i < E\phi / \phi(\bar{\theta})$, $0 < \mu^{CI}_i < E\theta / \bar{\theta}$, $i=1,\ldots,n$,
verify $\Pi\, \mu_i = E\phi / (ab)$, $\Pi\, \mu^{CI}_i = E\theta / (ab)$.

(b) There is i such that $x^{CI}_i(\underline{\theta}) < x^*_i(\underline{\theta})$ if $n \geq 2$.

Proof: From Proposition 11, $\mathbf{x}^*(\cdot)$ is an interior optimal performance function if and only if:

$$ab\exp[-bx_i^*(\theta_i)]E[\![\, \exp[-b\sum_{k=1,k\neq i}^{n} x_k^*(\theta_k)]\,]\!] = \phi(\theta_i),\ \forall\theta_i,\ \forall i.$$

As types are independent, denoting $\mu_i = E\{\exp[-b\, x^*_i(\theta_i)]\}$ $\forall i$, the above equalities are equivalent to:

$$ab\exp[-bx_i^*(\theta_i)] \prod_{k=1,k\neq i}^{n} \mu_k = \phi(\theta_i),\ \forall\theta_i,\ \forall i \tag{19}$$

and utilizing the mathematical expectation, we obtain $\Pi\mu_i = E\phi/(ab)$. Therefore, we have

$$\prod_{k=1, k\neq i}^{n} \mu_k = \frac{E\phi}{ab\mu_i}, \ \forall i,$$

and substituting into (19) and calculating $x^*_i(\theta_i)$, we see that the conditions in part (a) are necessary and sufficient for $x^*(\cdot)$ being an interior optimal performance function. On the other hand, the values μ_i defined above exist from assumptions.

For $x^{CI}(\cdot)$, we can use a similar argument because $x^{CI}(\cdot)$ has to be a solution of program CIP' (which coincides with VP' when $\lambda = 0$). The part (a) holds.

In order to prove the part (b), suppose $x^{CI}_i(\underline{\theta}) \geq x^*_i(\underline{\theta})$ $\forall$i. Then, by the part (a), we obtain $E\theta/\mu^{CI}_i \geq E\phi/\mu_i$ $\forall$i. This implies $(E\theta)^n/\Pi\mu^{CI}_i \geq (E\phi)^n/\Pi\mu_i$ which is equivalent to $(E\theta)^{n-1} \geq (E\phi)^{n-1}$ contradicting $n \geq 2$. □

The intuition is the following. In principle, the principal can decrease the informational rent of an agent by reducing his performance. But given the supposed virtual income, if he decreases the performance asked for an agent, preserving the income level, he has to increase the performances asked for other agents and this implies a rise of their informational rents.

Corollary 3 shows that the principal prefers increase the solicited performance, in relation to the one of complete information, for a very efficient agent in order to decrease informational rents of others less efficient agents.

6.- CONCLUSIONS

In this paper we study an adverse selection model, with a principal and several agents, where contracting is under asymmetric information. The agents' number is finite and types are "continuous" and independent. We analyze two settings. In the first one, the performance functions of mechanisms may depend on all the reported types. In the second one, each performance function depends only on the respective announced type.

Our first setting represents situations where an economic agent (the principal) proposes personalized take-it-or-leave-it "prices" and "quantities" to another agents, so that each individual "menu" of contracts depends on every announced type. Think of a firm purchasing an input, which is unfamiliar with the suppliers' productivity. Another example is the regulation of a good produced by an oligopoly with hidden efficiencies or the regulation of a monopolist with several independent divisions (or of a group of firms), each one producing a different good. Such a menu of contracts may represent also, for instance, several classes of managerial compensations.

Under the standard hypotheses in the basic one-agent adverse selection model and the *independence* assumption, there is not loss of generality if the principal considers only mechanisms for which every agent reports his true type as a dominant strategy. The former "equivalence" between the Bayesian implementation and the dominant strategy one stands firm in some cases, although the utility function of each agent is not monotone in his type.

Supposing that the principal's "virtual income" depends on the agents' performances only through the aggregate total performance (which is natural in the context of regulation of the good produced by an oligopoly), unlike the standard properties of the optimal mechanisms in the basic one-agent adverse selection model, in our model the optimal mechanism may ask very efficient agents for an individual performance higher than the one of complete information. The intuition is that the principal "saves" informational rent of an agent inducing the truthful revelation of others agent's types. In the frame of regulation of a monopolist with several independent divisions (or the one of a group of firms), each one producing a different good, when goods are substitutes the optimal regulatory mechanism may ask the most efficient divisions (or firms) for an individual production higher than the one of complete information. Nevertheless, when goods are complements, all individual production is lower than the one of complete information. The explanation is that, when goods are substitutes, informational rents and marginal social values of goods go in opposite directions: if the production of a good decreases, the marginal social value of the other goods increases and the optimal distortion, with regard to the complete information solution, has to make a trade-off between the reductions in rent of divisions.

Regarding the agents' ex ante optimal number (which in the frame of regulation of the private good produced by an oligopoly would represent an ex ante limit on the regulated firms' number), when agents are symmetrical, the principal may prefer ex ante to hire more than one agent because the distribution of the agents' virtual costs improves with the agent's number, according to the first order stochastic dominance.

The second setting, where feasible mechanisms have each individual performance based only on the type reported by the respective agent, is justifiable under two conditions which are verified in plenty of economic situations: contracts depending on performances are the only feasible, and every agent takes a hidden action, after privately observing his type, which determines univocally his performance. Conclusions in this second setting are similar to the ones of the first setting.

Appendix 1: Proof of Lemma 2

By the integral operator properties, each program $[P(\tau_i)]$ is equivalent to the following ones. Note that there is one for each value of θ_i: if $\tau_i=\bar{\theta}_i$ we consider only the first program, whereas if $\tau_i=\underline{\theta}_i$, only the second one.
For each $\theta_i \in [\underline{\theta}_i, \tau_i[$:

$$[\bar{P}(\theta_i;\tau_i)] \qquad \max_{x_i(\cdot,\theta_i)} \bar{T}[x_i(\cdot,\theta_i);\theta_i,\tau_i] \qquad \text{s.t. } \bar{G}[x_i(\cdot,\theta_i);\theta_i,\tau_i] \geq 0$$

for each $\theta_i \in]\tau_i, \bar{\theta}_i]$:

$$[\underline{P}(\theta_i;\tau_i)] \qquad \max_{x_i(\cdot,\theta_i)} \underline{T}[x_i(\cdot,\theta_i);\theta_i,\tau_i] \qquad \text{s.t. } \underline{G}[x_i(\cdot,\theta_i);\theta_i,\tau_i] \geq 0$$

The functionals $\bar{T}[\cdot;\theta_i,\tau_i]$, $\underline{T}[\cdot;\theta_i,\tau_i]$, $\bar{G}[\cdot;\theta_i,\tau_i]$, $\underline{G}[\cdot;\theta_i,\tau_i]$, are defined on the space Z_i of bounded and continuously differentiable almost everywhere functions from Θ_{-i} to $\mathbb{R}$ (extending in a differentiable fashion functions W_i and φ_i for $x \leq 0$), in the following manner:

$$\bar{T}[z(\cdot);\theta_i,\tau_i] = E^i\{W_i[z(\theta_{-i})] - \beta_i(\theta_i;\bar{\theta}_i)\,\varphi_i[z(\theta_{-i})]\}, \quad \bar{G}[z(\cdot);\theta_i,\tau_i] = E^i\varphi_i[z(\theta_{-i})] + \Psi_i'(\tau_i)$$

$$\underline{T}[z(\cdot);\theta_i,\tau_i] = E^i\{W_i[z(\theta_{-i})] - \beta_i(\theta_i;\underline{\theta}_i)\,\varphi_i[z(\theta_{-i})]\}, \quad \underline{G}[z(\cdot);\theta_i,\tau_i] = -E^i\varphi_i[z(\theta_{-i})] - \Psi_i'(\tau_i).$$

It is easy to show that Gateaux differentials of the above functionals are linear in their increments.

Differentiable extension of W_i and φ_i can be chosen such that the following inequalities hold:

$$W_i(x) - \beta_i(\theta_i;\bar{\theta}_i)\,\varphi_i(x) < W_i(x^*_i) - \beta_i(\theta_i;\bar{\theta}_i)\,\varphi_i(x^*_i) \quad \forall\, \theta_i \in \Theta_i,\ \forall\, x \leq 0$$

$$W_i(x) - \beta_i(\theta_i;\underline{\theta}_i)\,\varphi_i(x) < W_i(x^*_i) - \beta_i(\theta_i;\underline{\theta}_i)\,\varphi_i(x^*_i) \quad \forall\, \theta_i \in \Theta_i,\ \forall\, x \leq 0$$

Any solution of program $[\bar{P}(\theta_i;\tau_i)]$ must be strictly positive on some subset of positive measure in Θ_{-i}. If a solution was negative almost everywhere, the objective function value would be lower than $W_i(x^*_i) - \beta_i(\theta_i;\bar{\theta}_i)\,\varphi_i(x^*_i)$. As $\varphi_i(x^*_i) > \varphi_i(0)$, the constant functional equal to x^*_i would be feasible for the program with an objective function value lower than the one of the solution: a contradiction.

Thus, every solution of $[\bar{P}(\theta_i;\tau_i)]$ has to be a regular point for the constraint $\bar{G}[x_i(\cdot,\theta_i);\theta_i,\tau_i] \geq 0$ [according to definition in Luenberger (1969), page 248].

An analogous argument, taking into account $\varphi_i(x^*_i) < -\Psi_i'(\underline{\theta}_i)$, $\Psi_i'' < 0$, may be applied for $[\underline{P}(\theta_i;\tau_i)]$, and therefore each one of its solutions is a regular point.

Using the generalized Kuhn-Tucker theorem [page 249 of Luenberger (1969)], we have that given a solution $x_i(\cdot,\theta_i)$ of $[\overline{P}(\theta_i;\tau_i)]$ there is $\alpha(\theta_i,\tau_i) \geq 0$ such that the associated Lagrangian:

$$\overline{T}[z(\cdot);\theta_i,\tau_i] + \alpha(\theta_i,\tau_i)\,\overline{G}[z(\cdot);\theta_i,\tau_i]$$

is stationary at $x_i(\cdot,\theta_i)$. This implies that the equality:

$$E^i\{\,[\,W_i'[z(\theta_{-i})] + [\alpha(\theta_i,\tau_i) - \beta_i(\theta_i;\overline{\theta}_i)]\,\varphi_i'[z(\theta_{-i})]\,]\,h(\theta_{-i})\,\} = 0, \quad \forall\, h \in Z_i$$

holds for each solution $x_i(\cdot,\theta_i)$ and then, we get

$$W_i'[x_i(\cdot,\theta_i)] + [\alpha(\theta_i,\tau_i) - \beta_i(\theta_i;\overline{\theta}_i)]\,\varphi_i'[x_i(\cdot,\theta_i)] = 0, \quad \text{a.e. on } \Theta_{-i}$$

for all solution. Since $W_i'(\cdot)\,/\,\varphi_i'(\cdot)$ is strictly decreasing, any solution of $[\overline{P}(\theta_i;\tau_i)]$ has to be almost everywhere constant (independent on types θ).

With a similar line of argument, we can prove that any solution of program $[\underline{P}(\theta_i;\tau_i)]$ is a.e. constant with regard to types θ_{-i}.

We have just shown, then, that the above programs can be reduced, without loss of generality, defining them on the real number space (with $x_i \in \mathbb{R}$). Let $\overline{\phi}_i(\cdot)$ and $\underline{\phi}_i(\cdot)$ be the continuously differentiable, strictly decreasing and positive functions which are unique solutions respectively of

$$W'[\overline{\phi}_i(\theta)] = \beta_i(\theta;\overline{\theta}_i)\,\varphi_i'[\overline{\phi}_i(\theta)] \;\; \forall\, \theta \in \Theta_i, \qquad W'[\underline{\phi}_i(\theta)] = \beta_i(\theta;\underline{\theta}_i)\,\varphi_i'[\underline{\phi}_i(\theta)] \;\forall\, \theta \in \Theta_i$$

These functions exist by the properties of W_i and φ_i because moreover we have $\beta_i(\cdot;\cdot)>0$. On the other hand, we have $\beta_i(\cdot;\overline{\theta}_i) > \beta_i(\cdot;\underline{\theta}_i)$ and this implies $\overline{\phi}_i(\cdot) < \underline{\phi}_i(\cdot)$.

Therefore, the unique solutions of programs $[\overline{P}(\theta_i;\tau_i)]$ and $[\underline{P}(\theta_i;\tau_i)]$ are, respectively:

$$\overline{x}_i(\theta_i;\tau_i) := \max[\,\overline{\phi}_i(\theta_i),\, \rho_i(\tau_i)\,], \quad \underline{x}_i(\theta_i;\tau_i) = \min[\,\underline{\phi}_i(\theta),\, \rho_i(\tau_i)\,]$$

and, thus, the only solution of program $[P(\tau_i)]$ is:

$$x_i(\theta;\tau_i) = \overline{x}_i(\theta_i;\tau_i) \text{ if } \theta_i \in [\underline{\theta}_i, \tau_i[, \quad x_i(\theta;\tau_i) = \underline{x}_i(\theta_i;\tau_i) \text{ if } \theta_i \in]\tau_i, \overline{\theta}_i].$$

We can prove easily that the properties of Lemma 2 hold. □

Appendix 2: Proof of the properties of the Example

When $f(\tau)=1/\beta$; $0<\tau<\beta$, we have $\phi(\tau)=2\tau$, $G_n(\alpha)=1-(1-\alpha/2\beta)^n$ for $\alpha \in [0,2\beta]$. The principal's expected utility hiring n agents will be:

$$E[\frac{(a-c-\alpha(\theta))^2}{4}]-n(\Psi(\theta)+u)=\frac{1}{4}\int_0^{2\beta}(a-c-\alpha)^2\frac{n}{2\beta}(1-\frac{\alpha}{2\beta})^{n-1}d\alpha-n(\Psi(\theta)+u)=$$

$$=\frac{(a-c)^2}{4}+n\beta\,[\![\,\beta\int_0^1 z^2(1-z)^{n-1}dz-(a-c)\int_0^1 z(1-z)^{n-1}dz\,]\!]-n(\Psi(\theta)+u)=$$

$$=\frac{(a-c)^2}{4}+n\beta\,[\![\,\beta\frac{2(n-1)!}{(n+2)!}-(a-c)\frac{(n-1)!}{(n+1)!}\,]\!]-n(\Psi(\theta)+u)=$$

$$=V(a,c,\beta,n)-n(\Psi(\theta)+u)$$

where

$$V(a,c,\beta,n):=\frac{(a-c)^2}{4}+\frac{\beta}{n+1}[\frac{2\beta}{n+2}-a+c].$$

We prove easily that $\partial_n V>0$, $\partial_{nn}V<0$, $\partial_n V|_{n=1}=\bar{u}$, $\partial_n V|_{n=\infty}=0$, and then, the properties (a) and (b) of the example hold.

Let n^* denote the function satisfying $\varphi(a,c,\beta,u,n^*)\equiv 0$, where

$$\varphi(a,c,\beta,u,n)=\partial_n V(a,c,\beta,n)-(\Psi(\theta)+u).$$

From the implicit function theorem, we get that n^* is continuously differentiable.

As $\partial_n\varphi<0$, $\partial_\alpha\varphi=-\partial_c\varphi>0$, $\partial_u\varphi<0$, and $\partial_\beta\varphi>0$ (if $n\geq 2$) relations $\partial_\alpha n^*=-\partial_\alpha\varphi/\partial_n\varphi>0$, $\partial_c n^*=-\partial_c\varphi/\partial_n\varphi<0$, $\partial_u n^*=-\partial_u\varphi/\partial_n\varphi<0$, $\partial_\beta n^*=-\partial_\beta\varphi/\partial_n\varphi>0$ (if $n^*\geq 2$) hold.

Finally, given $B(a,c,\beta,u):=V(a,c,\beta,n^*)-(\Psi(\theta)+u)n^*$ we have $\partial_\beta B=\partial_\beta V(a,c,\beta,n^*)<0$. □

REFERENCES

BARON, D.P. (1989). *Design of regulatory mechanisms and institutions.* Handbook of Industrial Organization, Volume II, Chapter 24.

CAILLAUD, B., GUESNERIE, R., REY, P. & TIROLE., J. (1988). *Government intervention in production and incentives theory: a review of recent contributions.* RAND Journal of Economics, Vol.19, N.1, 1-26.

CRÉMER, J. & MCLEAN, R. (1988). *Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions.* Econometrica, 56, 1247-1257.

DEMSKI & SAPPINGTON (1984). *Optimal incentive contracts with multiple agents.* Journal of Economic Theory, 33, 152-171.

FUDENBERG, D. & TIROLE, J. (1991). *Game theory.* MIT Press, Cambridge.

GUESNERIE, R. & LAFFONT, J.J. (1984). *A complete solution to a class of principal-agent problems with an application to the control of a self-managed firm.* Journal of Public Economics, 25, 329-369.

LAFFONT, J.J. & TIROLE, J. (1987). *Auctioning Incentive Contracts.* Journal of Political Economy, vol.95, no.5, 921-937.

LAFFONT, J.J. & TIROLE, J. (1993). *A Theory of Incentives in Procurement and Regulation.* The MIT Press.

LEWIS, T.R. & SAPPINGTON, D.E.M. (1989). *Countervailing Incentives in Agency Problems.* Journal of Economic Theory, 49, 294-313.

LUENBERGER, D. G. (1969). *Optimization by vector space methods.* John Wiley & Sons, Inc.

MILGROM, P. & ROBERTS, J. (1992). *Economics, organization and management.* London: Prentice- Hall International.

MOOKHERJEE, D. & REICHELSTEIN, S. (1992). *Dominant Strategy Implementation of Bayesian Incentive Compatible Allocation Rules.* Journal of Economic Theory, 56, 378-399.

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