BARGAINING WITH CLAIMS IN ECONOMIC ENVIRONMENTS*

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A B S T R A C T

In this paper a reconstruction of the theory of bargaining with claims in economic environments is addressed. The spirit of that reconstruction is similar to that made by Roemer of the standard bargaining theory.

Keywords: Bargaining; Bargaining with Claims; Economic Environments.

1. INTRODUCTION

Chun & Thomson (1992) enriched the traditional model of bargaining [Nash (1950)], by adding to the disagreement point and the feasible set a point representing claims (or expectations) that agents may have when they come to the bargaining table. Chun and Thomson proposed the so called *Proportional Solution* which assigns to any problem with claims the optimal point of the feasible set lying in the straight line passing through the disagreement and the claims point, and provide with a number of characterization results for this solution by using different combinations of axioms. Bossert (1993) presents a new solution concept, namely, the *Claim-Egalitarian Solution*, which equalizes losses from the claims point across agents. A main shortcoming of the claim-egalitarian solution is that it can fail to be individually rational for more than two agents. In order to avoid that, Bossert (1993) also presented a modification of that solution, namely, the *Extended Claim-Egalitarian* solution, and provided with an axiomatic characterization of that proposal. The Extended Claim-Egalitarian solution equalizes losses from the claims point only for those agents which are kept above their disagreement level in the equal-decreasing procedure; for the rest of the agents, the solution provides them with their status-quo level. An alternative characterization of the extended claim-egalitarian solution is proposed in Marco (1994a). The aforementioned solutions are not, in general, Pareto optimal. Suitable modifications in order to ensure Pareto optimality appear in Marco (1994b).

As it is the case both in the traditional model of bargaining and in the rights problem, in that of bargaining with claims, reference to economic environments is usually made as a way of motivating the problems themselves as well as those axioms used in the characterization of the proposed solutions.

Roemer (1988) studies the traditional bargaining problem focusing on economic environments, and reconstructs the bargaining theory from this point of view. He argues that the axioms of the traditional theory impose stronger restrictions than can be supported by their motivating economic intuitions. Weaker alternative axioms, making explicit use of economic information, are used to characterize the standard bargaining solutions. The key idea in obtaining the "reconstruction" of the traditional bargaining theory from an economic perspective consists in properly enlarge the space of application of the axioms as a way of obtaining the axiom of welfarism, crucial in getting characterization results with a small number of conditions. Another unavoidable ingredient of the reconstruction consists of assuming "fullness" of the solution, that is, 'any feasible outcome providing the utility of the solution outcome can also be taken as a solution outcome'.

In this paper we develop a similar reconstruction to Roemer's, but in the bargaining with claims context. This reconstruction is limited to the Proportional and the Extended Claim-egalitarian solutions. In Section 2 we present the set-up, as well as the main axioms used in the aforementioned reconstruction of the theory. Section 3 presents characterization results for a fixed number of agents. Section 4 is devoted to the corresponding characterizations for a variable population. Some final comments and remarks, in Section 5, close the paper.

2. ECONOMIC ENVIRONMENTS AND BARGAINING WITH CLAIMS.

Let $\mathcal{U}^{(\ell)}$ be the set of all real valued monotone concave continuous functions defined on $\mathbb{R}_+^\ell$. An *economic environment* is a tuple

$$\xi = \langle n,u,\ell,w,c\rangle$$

where n, $\ell \geq 1$ are integer numbers, $w \in \mathbb{R}_+^\ell$, $c \in \mathbb{R}_+^{\ell n}$, $c = (c_1,\dots,c_n)$, such that $c_i \in \mathbb{R}_+^\ell$ for all $i = 1,\dots,n$, and $u = (u_1,\dots,u_n)$, where $u_i \in \mathcal{U}^{(\ell)}$ for all $i= 1,\dots,n$. The interpretation is that we deal with a problem involving n agents, each one characterized by his utility function (u_i), having claims on ℓ commodities (c_i), but the available amounts of the commodities to be divided among them is w. If for some $j = 1,\dots,\ell$, $\sum_{i=1}^{n} c_{ij} > w_j$, then a conflict arises, which shall be called as a conflict of bargaining with claims. The question is how to allocate w among the n individuals in the previous problem.

We start by dealing with a family of problems in which n (the number of agents) is fixed, but ℓ (the number of goods to be distributed) varies. Let us call $\sum^n$ the class of all admissible environments for n agents. In this section, when we specify an economic environment $\xi \in \sum^n$, no explicit mention to the number of agents will be made, that is, $\xi = \langle u,\ell,w,c\rangle$.

If $\xi \in \sum^n$, an allocation is a vector $x = (x_1,\dots,x_n)$, where $x_i \in \mathbb{R}_+^\ell$. x is feasible if $\sum_{i=1}^{n} x_i \leq w$. Let us call $Z(\xi)$ the set of feasible allocations for ξ, and $\mathcal{A}(\xi)$ the utility possibility set of ξ,

$$Z(\xi) = \{x = (x_1,\ldots,x_n),\ x_i \in \mathbb{R}^{\ell}_{+} \mid \sum_{i=1}^{n} x_i \leq w\}$$

$$\mathcal{A}(\xi) = \{(\bar{u}_1,\ldots,\bar{u}_n) \in \mathbb{R}^n \mid \exists\ \bar{x}_1,\ldots,\bar{x}_n \in Z(\xi),\ u_i(\bar{x}_i) \geq \bar{u}_i\}$$

$\mathcal{A}(\xi)$ is a closed, comprehensive, convex set in $\mathbb{R}^n$, containing the point u(0). Convexity comes from concavity of the utility functions; comprehensiveness and closedness come from freely disposability of the goods and continuity of the utility functions.

An *allocation mechanism* F is a correspondence which associates to each economic environment $\xi \in \sum^n$, a set of feasible allocations. It will be assumed, however, that F induces a function in utility space, that is, if $x = (x_1,\ldots,x_n)$, $y = (y_1,\ldots,y_n) \in F(\xi)$, then $u(x) = u(y)$, [where $u(x) = (u_1(x_1),\ldots,u_n(x_n))$].

Call the induced utility function $\mu_F(\xi) = u(F(\xi))$, where $F(\xi)$ is the set of bundles which F assigns to every agent. Furthermore, it is assumed that F chooses all the feasible allocations associated with a given point in the utility space. Roemer (1988) call this type of correspondence a "full" correspondence.

Associated to any *economic environment* $\xi \in \sum^n$, we can define *a classical bargaining with claims problem* in a natural way: If $\xi = \langle u,\ell,w,c \rangle$, take $\mathcal{A}(\xi)$ as the feasible set, u(0) as the disagreement point and u(c) as the claims point. Thus, we may look at ξ as a classical bargaining problem with claims.

Two main ingredients are necessary in order to reconstruct the classical bargaining with claims theory as a theory on economic environments: (1) To ensure that the information contained in the utility possibilities set, the utilities of the origin and the utilities of the claims is all that matters, (2) To ensure that any triple (S,d,c), where S is a convex comprehensive subset of $\mathbb{R}^n$, $d \in S$ and $c \in \mathbb{R}^n$, $c \notin S$ comes from a suitable economic environment $\xi \in \Sigma^n$.

In order to fulfill point (1) above, we need the following axiom:

Axiom of Welfarism (W).- Let $\xi = \langle u,\ell,w,c \rangle$, and $\xi' = \langle u',\ell',w',c' \rangle$ two problems in Σ^n such that $\mathcal{A}(\xi) = \mathcal{A}(\xi')$, $u(0) = u'(0)$ and $u(c) = u'(c')$. Then, $\mu_F(\xi) = \mu_F(\xi')$.

Axiom W says that an allocation mechanism F must treat identically (in terms of utility) any two economic environments which have the same utility possibilities set and in which the utility levels of the origin and of the claims point are identical for all individuals. W is extremely strong, since it has to hold even in circumstances in which the dimension of the goods space are different.

The welfarist axiom is so named because it requires the allocation mechanism to ignore all information about an environment not summarized both in the utility possibilities set and in the utilities of the origin and of the claims point.

In order to present the next axiom we need the concept of *personal good*. Let $\xi \in \sum^n$ be an economic environment, $\xi = \langle u,\ell,w,c\rangle$, $i \in N$, $j \in \{1,2,..,\ell\}$. We shall say that good jth is a *personal good* for agent ith if for any $k \in N$, $k \neq i$, for any $x \in \mathbb{R}^{\ell}_{+}$, and for any $\alpha \geq 0$, if $\bar{\alpha}^j$ stands for a vector in $\mathbb{R}^{\ell}$ having all components equal to zero but the jth component, equal to α, we get that $u_k(x) = u_k(x + \bar{\alpha}^j)$.

Axiom of Consistency (CON).- Let $\xi \in \sum^n$, $\xi = \langle u,\ell+\ell',(w,w'),(c,c')\rangle$ be an environment such that $(\bar{x},\bar{y}) \in F(\xi)$, where any of the y-goods ($k = \ell+1,...\ell+\ell'$) is a personal good for at most one of the agents. Define v by $v_i(x_i) = u_i(x_i,\bar{y}_i)$. Consider now the environment $\xi' = \langle v,\ell,w,c\rangle$. If $\xi' \in \sum^n$, and $\mathcal{A}(\xi) = \mathcal{A}(\xi')$, $v(0) = u(0)$, $v(c) = u(c,c')$, then $\bar{x} \in F(\xi')$.

CON asks for a particular type of consistency in the allocation procedure. First, the mechanism allocates those commodities which are only liked for a particular agent (personal goods). Next, the remainig commodities are allocated. The mechanism satisfies this property whenever the allocation is immune to this two steps procedure. The interest of the axiom CON is that it will imply the axiom of welfarism, even though it is an axiom relying on the economic data of the problem.

The main results of this section are contained in the next two lemmas:

Lemma 1.- W and fullness of F imply CON.

Proof: CON says that the induced utility mapping μ_F assigns the same utility point to environments ξ and ξ' related by a dimensional contraction, whenever they have identical utility possibility sets, utility in the origin and utility claims. W, on the other hand requires μ_F to treat identically any pair of environments generating the same utility set, disagreement utilities and utility claims.■

Lemma 2.- CON and fullness of F imply W.

<u>Proof:</u> Let ξ, $\xi' \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$; $\xi' = \langle u',\ell',w',c'\rangle$ such that $\mathcal{A}(\xi) = \mathcal{A}(\xi')$, $u(0) = u'(0)$, $u(c) = u'(c')$. It must be proven that $\mu_F(\xi) = \mu_F(\xi')$.

1. Construct $\xi^* = \xi \otimes \xi'$. $\xi^* = \langle v,\ell+\ell',(w,w'),(c,c')\rangle$, where $v_i(x_i,y_i) = \min \{u_i(x_i),\ u'_i(y_i)\}$. By Billera & Bixby (1973), $\mathcal{A}(\xi^*) = \mathcal{A}(\xi) = A(\xi')$; $v(0) = u(0) = u'(0)$, $v(c,c') = u(c) = u'(c')$.

We shall prove that $\mu_F(\xi^*) = \mu_F(\xi)$. Analogously, $\mu_F(\xi^*) = \mu_F(\xi')$, and the lemma follows.

2. Consider now $\bar{\xi} = \langle \bar{u},\ell+\ell',(w,w'),(c,0)\rangle$, where $\bar{u}_i(x_i,y_i) = u_i(x_i)$, $i=1,\dots,n$. Obviously, $\mathcal{A}(\xi) = \mathcal{A}(\bar{\xi})$, $u(0) = \bar{u}(0)$, $u(c) = \bar{u}(c,0)$. Moreover, we can apply CON to ξ, $\bar{\xi}$, and then we get $\mu_F(\xi) = \mu_F(\bar{\xi})$.

3. Notice that $\bar{u}_i(x_i,y_i) \geq v_i(x_i,y_i)$ for all $i = 1,\dots,n$. Now by Howe (1987), Prop. 3, it is possible to find functions U_i: $\mathbb{R}^{\ell+\ell'+1}$ ---> $\mathbb{R}$, such that $U_i(x_i,y_i,1) = \bar{u}_i(x_i,y_i)$, $U_i(x_i,y_i,0) = v_i(x_i,y_i)$.

Now, construct the functions $V_i:\mathbb{R}^{\ell+\ell'+n}$ ---> $\mathbb{R}$, $V_i(x,y,z) = V_i(x,y,z_i)$, for $z \in \mathbb{R}^n$, $i = 1,\dots,n$.

4. Consider now the environment: $\zeta = \langle V,\ell+\ell'+n,(w,w',1,\dots,1),(c,0,f)\rangle$, where $f_i = e_i$, the ith vector of the canonical basis in $\mathbb{R}^n$. Thus, $\mathcal{A}(\zeta) = \mathcal{A}(\bar{\xi}) = \mathcal{A}(\xi)$. Moreover, $V_i(0,0,e_i) = U_i(0,0,1) = \bar{u}_i(0,0) = u_i(0)$, $V_i(c_i,0,e_i) = U_i(c_i,0,1) = \bar{u}_i(c_i,0) = u_i(c_i)$. By CON, $\mu_F(\zeta) = \mu_F(\bar{\xi}) = \mu_F(\xi)$.

5. Now, construct the environment $\bar{\xi}* = <V,\ell+\ell'+n,(w,w',0);(c,c',0)>$. Thus, $\mathcal{A}(\xi*) = \mathcal{A}(\bar{\xi}^*)$. Moreover, $V(0,0,0) = U_i(0, 0,0) = v_i(0,0)$, $V(c_i,c'_i,0) = U_i(c_i, c'_i,0) = v_i(c_i,c'_i)$. By CON, $\mu_F(\xi*) = \mu_F(\bar{\xi}*)$.

6. From previous constructions, $\mathcal{A}(\bar{\xi}*) = \mathcal{A}(\zeta)$. Consider an allocation in $F(\zeta)$, $z_i = (x_i,y_i,e_i)$. Then, there must be an allocation $t_i = (x'_i,y'_i,0)$ in $Z(\bar{\xi}*)$, inducing the same utility point, i.e., $V(z_i) = V(t_i)$. Thus, since t_i is feasible in ζ, and because of fullness of F, $t \in F(\zeta)$.

Now, by CON, $(x'_i,y'_i)_{i=1,\ldots,n} \in F(\xi*)$, obtaining that $\mu_F(\xi*) = \mu_F(\zeta)$. And therefore $\mu_F(\xi*) = \mu_F(\xi)$.■

3. BARGAINING WITH CLAIMS AND ECONOMIC INFORMATION WITH A FIXED NUMBER OF AGENTS.

We shall now formulate some axioms of classical bargaining with claims theory, but viewed as applying to mechanisms acting on economic environments.

For any $\xi \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, consider the following sets:

$WPO(\xi) = \{ x \in Z(\xi) |$ there is no $y \in Z(\xi)$ such that $u(y) >> u(x)\}$

$PO(\xi) = \{ x \in Z(\xi) |$ there is no $y \in Z(\xi)$ such that $u(y) > u(x)\}$

Axiom of Efficiency (PO) $\forall\ \xi \in \sum^n$, $F(\xi) \subset PO(\xi)$.

Axiom of Weak Efficiency (WPO) $\forall\ \xi \in \sum^n$, $F(\xi) \subset WPO(\xi)$.

Axiom of Symmetry (Sy). Let $\xi \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$ be such that for all $i,j \in N$, $u_i = u_j$, $c_i = c_j$. Then, $(w/n,\dots,w/n) \in F(\xi)$.

Let Π^n be the set of all permutations π of the set $\{1,\dots,n\}$.

Axiom of Anonymity (An). $\forall\ \xi \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, and $\forall\ \pi \in \Pi^n$, if we call $\pi(\xi) = \langle\pi(u),\ell,w,\pi(c)\rangle$, then, $x \in F(\xi)$ iff $\pi(x) \in F[\pi(\xi)]$.

PO and WPO ask the solution allocations for different degrees of efficiency. Sy stablishes that whenever we face identical individuals, then the equal division of the available resources is a solution allocation. An points out that the name of the agents is irrelevant when solving a problem.

Let Λ^n be the class of transformations $\lambda:\mathbb{R}^n \longrightarrow \mathbb{R}^n$ such that for any $i \in N$, there exist $a_i \in \mathbb{R}_{++}$, $b_i \in \mathbb{R}$, such that for all $x \in \mathbb{R}^n$, $\lambda_i(x) = a_i x_i + b_i$. For $\lambda \in \Lambda^n$, define $\lambda(S) = \{ y \in \mathbb{R}^n | \exists\ x \in S \text{ with } y = \lambda(x) \}$. Let T^n be the subclass of Λ^n with $b_i = 0$ for all $i \in N$.

Consider now the following axioms:

Axiom of Cardinality and Non Comparability (CNC). Let ξ, $\xi' \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, $\xi' = \langle u',\ell,w,c\rangle$, with $u' = \lambda u$, $\lambda \in \Lambda^n$. Then, $F(\xi) = F(\xi')$.

Axiom of Ordinal Level Comparability (OLC). Let ξ, $\xi' \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, $\xi' = \langle u',\ell,w,c\rangle$, with $u' = \tau u$, $\tau \in T^n$. Then, $F(\xi) = F(\xi')$.

CNC and OLC deal with two problems in which all the physical data are identical and the only difference relies in the utilities. If these utilities are related by allowed transformations, then the solutions must be identical. In CNC the allowed transformations belong to Λ^n, and therefore we are confined to a universe in which utilities are cardinal and no interpersonal comparability can be performed. Under OLC, the transformations belong to T^n, and in consequence, interpersonal comparability is possible, and utilities are ordinal. These axioms play the role of Scale and Translation Invariance, respectively.

*Axiom of Boundedness (BDD).*For all $\xi \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, $u(0) \leq \mu_F(\xi) \leq u(c)$.

Previous axiom has the same meaning as in traditional bargaining problems with claims. BDD requires that at any solution allocation no agent is either better off than he is at the claims point or worse off than he is under no goods.

The axioms of consistency we present below are two different strengthening of CON.

Axiom of Strong Consistency (CON).-* Let $\xi \in \sum^n$, $\xi = \langle u,\ell+\ell',(w,w'),(c,c')\rangle$ be an environment such that $(\bar{x},\bar{y}) \in F(\xi)$, where any of the y-goods $(k=\ell+1,\dots,\ell+\ell')$ is a personal good for at most one of the agents. Define v by $v_i(x_i) = u_i(x_i,\bar{y}_i)$. Consider now the environment $\xi' = \langle v,\ell,w,c\rangle$. If $\xi' \in \sum^n$, and $v(0) = u(0)$, $v(c) = u(c,c')$, then $\bar{x} \in F(\xi')$.

Axiom of Rational Strong Consistency (RCON).-* Let $\xi \in \sum^n$, $\xi = \langle u,\ell+\ell',(w,w'),(c,c')\rangle$ be an environment such that $(\bar{x},\bar{y}) \in F(\xi)$, where any of the y-goods $(k=\ell+1,\dots,\ell+\ell')$ is a personal good for at most one of the agents. Define v by $v_i(x_i) = u_i(x_i,\bar{y}_i)$. Consider now the environment $\xi' = \langle v,\ell,w,c\rangle$. If $\xi' \in \sum^n$, and $v(0) \geq u(0)$, $v(c) = u(c,c')$, $u(\bar{x},\bar{y}) \geq v(0)$, then $\bar{x} \in F(\xi')$.

CON* will play the role of *Independence of Irrelevant Alternatives* [Chun & Thomson (1992)], and RCON* is the version of *Rational Contraction*

Independence other than claims point [Marco (1994c)] in economic environments.

As for the monotonicity axiom, next is the version in economic environments:

Axiom of Resource Monotonicity (R.MON).- Let ξ, $\xi' \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, $\xi' = \langle u,\ell,w',c\rangle$, with $w' \geq w$. Then $\mu_F(\xi') \geq \mu_F(\xi)$.

We can now define the *Proportional Mechanism* P in economic environments: For any $\xi \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, $P(\xi) = \{\ x \in Z(\xi) |\ u(x) = \lambda u(c) + (1-\lambda)u(0),\ 0 \leq \lambda \leq 1,\ x \in WPO(\xi)\ \}$.

Similarly, we may consider the *Claim-Egalitarian Mechanism* E, as follows: For any $\xi \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, $E(\xi) = \{\ x \in Z(\xi) |\ u_i(c_i) - u_i(x_i) = u_j(c_j) - u_j(x_j),\ i,j=1,\ldots,n,\ x \in WPO(\xi)\ \}$. Now, the *Extended Claim-Egalitarian Mechanism* EE, is defined in the following way: For any $\xi \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, $EE(\xi) = \{\ x \in Z(\xi) |$ if for some $i,j \in N$, $u_i(c_i) - u_i(x_i) > u_j(c_j) - u_j(x_j)$, thus $u_j(x_j) = 0$, $x \in WPO(\xi)\ \}$.

By means of previous axioms the following characterization results are obtained [see Appendix]:

Theorem 1.- The proportional mechanism is the unique full mechanism in $\sum^n$, satisfying Weak Efficiency, Consistency, Resources Monotonicity, Symmetry and Cardinality and Non Comparability.

Theorem 2.- The extended claim-egalitarian mechanism is the unique full mechanism in $\sum^n$ satisfying Weak Efficiency, Ordinal Level Comparability, Strong Consistency, Resources Monotonicity, Symmetry and Boundedness.

4. BARGAINING WITH CLAIMS IN ECONOMIC ENVIRONMENTS WITH VARIABLE POPULATION.

Let us consider now an infinite set of potential agents $I = \{1,2,...\}$, such that only a finite group of them are present in every concrete problem. Let $\mathcal{M}$ be the class of finite subsets of I. For a given $M \in \mathcal{M}$, $\mathbb{R}^M$ denotes the cartesian product of $|M|$ copies of $\mathbb{R}$ indexed by the elements of M. Let Σ^M be the class of economic environments for $|M|$ agents indexed by the elements of M. Let $\Sigma = \bigcup_{M \in \mathcal{M}} \Sigma^M$. From now on, an economic environment is a tuple $\xi = \langle M,u,\ell,w,c \rangle$, where $M \in \mathcal{M}$, and the environment $\langle u,\ell,w,c \rangle \in \Sigma^m$, where $m = |M|$.

An allocation mechanism F is a full correspondence defined on Σ associating to any economic environment $\xi \in \Sigma$ a set of feasible allocations.

Consider now the following axiom:

Axiom of Population Monotonicity (Pop.Mon). For any $M,N \in \mathcal{M}$, for any $\xi = \langle M,u,\ell,w,c \rangle \in \Sigma^M$, $\xi' = \langle N,u',\ell',w,c' \rangle \in \Sigma^N$, if $M \subset N$, $u'_M = u$, $c'_M = c$, then $\mu_{FM}(\xi') \leq \mu_F(\xi)$.

A new version of anonymity is now needed:

Axiom of Anonymity (An). For all M, M', with $|M| = |M'|$, for any $\xi \in \Sigma^M$, $\xi = \langle u,\ell,w,c \rangle$ and for any one to one mapping $\gamma: M \longrightarrow M'$, if $\xi' \in \Sigma^{M'}$, is such that $\xi' = \langle \gamma(u),\ell,w,\gamma(c) \rangle$, then $x \in F(\xi)$ iff $\gamma(x) \in F(\xi')$.

Axiom of Continuity (Con). For all N, for all sequence $\{\xi^t\}$, $\xi^t \in \sum^n$, $\xi^t = \langle u^t, \ell, w^t, c^t \rangle$, $\xi = \langle u, \ell, w, c \rangle \in \sum^n$, if $u^t \to u$ in the pointwise convergence topology, $w^t \to w$, $c^t \to c$ in the Hausdorff topology, then $\mu_F(\xi^t) \to \mu_F(\xi)$ in the pointwise convergence topology.

Now, the following characterization results are obtained:

Theorem 3.- The proportional mechanism is the unique full mechanism in $\sum$ satisfying Weak Efficiency, Boundedness, Continuity, Anonymity, Cardinality and non Comparability, Strong Consistency and Population Monotonicity.

Theorem 4.- The extended claim-egalitarian mechanism is the unique full mechanism in $\sum$ satisfying Weak Efficiency, Boundedness, Continuity, Symmetry, Ordinal Level Comparability, Rational Strong Consistency and Population Monotonicity.

5. FINAL REMARKS.

The reconstruction presented is very much in the spirit of Roemer's reconstruction of the theory of bargaining, and so are the proofs of the results. A key element in previous reconstruction is the assumption of allowing for *any number of goods*. This assumption is essential in obtaining a class of problems of the "same size" as the class of bargaining problems with claims in the classical setting, and thus it allows to characterize the different solutions with a set of properties similar to that used in the other case.

Generically, the problem of bargaining with claims can also be looked at as an extension of the "rights" or "bankruptcy" problems for the case of nontransferable utility among agents. By reference to the bankruptcy problems, Herrero (1994a) introduces a "natural way" of defining a *reference point* in any problem of bargaining with claims. Roughly speaking, starting from individual claims some natural concessions arise, giving rise to the natural reference point. This idea also appears in the *minimal right* concept of every player in NTU games in Borm et al. (1992). By suitably choosing the reference point, modifications of the aforementioned solutions can be obtained. In Herrero (1993), the modification of the proportional solution, namely the *adjusted proportional solution* is proposed and several characterizations of this solution are provided. Also, egalitarian solutions from this perspective are analyzed in Herrero (1994b). Nonetheless, the reconstruction of these late solutions in economic environments presents some specific dificulties, and we did not address this topic here.

APPENDIX: PROOFS OF THE THEOREMS

By using the characterizations of the proportional and the extended claim-egalitarian solutions in the classical setting, we may prove our characterization results, by proving that the right combination of axioms on economic environments imply the corresponding axioms in utility terms.

Chun & Thomson (1992), theorem 1, characterize the proportional solution in utility terms by means of WPO, SY, Sc. INV. and ST.MON.

Lemma 3.- Fullness, R.Mon and CON imply ST.Mon.

Proof:

1. Suppose F is a solution satisfying W and R. Mon. Consider two environments $\xi = \langle u,\ell,w,c \rangle$, $\xi' = \langle u',\ell',w',c' \rangle$ such that $u(0) = u'(0)$, $u(c) = u'(c')$, and $\mathcal{A}(\xi) \geq \mathcal{A}(\xi')$. We have to show that $\mu_F(\xi) \geq \mu_F(\xi')$.

2. Consider $\xi^* = \xi \otimes \xi'$, $\xi^* = \langle v,\ell+\ell',(w,w'),(c,c') \rangle$. We have $\mathcal{A}(\xi^*) = \mathcal{A}(\xi')$, and $v(0,0) = u(0) = u'(0)$, $v(c,c') = u(c) = u'(c')$. By W, $\mu_F(\xi^*) = \mu_F(\xi')$. Consider now $\bar{\xi}$, as in lemma 1. Again, $\mathcal{A}(\xi) = \mathcal{A}(\bar{\xi})$, $u(0) = \bar{u}(0,0)$, $u(c) = \bar{u}(c,0)$. Thus, by W, $\mu_F(\xi) = \mu_F(\bar{\xi})$.

3. By construction, $\bar{u}_i(x_i,y_i) \geq v_i(x_i,y_i)$, for all $i = 1,\ldots,n$. Construct now the Howe extensions $U_i \in \mathcal{U}^{(\ell+\ell'+1)}$, $U_i(x,y,1) = \bar{u}_i(x,y)$; $U_i(x,y,0) = v_i(x,y)$, and the flat extensions of the U_i, $V_i \in \mathcal{U}^{(\ell+\ell'+n)}$, $V_i(x,y,z) = U_i(x,y,z^i)$.

4. Consider now the environments

ζ = <V,ℓ+ℓ'+n,(w,w',1,...,1),(c,c',f)>, where $f_i = e_i$, and

$\bar{\xi}^*$ = <V,ℓ+ℓ'+n,(w,w',0);(c,c',0)>. Notice that by W, $\mu_F(\zeta) = \mu_F(\xi)$. Analogously, $\mu_F(\xi^*) = \mu_F(\bar{\xi}^*)$.

Now, consider ζ^* = <V,ℓ+ℓ'+n,(w,w',0),(c,c',f)>. If we compare ζ and ζ^*, we notice that the only difference is that in ζ there are more resources available. So, by R.Mon., $\mu_F(\zeta^*) \leq \mu_F(\zeta)$. On the other hand, $\mathcal{A}(\zeta^*) = \mathcal{A}(\bar{\xi}^*)$. We can perform the CON restriction of ζ^* to ξ^* by fixing the last goods at 0 level. Then, $\mu_F(\zeta^*) = \mu_F(\zeta^*)$, and therefore $\mu_F(\xi) \geq \mu_F(\xi')$. $\blacksquare$

Lemma 4.- Fullness, W and Sy imply SY.

<u>Proof:</u>

Suppose ξ = <u,ℓ,w,c> is such that for any permutation π:{1,...,n} --> {1,...,n}, $\mathcal{A}(\xi) = \mathcal{A}[\pi(\xi)]$, u(0) = π(u)[π(0)] and u(c) = π(u)[π(c)]. We have to see that, if x $\in$ F(ξ), then $u_i(x_i) = u_j(x_j)$, for all i,j = 1,...,n.

The idea is to construct another environment, ζ = <v,ℓ',w',c'> such that $\mathcal{A}(\xi) = \mathcal{A}(\zeta)$, u(0) = u'(0), u(c) = u'(c'), and $v_i = v_j$, $c'_i = c'_j$, for all i,j=1,...,n. As a way of illustration, consider the case n = 2, and let the Pareto frontier of $\mathcal{A}(\xi)$ be defined by the concave function $u_2 = f(u_1)$. Consider the environment ζ = <(v,v),1,(1,1),(c,c)>, defined by

$$v(x) = \begin{cases} 2ax & 0 \leq x \leq 1/2 \\ f(2a(1-x)) & 1/2 \leq x \leq 1 \end{cases}$$

where (a,a) is the symmetric point on the Pareto frontier of A(ξ), and let c such that v(c) = c'_i. v is concave, continuous and monotone. The Pareto frontier of $\mathcal{A}(\xi)$ is the set of points {[v(1-x),v(x)], o≤x≤1}. It is immediate to check that $\mathcal{A}(\xi) = \mathcal{A}(\zeta)$. $\blacksquare$

Lemma 5.- Fullness, CNC and W imply SC.Inv.

Proof:

Let ξ, $\xi' \in \sum^n$, $\xi = \langle u,\ell,w,c\rangle$, $\xi' = \langle u',\ell',w',c'\rangle$ such that there exist $\lambda \in \Lambda^n$ such that $z \in Z(\xi)$ iff $\lambda z \in Z(\xi')$. To prove: $\mu_F(\xi') = \lambda[\mu_F(\xi)]$.

Construct the environment $\zeta = \langle v,\ell,w,c\rangle$, where $v = \lambda u$. By CNC, $F(\zeta) = F(\xi)$. On the other hand, $\mathcal{A}(\zeta) = \mathcal{A}(\xi')$, $v(0) = u'(0)$, $v(c) = u'(c')$. Thus, by W, $\mu_F(\zeta) = \mu_F(\xi')$. Therefore, $\mu_F(\xi') = \mu_F(\zeta) = v(x_i) = \lambda u(x) = \lambda[\mu_F(\xi)]$, since $x \in F(\xi)$. ∎

From Lemmas 2, 3, 4, 5 and theorem 1 in Chun & Thomson (1992), the next result follows:

Theorem 1.- The proportional mechanism is the unique full mechanism in $\sum^n$, satisfying Weak efficiency, Consistency, Resources Monotonicity, Symmetry and Cardinality and Non Comparability.

Bossert (1993), theorem 1, characterizes the extended claim-egalitarian solution by means of WPO, SY, T.INV., ST.MON, BDD e IND.

Lemma 6.- CON* and fullness imply FCCI and IND.

Proof: Let us prove that IND follows. The other implication goes similarly.

1. Let $\xi = \langle u,\ell,w,c\rangle$, $\xi' = \langle u',\ell',w',c'\rangle$ such that $u(c) = u'(c')$, and $WPOB(\xi) \subseteq WPOB(\xi')$. Let $x' \in F(\xi')$ such that $u'(x') \in WPOB(\xi)$. To prove: $\mu_F(\xi) = \mu_F(\xi')$.

2. Construct $\xi^* = \xi \otimes \xi'$. $\xi^* = \langle v, \ell+\ell', (w,w'), (c,c')\rangle$, where $v_i(x_i,y_i) = \min\{u_i(x_i), u'_i(y_i)\}$. WPOB($\xi^*$) = WPOB($\xi$)$\cap$WPOB($\xi'$) = WPOB($\xi$), and $v(c,c') = u(c) = u'(c')$.

3. Consider now $\bar{\xi} = \langle \bar{u}, \ell+\ell', (w,w'), (c,0)\rangle$, where $\bar{u}_i(x_i,y_i) = u_i(x_i)$, $i=1,\dots,n$. Obviously, WPOB(ξ) = WPOB($\bar{\xi}$), $u(c) = \bar{u}(c,0)$.

4. Notice that $\bar{u}_i(x_i,y_i) \geq v_i(x_i,y_i)$ for all $i = 1,\dots,n$. Now by Howe (1987), Proposition 3, it is possible to find functions U_i: $\mathbb{R}^{\ell+\ell'+1} \longrightarrow \mathbb{R}$, such that $U_i(x_i,y_i,1) = \bar{u}_i(x_i,y_i)$, $U_i(x_i,y_i,0) = v_i(x_i,y_i)$.

Now, construct the functions V_i:$\mathbb{R}^{\ell+\ell'+n} \longrightarrow \mathbb{R}$, $V_i(x,y,z) = V_i(x,y,z_i)$, for $z \in \mathbb{R}^n$, $i = 1,\dots,n$.

5. Consider now the environment: $\zeta = \langle V, \ell+\ell'+n, (w,w',1,\dots,1), (c,0,f)\rangle$, where $f_i = e_i$, the ith vector of the canonical basis in $\mathbb{R}^n$. Thus, WPOB(ζ) = WPOB($\bar{\xi}$) = WPOB(ξ). Moreover, $V_i(c_i,0,e_i) = U_i(c_i,0,1) = \bar{u}_i(c_i,0) = u_i(c_i)$.

6. Now, construct the environment $\bar{\xi}^* = \langle V, \ell+\ell'+n, (w,w',0); (c,c',0)\rangle$. Thus, WPOB($\xi^*$) = WPOB($\bar{\xi}^*$). Moreover, $V(c_i,c'_i,0) = U_i(c_i, c'_i,0) = v_i(c_i,c'_i)$.

7. From previous constructions, $\mathcal{A}(\bar{\xi}^*) = \mathcal{A}(\zeta)$. Consider an allocation in $F(\zeta)$, $z_i = (x_i,y_i,e_i)$. Then, there must be an allocation $t_i = (x'_i,y'_i,0)$ in $Z(\bar{\xi}^*)$, inducing the same utility point, i.e., $V(z_i) = V(t_i)$. Thus, since t_i is feasible in ζ, and because of fullness of F, $t \in F(\zeta)$.

Now, by CON*, $(x'_i,y'_i)_{i=1,\dots,n} \in F(\xi^*)$, obtaining that $\mu_F(\xi^*) = \mu_F(\zeta)$. And therefore $\mu_F(\xi^*) = \mu_F(\xi)$.■

Lemma 7.- OLC, W and fullness imply T.INV.

Proof: It goes similarly as that of Lemma 5.■

By using lemmas 2, 3, 4, 6, 7 and theorem 1 in Bossert (1993), the following result is obtained:

Theorem 2.- The extended claim-egalitarian mechanism is the unique full mechanism in $\sum^n$ satisfying Weak efficiency, ordinal level comparability, strong consistency, Resources monotonicity, Symmetry and boundedness.

Chun & Thomson (1992), theorem 6, characterizes the proportional solution for a variable population by means of WPO, BDD, CONT, AN, SC.INV, FCCI and POP.MON.

In a similar way to previous lemmas we obtain now:

Lemma 8.- Fullness of F, W and Pop.Mon imply p.Mon.

Lemma 9.- Fullness of F, W and RCON* imply RCIC.

Lemma 10.- Fullness of F, W and Cont. imply Con.

Lemma 11.- Fullness of F, An and W imply A.

From lemmas 2, 5, 6, 8, 10, 11 and theorem 6 in Chun & Thomson (1992), the following result is obtained:

Theorem 3.- The proportional mechanism is the unique full mechanism in $\sum$ satisfying weak efficiency, boundedness, continuity, anonymity, cardinality and non comparability, strong consistency and population monotonicity.

Marco (1994b), theorem 1 characterizes the extended claim-egalitarian solution for a variable population by means of WPO, SY, T.INV., CONT., BDD and RCIC.

From lemmas 2, 4, 7, 8, 9, 10 and theorem 1 in Marco (1994b), we obtain:

Theorem 4.- The extended claim-egalitarian mechanism is the unique full mechanism in $\sum$ satisfying weak efficiency, boundedness, continuity, symmetry, ordinal level comparability, rational strong consistency and population monotonicity.

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