THREE KINDS OF UTILITY FUNCTIONS FROM THE MEASURE CONCEPT*

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WP-AD 95-11

* This research was made while the authors were visiting the University of Duke. We wish to thank Professor H. Moulin and the Department of Economics of this University for their hospitality, we also thank Professor E. Induráin for helpful comments in a earlier version. Financial support from the Spanish DGICTY, under project PB92-0342 is also acknowledged.

** University of Alicante.

Editor: **Instituto Valenciano de Investigaciones Económicas, S.A.**
Primera Edición Mayo 1995.
ISBN: 84-482-0991-5
Depósito Legal: V-1950-1995
Impreso por Copisteria Sanchis, S.L.,
Quart, 121-bajo, 46008-Valencia.
Printed in Spain.

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A B S T R A C T

In this paper we analyze the existence of three different kinds of continuous numerical representations for binary relations by using a finite measure defined on the space of alternatives. Each one of these kinds of representation is suitable for a different class of binary relations. Thus we obtain the traditional utility function for continuous preorders and weaker representations by dropping continuity or transitivity conditions (in particular, for lexicographical orders or acyclic relations).

Keywords: Utility Function; Measure; Numerical Representation.

1. INTRODUCTION.

The problem of the existence of numerical representations for preference relations appears when it is needed to assign a real function to a binary relation in a way that gives useful information about the preferences. There are different kinds of numerical representations depending both on the kind of preferences that they have to represent and on the form of the representation itself (single valued functions, multivalued functions, bilinear skew-symmetric forms).

In the literature exist several characterizations of the existence of utility function (see for instance Eilenberg (1941), Debreu (1954), Fishburn (1970) among others). This existence implies that the relation is a continuous preorder. For non continuous preorders, quasiorders or acyclic relations, only weaker representations exist (see for example Peleg (1970) for quasiorders, Subiza (1994) and Peris and Subiza (1995) for acyclic preference relations, or Roberts (1980) for non continuous preorders).

In this paper we propose a homogeneous study of the existence of three kinds of utility representations for different classes of preference relations, by defining all of them in the same way. The basic idea comes from the context of finite alternative sets; in this case to obtain a numerical representation for a quasiorder it is enough to account the number of alternatives less preferred than x. With an analogous principle, we will obtain in the non finite case a numerical representation by measuring the lower contour sets of the relation.

The concept of measure has already been used in the literature to obtain utility representations (Neuefeind (1972), Chichilnisky (1980), for continuous separable preorders; Candeal and Induráin (1993,1994) for continuous and separable quasiorders). By using this idea we obtain the representability of a wide class of binary relations. In particular, with respect to the above mentioned works, we weaken the transitivity condition, analyzing acyclic relations and, on the other hand, we also study the case of non separable and non continuous preferences, obtaining a representation which is suitable, for example, for the lexicographical order defined in $\mathbb{R}\times\{0,1\}$ (non separable) or defined in the euclidean space $\mathbb{R}^n$ (non continuous).

2. PRELIMINARIES.

Through the paper let X be a topological space, P an asymmetric binary relation defined on X, R the weak relation associated to P [x R y if and only if (not y P x)] and I the indifference relation [x I y if and only if x R y and y R x]. We will denote by $L_P(x)$ and $U_P(x)$ the lower and upper contour sets of x respect to the relation P:

$$L_P(x) = \{ z \in X \mid x P z \}$$
$$U_P(x) = \{ y \in X \mid y P x \}$$

The relation P is said to be *continuous* if $L_P(x)$ and $U_P(x)$ are open sets for all x in X and P is *separable* if there is a countable subset $D = \{ d_i , i \in \mathbb{N} \}$ of X such that if x P y then there exists $d_i \in D$ verifying x P d_i P y. We will denote by $\bar{P}$ the *transitive closure* of P, defined by

$$x \bar{P} y \iff \exists z_1, z_2, \ldots, z_n \in X \mid x = z_1 P z_2 P \cdots P z_n = y$$

When P is an acyclic relation $\bar{P}$ is a quasiorder (asymmetric and transitive relation).

When the set of alternatives is a topological space, the continuity of the function which represents the binary relation is a very desirable property. In such a point of view, some authors define utility functions as numerical representations satisfying continuity (see, for instance, Peleg (1970)). We also consider this point of view.

We will distinguish three different kinds of numerical representations that appear in the literature. A *utility function* for the binary relation P is a continuous real function u: $X \longrightarrow \mathbb{R}$ such that x P y if and only if u(x) > u(y). A *weak-utility function* is a continuous real function v: $X \longrightarrow \mathbb{R}$ such that x P y implies v(x) > v(y). Finally, a *pseudo-utility function* is a continuous real function w: $X \longrightarrow \mathbb{R}$ such that w(x) > w(y) implies x P y. We give the name utility representation indistinctly to any of the aforementioned kinds of utility functions.

Utility representations are useful due to the information they provide about the relation and the advantage of working with real values instead of alternatives. The information provided by the utility functions is complete and there is no difference between knowing the binary relation or the utility function. A weak-utility function has the disadvantage, with respect to a utility function, that when v(x) > v(y) the alternative x could be preferred or indifferent to the alternative y. The information given by the third kind of utility representation, the pseudo-utility functions, could be irrelevant (note that each constant function is a pseudo-utility function for every binary relation). In order to ensure that this kind of numerical representation also gives "valuable" information about the binary relation, we introduce a restricted class of this kind of representation.

Definition 1.

Let P be a binary relation defined on a topological space X. A continuous real function w: $X \longrightarrow \mathbb{R}$ is said to be a *nontrivial pseudo-utility function* for P if

a) $w(x) > w(y)$ implies $x \, P \, y$

b) if there is some $z \in X$ and some $U \in \mathcal{E}(z)$ such that

$$x \, P \, z' \quad \text{and} \quad z' \, P \, y \quad \text{for all } z' \in U$$

then $w(x) > w(y)$

where $\mathcal{E}(z)$ stands by the family of open neighborhoods of z.

The idea is that when x is "preferred enough" to y, this fact must be represented by the pseudo-utility function.

In this paper we propose a common study of these three utility representations by measuring the lower contour sets. In order to do this we suppose that there is a finite measure μ defined on a topological space such that any open set U is a measurable set and $\mu(U) > 0$. Formally, given a binary relation P defined on a topological space, with a finite measure μ, we define the real function α: $X \longrightarrow \mathbb{R}$ in the following way:

$$\text{for any } x \in X \quad \alpha(x) = \mu[L_P(x)]$$

The utility representations that we are going to obtain in this work are based in this function $\alpha(x)$ and first we are interested in its continuity. To obtain this continuity we need to introduce an additional property about the behavior of the lower contour sets. Similar conditions are introduced in the literature in order to avoid the fact that the indifferent classes have positive measure (Neuefeind (1972) directly requests this condition, $\mu[\{ a \mid a \, I \, x \}] = 0$, $\forall x$; Candeal and Induráin (1993) introduce a regularity condition with the same idea). The hypothesis we introduce is similar to these (in fact, it is not hard to prove that if the relation is continuous the condition in Neuefeind implies ours, while in any case the one proposed by Candeal and Induráin is equivalent to our condition).

Definition 2.

A binary relation defined on a topological space with a finite measure μ is said to verify the condition of *uniformity* with respect to the measure μ if for any convergent net $\{x_j\}_{j\in J}$, $\lim_{j\in J} x_j = x$, then the nets of real numbers

$$\alpha_j = \mu[\{ a \mid x \, P \, a \, , \, a \, R \, x_j \}]$$
$$\beta_j = \mu[\{ a \mid x_j \, P \, a \, , \, a \, R \, x \}]$$

converge to 0.

The next result proves the continuity of the function $\alpha(x)$ when P is a quasiorder verifying the uniformity condition.

Theorem 1.

Let X be a topological space with a finite measure μ and let P be a quasiorder on X such that it satisfies the uniformity condition. Thus the real function α: $X \longrightarrow \mathbb{R}$ defined as

$$\alpha(x) = \mu[L_P(x)]$$

is continuous.

Proof:

For any convergent net $\{x_j\}_{j\in J}$, $\lim_{j\in J} x_j = x$, then by using measure properties,

$$|\alpha(x) - \alpha(x_j)| = |\mu[L_P(x)] - \mu[L_P(x_j)]| \leq$$

$$\leq \mu[L_P(x) - L_P(x_j)] + \mu[L_P(x_j) - L_P(x)] =$$

$$= \mu[\{ a \mid a \; R \; x_j \text{ and } x \; P \; a \}] + \mu[\{ a \mid a \; R \; x \text{ and } x_j \; P \; a \}] =$$

$$= \alpha_j + \beta_j$$

and uniformity implies that these numbers can be taken as small as we wish. Thus function $\alpha(x)$ is continuous.■

In order to obtain the existence of a utility representation of a binary relation, a usual required condition is the continuity of this relation. We will introduce a weaker property with the same purpose.

Definition 3.

A binary relation P defined on a topological space X is said to be *weak-continuous* if whenever $x \; P \; z \; P \; y$ there are $z_1, z_2 \in X$ (probably $z_1 = z_2 = z$) and $U \in \mathcal{E}(x)$, $V \in \mathcal{E}(y)$ such that

$$\forall a \in U \qquad a \; P \; z_1 \; P \; y$$

$$\forall b \in V \qquad x \; P \; z_2 \; P \; b$$

It is clear that continuity implies weak continuity. The converse, even by adding separability, is not true. The next example shows this fact, and the following Lemma establishes the relationship between both concepts of continuity.

EXAMPLE 1.

Let $X \subseteq \mathbb{R}^2_+$ far enough from the origin and let P be the binary relation defined by

$$x,y \in X \qquad x\ P\ y \iff \sum x_i > \sum y_i \text{ and } \|x-y\| \geq 1$$

where $\|a\|$ stands by the euclidean norm of a vector. This relation is acyclic, separable ($D = X \cap (\mathbb{Q}\times\mathbb{Q})$) and weak-continuous. Nevertheless it is not continuous.

Lemma 1.

Let X be a topological space and let P be a separable weak-continuous acyclic relation defined on X. Then $\bar{P}$ is continuous.

Proof:

Let $y \in L_{\bar{P}}(x)$. Thus there exist $z_1,\dots,z_{n-1},z_n$ such that

$$x = z_1\ P \cdots P\ z_{n-1}\ P\ z_n = y$$

Note that by the separability condition we can obtain a chain of strict preferences between x and y which is as long as we wish. Applying weak-continuity to the sequence

$$z_{n-2}\ P\ z_{n-1}\ P\ z_n = y$$

there is some $z \in X$ and some $U \in \mathcal{E}(y)$ such that

$$\forall\ b \in U \qquad\qquad z_{n-2}\ P\ z\ P\ b$$

so x $\bar{P}$ b , $\forall$ b $\in$ U, which implies U $\subseteq$ $L_{\bar{P}}(x)$, and thus the lower contour set of $\bar{P}$ is open. With an analogous argument, $U_{\bar{P}}(x)$ is open and $\bar{P}$ is continuous.■

From the above Lemma, as every quasiorder coincides with its transitive closure, it is deduced immediately that for a separable quasiorder continuity and weak-continuity are equivalent conditions.

3. REPRESENTABILITY RESULTS.

Theorem 2.

Let X be a topological space with a finite measure μ and let P be a weak-continuous and separable preorder that satisfies uniformity. Then

$$u(x) = \mu[L_P(x)]$$

is a utility function.

Proof:

Theorem 1 provides the continuity of u(x) and Lemma 1 says that P is continuous. To prove that u(x) represents the relation P, let $x,y \in X$ such that x P y. The continuity and separability of P imply that there is an open set U such that

$$U \subseteq L_P(x) - L_P(y)$$

and then, by using measure properties

$$u(y) < u(y) + \mu[U] = \mu[L_P(y) \cup U] \leq \mu[L_P(x)] = u(x).$$

In order to prove the converse, let $x,y \in X$ such that $u(x) > u(y)$. Note first that this implies x R y. If x I y then $L_P(x) = L_P(y)$ and thus $u(x) = u(y)$. So necessarily x P y.■

It is well known that there are continuous preorders that can be represented by a utility function and do not verify the uniformity condition (for instance, preorders in $\mathbb{R}^n$ with indifferent classes whose Lebesgue measure is positive). However it is possible to obtain a utility function by using a particular measure of the lower contour sets (see Candeal and Induráin (1993)).

The next result proves that when the transitivity of R is weakened we can obtain a weak-utility function which represents P.

Theorem 3.

Let X be a topological space with a finite measure μ and let P be an acyclic weak continuous and separable binary relation, such that the uniformity condition is verified by the transitive closure of P. Then the function

$$v(x) = \mu[L_{\bar{P}}(x)]$$

is a weak-utility function representing P.

Proof:

By using Lemma 1, relation $\bar{P}$ is a continuous quasiorder; the separability of P implies that $\bar{P}$ is also separable. Reasoning as in Theorem 2 we obtain that v(x) is a continuous weak-utility function for the relation $\bar{P}$. It is immediate that v(x) also is a weak-utility function for P.■

This theorem is a generalization of Theorem 3 in Candeal and Induráin (1993) since they use a continuous quasiorder to obtain a weak-utility function.

On the other hand, there are interesting relations that do not satisfy the conditions in Theorems 2 and 3; this is the case of non separable or non continuous preorders (important examples are the aforementioned lexicographical orders: it is well known that no utility nor weak-utility functions representing these orders exist). In the next theorem we present an existence result of nontrivial pseudo-utility function which is applicable to those examples.

Theorem 4.

Let X be a topological space with a finite measure μ and let P be a preorder such that the uniformity condition is verified. Then the function

$$w(x) = \mu[L_P(x)]$$

is a nontrivial pseudo-utility function representing P.

Proof:

Theorem 1 gives us the continuity of the function w(x). Let be $x,y \in X$ such that $w(x) > w(y)$. If y R x, as R is a preorder $L_P(x) \subseteq L_P(y)$ and then $w(x) \leq w(y)$ which is not possible. Then x P y. Thus it only remains to prove that the representation is nontrivial. To do that, if we suppose that x P y and that there exist some $z \in X$ and some $V \in \mathcal{E}(z)$ such that for all $z' \in V$, x P z', z' P y then

$$V \subseteq L_P(x) \quad \text{and} \quad V \cap L_P(y) = \emptyset$$

as $L_P(y) \cup V \subseteq L_P(x)$ and $\mu[V] > 0$

$$w(y) = \mu[L_P(y)] < \mu[L_P(y)] + \mu[V] = \mu[L_P(y) \cup V] \leq$$

$$\leq \mu[L_P(x)] = w(x)$$

and this is a nontrivial representation.■

Finally it could be interesting to analyze converse results of Theorems 2, 3 and 4: which conditions on the binary relation are necessary for the existence of the different kinds of utility representations?. In the first two cases there are known results: when a utility function representing a binary relation exists, this fact implies that the relation is a continuous preorder and, if X is a connected topological space, separability is also a necessary condition; in a similar way, when a binary relation is representable by a weak-utility function it is an acyclic relation. However, the existence of a nontrivial pseudo-utility function does not imply any of the aforementioned conditions on the relation P, as the following example shows.

EXAMPLE 2.

Let $X = [0,4]$ and let the binary relation defined by:

$a \in [0,1]$, $b \in (1,2]$	implies	$b \; P \; a$
$b \in (1,2]$, $c \in (2,3]$	implies	$c \; P \; b$
$c \in (2,3]$, $a \in [0,1]$	implies	$a \; P \; c$
$d \in (3,4]$, $x \in [0,3]$	implies	$d \; P \; x$

Thus the function

$$w(x) = 0 \quad x \in [0,3] \ , \ w(x) = x - 3 \quad x \in (3,4]$$

is a continuous nontrivial pseudo-utility function, although P is non acyclic, non separable and non continuous.

When X is a connected topological space and the relation is continuous, the existence of a nontrivial pseudo-utility function implies that the relation is a separable preorder.

Theorem 5.

Let X be a connected topological space and let P be a continuous binary relation such that a nontrivial pseudo-utility function representing it exists. Then P is a separable continuous preorder.

Proof:

Let P* be the binary relation defined by

$$x \; P^* \; y \quad \text{if and only if} \quad w(x) > w(y)$$

where w(x) is the pseudo-utility function representing P. Being w(x) a continuous function, P* is a continuous and separable ($D = w^{-1}(\mathbb{Q})$) preorder. We will prove that P* coincides with P.

If x P* y, thus w(x) > w(y) and since w(x) is a pseudo-utility for P, x P y. On the other hand, if x P y, there is some $z \in X$ such that x P z P y (this property is obtained from the continuity of P and the connectedness of X; note that this does not imply separability since a countable subset which satisfies this property does not necessarily exist). Thus by continuity there is some $U \in \mathcal{E}(z)$ such that $x \; P \; z' \; P \; y$ for all $z' \in U$ and, by nontriviality of the pseudo-utility function, $w(x) > w(y)$; that is, $x \; P^* \; y$.■

It must be noted that in the proof of this theorem we have not used the measure definition of the pseudo-utility function and it is, then, a general result.

FINAL COMMENTS.

It would be interesting to obtain a "regularity" condition independent of the measure used in the construction of the utility function. In order to do so, the following condition about convergence of sets could be used:

If A_1, A_2,... are a sequence of subsets we define the following set operations:

$$\limsup A_n = \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$$

$$\liminf A_n = \bigcup_{n=1}^{\infty} \bigcap_{n=k}^{\infty} A_k$$

If $\limsup A_n = \liminf A_n = A$, A is said to be the *limit* of the sequence and we write $A = \textit{limit}\ A_n$.

Note that this definition of set convergence does not coincide, in general, with the usual notion of set convergence in the Hausdorff topology. By using this set convergence, we will say that a binary relation defined in a metric space X has *convergent lower sets* if for any convergent sequence $\{x_n\}$, $\lim x_n = x$, then

$$\textit{limit}\ [L_P(x_n)] = L_P(x)$$

It is now immediate, from the properties of the measure (see, for instance Ash (1972)), that if a binary relation P has convergent lower sets then for any finite measure μ defined in the space of alternatives X the function

$$\alpha(x) = \mu[L_P(x)]$$

is continuous.

To require this property is, in general, a much stronger assumption than the regularity or uniformity conditions. Although the results we obtain are presented using the uniformity condition, all of them can obviously be rewritten in terms of the convergence of lower sets.

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