EXCHANGE-PROOFNESS OR DIVORCE-PROOFNESS? STABILITY IN ONE-SIDED MATCHING MARKETS*

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Exchange-Proofness or Divorce-Proofness? Stability in One-Sided Matching Markets

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Abstract

Two stability concepts for one-sided matching markets are analyzed: Gale-Shapley stability and ξ-stability. The first one applies best to markets were no status quo allocation is considered, whereas the second one is a solution to be used when property rights are allowed. A common problem of existence is shared by both solution concepts. Hence, we study economic environments where this problem does not exist, and present a family of agents' preferences for which existence is guaranteed for both Gale-Shapley stable and ξ-stable outcomes.

Keyworks: Matching markets, Roommate problem, Stability.

1. Introduction

This paper studies allocation problems for one-sided matching markets. These markets were introduced by Gale and Shapley [8] as the "roommate market problem.[1]" They consider a set of students to be allocated in different rooms, in such a way that each one must share a room with another student. Given an allocation, the utility it produces to each student is uniquely determined by who her roommate is.

We can describe several other situations where the same type of allocation problem arises. The chess matching problem, in which some agents are matched for playing some chess matches; or the rural school problem, where some schools should be shared by rural populations, are additional examples for one-sided matching markets. Because there is a great diversity of problem so qualified, it is difficult to find a single suitable definition to describe what a "good allocation" should be for any of them.

The main solution concept analyzed by the literature on one-sided matching was introduced by Gale and Shapley [8]. These authors propose some properties to be satisfied by any allocation to be considered a satisfactory solution. The main solution they analyze is a stability property for the allocations. They propose a concept of stability involving an idea of collective rationality. This solution, we will refer as GS-stability, is also proposed for marriage problems. This is why we say that GS-stability is a "divorce-proof" based solution.

The central hypothesis for their condition is that any pair of agents can become roommates if they both wish to. We consider this hypothesis satisfactory only when no allocation provides property rights to the agents. Yet, there are one-sided matching markets where each allocation gives some property rights to the agents. Then, the actions of any pair of agents may have an impact on third parties, and these may have a say if their property rights are affected. Since GS-stability does not allows for such considerations, we challenge its general usefulness and explore an alternative solution concept. ξ-stability should be useful to describe a collective rationality idea for contexts where property rights are allowed. Allocations satisfying this requirement will conform to a notion of "exchange-proofness" within markets.

This concept does not want to be an alternative to the GS-stability concept, but rather complement it in order which will be suitable to cover those types of one-sided matching markets where GS-stability does not fare well. That is, given any one-sided matching market, any satisfactory allocation should be either ξ-stable if it allows property rights to the agents or GS-stable if not.

Both solution concepts present some good properties, but also share a common prob-

[1]Roommate markets and one-sided matchings are terms used interchangeably throughout the paper.

lem. Given a one-sided matching market, both the set of its GS-stable outcomes and the set of its ξ-stable allocations may well be empty. The direction we explore to avoid this problem is extensively employed in Economic Theory. We look for sufficient conditions on economic environments under which positive results to stability problems become possible. We will present classes of individual preferences where neither the set of GS-stable allocations nor the set of ξ-stable outcomes are empty. These are economic environments in which agents' preferences are restricted in plausible ways. One of these environments considers agents who present preferences for diversity. Another one introduces agents who are narcissistic. In a third case, agents have identical preferences, which are defined in a general consensus on how agents should be ordered. In all three cases, there exists a unique GS-stable allocation, which moreover is ξ-stable. Therefore a matchmaker can ensure ξ-stability of the allocation she proposes whenever it is GS-stable, reconciling both exchange-proof and divorce-proof ideas for allocations in one-sided matching markets.

The remainder of the paper is organized as follows. Section 2 is devoted to an introduction of the model, and gives some examples for roommate markets. Section 3 presents and discusses the stability concepts above mentioned. Section 4 presents three economic environments where either GS-stable and ξ-stable allocations always exist. Section 5 presents the main results. Conclusions are gathered in Section 6.

2. One-sided matching models

Let N = {1, 2, ..., n} be the set of agents. This set stays fixed throughout the paper. Each agent i is endowed with a complete asymmetric transitive preference relation on $N\backslash\{i\}$. We denote agent i preferences by P(i), and represent it by an ordered list of agents on $N\backslash\{i\}$. In a four-agents roommate market, P(1) = 2 3 4 would mean that agent 1's best alternative is to be matched with agent 2; and having agent 3 as mate is better than to be matched with 4. An ordered list {P(1), P(2), ..., P(i), ..., P(n)} containing each agent' preferences is called a profile and denoted by $\tilde{P}$.

A roommate market can be fully described by a pair $\left\{N, \tilde{P}\right\}$ where N is the set of agents, and $\tilde{P}$ are their preferences.

Definition 2.1. *Let* $\left\{N, \tilde{P}\right\}$ *be a roommate market. A solution for this problem, called* a matching, *is a function* ρ *on N onto itself of order two.* ($\rho\,[\rho(i)\,] = i$ *for each i in N.)*

Next, we present some examples for one-sided matching markets. They will be useful in justifying why stability concepts, other that the proposed by Gale and Shapley [8], should be analyzed.

Example 2.2. *The Chess Matching Problem*

Consider a set of people who want to play some chess matches. Each agent has preferences on her rival. Because all the agents like to play chess, each agent prefers to share a chess board with somebody else rather than remaining with no opponent.

The problem to solve is how to match those agents who are looking for a chess rival.

Example 2.3. *The Roommate Market Problem*

Consider a set of agents having some rooms to share, in such a way that each room has to have exactly two people. For simplicity, consider that the rooms are all identical; this allows us to assume that an agent will prefer one allocation to another depending solely on who his roommate is.

The problem to solve here is how should people be allocated.

All the above examples are one-sided matching problems. Apparently they should be treated in a similar way, but we will provide arguments why sometimes we should give them a different treatment.

Our main considerations are based on the (possible) existence of property rights. We can find one-sided matching markets with a distinguished starting point (an initial allocation, or status quo), and others where no allocation has such a special status. We cannot analyze both cases with the same instruments.

In order to clarify the effect of property rights in one-sided matching markets, let us consider the chess matching problem introduced in Example 2.2. Assume that n is even and there is n/2 chess boards to be shared. Suppose that a matchmaker proposes an allocation to be implemented. How should this allocation be interpreted? Two answers can be expected to this question: (a) This is a way for the matchmaker to interact with the chess players. Given an allocation, agents can propose matchings, which are alternatives to this one. The allocation proposed by the matchmaker should or should not be implemented depending on the alternatives presented by the agents. The second interpretation we propose is (b) this allocation plays an initial endowment role, and provides some property rights to the agents. We can think on a chess matching as a function that assigns a place in a (specific) chess board, in such a way that no more than two agents are assigned to the same chess board. Finally, given an initial matching, or status quo, any group of agents can exchange their own property rights when they agree to it. Therefore, whe shall distinguishthe case where a matchmaker *proposes* a matching to be implemented from that where the matchmaker *implements* a matching.

According with the above classification, we think that GS-stability concept is the most suitable solution for problems of the first type; whereas the most useful solution concept for the second one is ξ-stability. Actually, in many cases where agents have to share a room, there is a status quo, and we find ξ-stability especially appropriate. Think of a department chairman trying to reallocate shared office space (or anything else, for that matter) among faculty members!

3. Stability for one-sided matching markets

This section is devoted to analyze two stability concepts for roommate markets. Each of these concepts reflects the cases where property rights are allowed and those in which property rights are not considered. The first case is considered in Section 3.b, under the concept of ξ-stability, whereas the second case is introduced by GS-stability, in Section 3.a.

3.a. Gale-Shapley stability

Consider the chess matching problem introduced in Example 2.2. Suppose that a match-maker proposes an allocation and asks to the agents how can she do to improve their utility. The question that arises is: when can she predict that this allocation will be supported by the agents?

Suppose that two agents prefer to share a match rather than to keep on in the present situation. This allocation should no last, because these agents shall change it if they both can left the market. We will say that such an allocation is GS-unstable, because these agents form a blocking pair for the proposed matching.

Definition 3.1. *Let $\left\{N,\, \tilde{P}\right\}$ be a roommate problem, and ρ be a matching for this market. We say that ρ is* GS-stable *for $\left\{N,\, \tilde{P}\right\}$ if there is no pair of agents in N each one preferring the other to her mate in ρ.*

$$\rho \text{ is GS-stable in } \left\{N,\, \tilde{P}\right\} \text{ iff } \forall\, i,\, j \in N, [j\ P(i)\ \rho(i)] \Longrightarrow [\rho(j)\ P(j)\ i] \qquad \text{(I)}$$

Given a roommate market $\left\{N,\, \tilde{P}\right\}$ we denote the set of its GS-stable allocations by $S(\tilde{P})$.

We next define the core of roommate markets in which no property rights are allowed.

Definition 3.2. *Let $\left\{N,\, \tilde{P}\right\}$ be a roommate problem, and let ρ and ρ' be two matchings for this market. We say that ρ' dominates ρ if there is a non-empty set of agents S in N, called a coalition, for which*

(i) $\rho'(i) \in S$, for each i in S, and

(ii) $\rho'(i)\ P(i)\ \rho(i)$, *for all i in S.*

Given a roommate market $\left\{N,\ \widetilde{P}\right\}$ *we define its* core, *and denote it by* $C(\widetilde{P})$, *as the set of its undominated matchings.*

Our first result is the equivalence between the collective rationality underlying the core concept and the equilibrium idea contained in GS-stability. This result extends Theorem 3.3 in Roth and Sotomayor [12] for the marriage problem case: "the core and the set of GS-stable allocations coincide for each marriage market." Since every marriage market can be seen as a special case of one-sided matching markets -Gusfield and Irving [10]- our Proposition 3.3 extends the previous result.

Proposition 3.3. *The core of each one-sided matching market coincides with the set of its GS-stable outcomes.*

This result reinforces the desirability of implementing GS-stable outcomes, but Gale and Shapley [8] warn us. Finding GS-stable outcomes for roommate markets can be an impossible job. They present a roommate market whose core is empty.

3.b. ξ-stability

In this section we are going to introduce ξ-stability, a different concept of stability for roommate markets. Through this concept we want to express a requirement of exchange-proofness for the case in which each matching endows agents with property rights.

Think of roommate problems as a class of exchange markets. Given an initial allocation, agents can exchange their "endowments" to improve their utility. An allocation will be called ξ-stable when no such exchanges are made by any group of agents.

Definition 3.4. *Let* $\left\{N,\ \widetilde{P}\right\}$ *be a roommate problem, and* ρ *be a matching for this market. We say that* ρ *is* ξ-stable *for* $\left\{N,\ \widetilde{P}\right\}$ *if no set of agents in N can improve their utility by exchanging their mates by* ρ.

$$\rho \text{ is } \xi\text{-stable iff } \nexists\ S \subseteq N,\ S \neq \emptyset,\ \text{s.t. } \forall\, i \in S\ \exists\, j \in S \text{ s.t. } \rho(j)\ P(i)\ \rho(i) \qquad \text{(II)}$$

Given a roommate market $\left\{N,\ \widetilde{P}\right\}$, *we denote the set of its* ξ*-stable matchings by* ξ-$S(\widetilde{P})$.

ξ-stability involves an idea of stability in exchange markets. Notice that those are markets with a high level of externalities affecting all agents. The role of an "initial endowment" is played by the agents themselves. This is perhaps the main reason for which the set ξ-S$(\widetilde{P})$ may be empty for some markets. Our Example 3.5 presents a roommate market whose ξ-stable set is empty.

Example 3.5. *Consider a four agents market where agents' preferences are summarized in the following table:*

$$P(1) = 2\ 3\ 4 \qquad P(2) = 4\ 1\ 3$$

$$P(3) = 1\ 4\ 2 \qquad P(4) = 3\ 2\ 1$$

The ξ-stable set for this market is empty. Observe that if agents 1 and 2 are mates, then agents 2 and 3 can improve their utilities by exchanging their own mates. When agent 1's mate is 3, then 1 and 4 can win in terms of utility by exchanging their own mates. If 1 and 4 are matched, any change should improve the utility of all the agents.

We can now establish the independence between the GS-stability and ξ-stability concepts.

Proposition 3.6. *The sets ξ-$S(\tilde{P})$ and $S(\tilde{P})$ are mutually independents.*

Proof. Let $\{N', \tilde{P'}\}$ be a four agents market where P′(1) = 2 3 4; P′(2) = 3 1 4; P′(3) = 1 2 4, and P′(4) = 1 2 3. Gale and Shapley [8] shown that S($\tilde{P'}$) is empty. It can be easily verified that the matching ρ', where $\rho'(1) = 3$, and $\rho'(2) = 4$ is ξ-stable for this market. Then ξ-S($\tilde{P'}$) is not contained in S($\tilde{P'}$).

On the other hand, Example 3.5 provides a market whose ξ-stable set is empty. Furthermore, the matching ρ where $\rho(1) = 2$, and $\rho(3) = 4$ is GS-stable for this market. ∎

4. Roommate environments with stable allocations

This section focuses on studying the existence of non trivial restrictions on the agents' preferences for which GS-stable and/or ξ-stable allocations always exist. The economic environments we analyze in this section describe agents with some kind of "standardized" behaviour. We will present three classes of preferences to be called snug, heterophilic and cloned, respectively. Snug preferences describe a kind of conservative agents. They always prefer the roommate whose traits are more similar to their own characteristics. On the contrary, heterophilic preferences describe adventurous agents. An heterophilic agent will always prefer a mate with opposite traits than one similar to her. Finally, all agents with cloned preferences will arrange their roommates in the same way. Each one of these types of preferences can be justified in different situations.

4.a. Snug preferences

Definition 4.1. *Let N be a set of agents, and H be a linear order on N.*[2] *We say that agent i's preferences are* snug *with respect to H if, for each pair of agents, j and k in* $N\setminus\{i\}$*, the following holds:*

$$\begin{array}{lll} (i) & k\,H\,j\,H\,i & \Longrightarrow\; j\,P(i)\,k, \text{ and} \\ (ii) & i\,H\,j\,H\,k & \Longrightarrow\; j\,P(i)\,k \end{array}$$

We say that a roommate market $\left\{N,\,\tilde{P}\right\}$ *is sung if all its agents are snug with respect to some order H.*

These preferences are presented by Bartholdi and Trick [3] as a class of single-peaked preferences. In fact, they describe this family of economic environments as those in which agents' preferences are single-peaked, and agents are also narcissistic.

The chess mate problem, Example 2.2, can be very useful to explain the idea underlying snug preferences. Consider a set of heterogeneous chess players who want to enjoy some matches. In general, a "good" chess match requires the level of play of both rivals not to be too different: bigger differences on the quality levels with players implies lower interest in the matches. Therefore, it is natural, in this case, to suppose that agents have snug preferences

[2] We can think about H as a "natural" order for the agents.

Remark 4.2. *Note that snug markets satisfy* a population consistency *property. Let* $\{N, \widetilde{P}\}$ *and* $\{N', \widetilde{P}'\}$ *be two one-sided matching markets, with* $N' \subset N$*; and for each* i *in* N'*, let* $P'(i)$ *be the restriction of* $P(i)$ *to* N'*. Then, snugness for* $\{N, \widetilde{P}\}$ *implies that market* $\{N', \widetilde{P}'\}$ *is also snug.*

4.b. Heterophilic preferences

Definition 4.3. *Let* N *be a set of agents, and* H *be a linear order on* N*. We say that agent* i*'s preferences are* heterophilic *with respect to* H *if, for each pair of agents,* j *and* k *in* $N \setminus \{i\}$*,*

$$\begin{array}{lll} (i) & j\ H\ k\ H\ i \implies & j\ P(i)\ k, \text{ and} \\ (ii) & i\ H\ k\ H\ j \implies & j\ P(i)\ k \end{array}$$

We say that a roommate market $\{N, \widetilde{P}\}$ *is heterophilic if all its agents are heterophilic with respect to some order* H*.*

Heterophilia can be seen as the intersection of two properties satisfied by agents' preferences. The first one is a property of local quasi-concavity of the preferences, whereas the second one is a property stating which is the worst alternative for each agent.

In order to present markets with agents having heterophilic preferences, let us consider the chess matching problem again. Suppose that agents are arranged on the level of quality play. A direct implication of the differences between agents' levels is the following: when two agents with different level play a match, the agent with the highest play level wins the match (and her quality play is not affected); whereas the agent with the worst level learns, and increases her play level, but does not win the match. For any player, her play level introduces a partition between her rivals. For a set of rivals, the relevant variable is "how much I will learn from the match," whereas to the other agents, the key question is "how difficult will it be for me to win the match." Consider, finally the following behaviour for agents: "If I should win, I would prefer a weaker rival than an stronger one; whereas if I should loose, I would prefer to play against the stronger one.[3]"

Notice that the population consistency property introduced in Remark 4.2 for snug preferences is also satisfied by heterophilic one-sided matching markets.

[3]The difference between heterophilic and snug agents can be seen as the difference between agents who think about chess as a play -and therefore want to enjoy it- and those who think in chess as a competition -and want to win it-.

4.c. Cloned preferences

In snug and heterophilic markets, differences in agents' preferences are based on differences between agents' characteristics. This consideration is no made for the cloned preferences case. Agents' differences are not based on the agents' preferences, but only on their own (physical) characteristics. This is the basis of the cloned preferences concept.

Definition 4.4. *Let N be a set of agents, and H be a linear order on N. We say that agent i's preferences $P(i)$, are* cloned *with respect to H if, for each pair of agents, j and k in $N \setminus \{i\}$, agent i preferences coincides with H.*

$$k\, P(i)\, j \iff j\, H\, k$$

Following with the analysis of some chess players' behaviour, consider now agents arranged by their level of quality play. Given two rivals for an agent, she prefers to play a match against the agent with the highest play quality: this is a better challenge to her. When all the agents have this behaviour, their chess market is said to be cloned.

Notice that cloned markets also satisfy the population consistency property (see Remark 4.2).

5. Some possibility results

In this section we show that in all the previous environments -snug, heterophilic and cloned markets- both the set of GS-stable and the set of ξ-stable allocations are always non-empty. Furthermore, we will see that there is a unique GS-stable allocation, which is also ξ-stable.

These results are based on two properties satisfied by these markets. The first one is the population consistence introduced in Remark 4.2. The second property is *P-reciprocity* on N. Agents i and j are said to be P-reciprocal on $N' \subseteq N$ whenever i is the P(j)-maximal element on N′, and j is the P(i)-maximal element on N′. In fact, both properties, when considered together, shape a sufficient condition for the existence of GS-stable and ξ-stable allocations for one-sided matching markets.

Lemma 5.1. *Let* $\left\{N,\ \tilde{P}\right\}$ *be a roommate market, and let i and j be two P-reciprocal agents on N. Let* ρ *be a matching for this market. Then,* ρ *is GS-stable (*ξ*-stable) for* $\left\{N,\ \tilde{P}\right\}$ *if it satisfies that:*

(i) $\rho(i) = j$, *and*

(ii) the restriction of ρ *to* $N\setminus\{i, j\}$ *is GS-stable (*ξ*-stable, resp.) for the market* $\left\{N\setminus\{i, j\},\ \tilde{P}\big|_{N\setminus\{i,j\}}\right\}$.

Furthermore, ρ *is GS-stable only if it satisfies conditions (i) and (ii) above.*

Proof. We concentrate on the proof for GS-stability. The proof is similar for the case of ξ-stability. Let ρ be a GS-stable matching for $\left\{N,\ \tilde{P}\right\}$. It is easy to see that ρ has to satisfy (i). Suppose that ρ does not satisfy (ii). Since the restriction of ρ to $N\setminus\{i, j\}$ is a matching for the restricted market, it should be a pair of agents in $N\setminus\{i, j\}$ that blocks ρ in $\left\{N\setminus\{i, j\},\ \tilde{P}\big|_{N\setminus\{i,j\}}\right\}$. Therefore, this pair blocks ρ for $\left\{N,\ \tilde{P}\right\}$. A contradiction.

On the other hand, let ρ be a matching for $\left\{N,\ \tilde{P}\right\}$ that satisfies (i) and (ii). If ρ is GS-unstable, there should be a pair that blocks it in $\left\{N,\ \tilde{P}\right\}$. Clearly, these agents are not P-reciprocal on N. Then, GS-instability for ρ is in contradiction with (ii). ∎

We present now α-reducibility, a property satisfied by all the roommate environments introduced in Section 4.

Definition 5.2. *Let* $\left\{N, \tilde{P}\right\}$ *be a roommate market. We say that this market is* α-reducible *if, for each* $N' \subseteq N$ *with* $\#N' \geq 2$*, it contains a pair of P-reciprocal agents on* N'*.*

The α-reducibility for snug, heterophilic and cloned preferences is showed in Proposition 5.3. Existence of either GS-stable and ξ-stable allocations in α-reducible environments is established in Theorem 5.4.

Proposition 5.3. *Let* $\left\{N, \tilde{P}\right\}$ *be a roommate market. If agents in this market are snug, heterophilic or cloned, then this roommate market is* α*-reducible.*

Proof. Because snug, heterophilic and cloned markets satisfy population consistency (see Remark 4.2), we only need to prove that each of these one-sided matching market has a pair of P-reciprocal agents on N.

(a) If $\left\{N, \tilde{P}\right\}$ is snug, then there is i and j which are P-reciprocal on N

Suppose that all the agents in N are snug with respect to H. Given any j in N, let H_j denote the set $\{i \in N \text{ such that } i \ H \ j\} \cup \{j\}$. Let F be a function mapping $\bigcup_{j \in N} H_j$ onto itself defined as

$$F(H_j) = H_k \cup \{j\}, \text{ where k is the P(j)-maximal on N}$$

It can be easily seen that F has a fixed point. In fact, H_n is a fixed point for F. Since the domain of F is finite, there is a least fix point for F; it is, there exists an agent, say j^*, such that every other fixed point for F, H_j, satisfies that $j^* \ H \ j$. We argue that j^* and $j' \equiv$ $P(j^*)$-maximal on N are P-reciprocal. Clearly j^* must satisfy that $j' \ H \ j^*$. Suppose that they are not P-reciprocal on N. Then, the $P(j')$-maximal on N, say j'', satisfies that $j'' \ H \ j'$. This implies that $H_{j'}$ is a fixed point for F, which contradicts the minimal property for j^*.

(b) If $\left\{N, \tilde{P}\right\}$ is heterophilic, then there is i and j which are P-reciprocal on N

Suppose that all the agents in N are heterophilic with respect to H. Clearly the pair formed by the H-minimal and the H-maximal elements on N satisfies this property.

(c) If $\left\{N, \tilde{P}\right\}$ is cloned, then there is i and j which are P-reciprocal on N

Suppose that all the agents in N are heterophilic with respect to H. The pair formed by $\hat{\imath}$, the H-maximal element on N and $\tilde{\imath}$, the H-maximal element on $N \setminus \{\hat{\imath}\}$ are P-reciprocal on N. ∎

Theorem 5.4. *Let* $\left\{N, \tilde{P}\right\}$ *be an* α*-reducible roommate market. Then, there is a unique GS-stable allocation. This allocation is also* ξ*-stable.*

Proof. The proof is based in an inductive argument on the size of the agents. For $\#N =$ 2, the statement is trivially satisfied. Consider that the statement is satisfied for $\#N =$ k-1.

Let $\#N = k$. By α-reducibility, there is a pair of P-reciprocal agents, say i and j. Let $\tilde{\rho}$ be the unique GS-stable matching for $\left\{N \setminus \{i, j\}, \widetilde{P}'\right\}$ where $\widetilde{P}'$ is the projection of $\widetilde{P}$ to $N \setminus \{i, j\}$.

Let ρ be a matching for $\left\{N, \widetilde{P}\right\}$ satisfying

(i) $\rho(i) = j$, and

(ii) $\forall\, k \in N \setminus \{i, j\}$, $\rho(k) = \tilde{\rho}(k)$.
By Lemma 5.1 we know that ρ is the unique GS-stable allocation for $\left\{N, \widetilde{P}\right\}$. ξ-stability of ρ is also stated in Lemma 5.1. ∎

Corollary 5.5. *Let $\left\{N, \widetilde{P}\right\}$ be a roommate market. If agents's preferences satisfy any of the snugness, heterophilia or cloned properties, then there exists a unique GS-stable allocation. This allocation is also ξ-stable.*

The statement for Corollary 5.5 is a direct consequence from Theorem 5.4 and Proposition 5.3.

6. Final remarks

The paper presents two concepts of stability in one-sided matching markets. Both concepts present the common problem that existence of stable outcomes cannot be guaranteed. A way to avoid this problem is to restrict the set of economic environments that are considered in our analysis. In such a case we find a general condition, we call α-reducibility, under which there exists both GS-stable and ξ-stable allocations. Furthermore, the set of GS-stable allocations is a singleton and is contained in the set of ξ-stable allocations. That is, in α-reducible environments a matchmaker can implement an allocation for which no set of agents can be better off by themselves; furthermore, the matching can allow property rights to be exchanged by the agents, knowing that no exchange should be made. As special cases of α-reducible environments, we present snug, heterophilic and cloned markets, that describe some types of agents' behaviours that would be satisfied in some contexts.

Two of the aspects studied in our paper can be considered as a starting point for further analysis in one-sided matching markets. The first one is the analysis of markets where property rights are allowed. This paper pointed out the necessity for studying useful solutions to allocation concepts in these markets. The second one is the study of economic environments for which some solution concepts are useful. Some extensions of the α-reducibility idea can be straightforwardly found; but a problem which is not trivial is to find any extension for the α-reducibility concept (or for snugness, heterophilia or cloned cases) describing the behaviour of reasonable agents.

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