COST MONOTONIC MECHANISMS*

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ABSTRACT

We study the existence of cost monotonic selections of the core in economies with several public goods. Under quasilinear utilities there is a cost monotonic core selection mechanism if and only if the agents order the bundles of public goods equally. If this is indeed the case, any such mechanism must choose an egalitarian equivalent allocation.

The equal ordering property is no longer required in the case of economies with quasi-linear separable utility functions and separable costs. In this set up, there is essentially only one cost monotonic mechanism. Furthermore, it has to select an egalitarian equivalent allocation.

KEYWORDS: Public good, cost monotonicity, core, egalitarian equivalent allocations.

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1 INTRODUCTION

A significant research effort has been dedicated to the problem of deciding the appropriate supply of a public good, and the way in which its cost should be distributed among the members of the society who enjoy its consumption. Most of the literature has concentrated on two aspects of this problem. First, there is the controversy of designing mechanisms which induce the agents to reveal their utilities; one would expect that, in most cases, the agents have strong incentives to hide their true utility regarding the public goods. Second, as in the present work, there is the issue of selecting an optimal bundle of public goods and distributing the cost of financing their production plan among the members in the Economy.

To address this problem we take the normative approach: The solution is determined by considering some "equitable" properties which are agreed upon by the agents and express their sense of fairness. Once the relevant "ethical guidelines" have been acknowledged, one tries to pinpoint a solution complying with them. If there is one, then it is applied to the problem at hand.

A universally accepted property is Pareto Optimality. One should not consider allocations for which it is possible to improve the welfare of some agents without making the rest worse off. This property by itself has one major drawback: it is not single valued. Even worse, it contains proposals such as "one agent absorbs all the surplus" which might be objectable as unequitable.

The core property is another of the most widely accepted requirements for a solution to have. Since, the technology is jointly owned by all members of the society, it seems reasonable to require that the optimal production plan and its financing should enjoy a certain degree of unanimity. In this framework, this corresponds to the core property; a possible allocation will be objected by some coalition whenever operating the technology on its own, could improve the utility of its members.

There are, however, two main reasons due to which the core itself is not an entirely satisfactory solution concept. First, it may be empty, rendering it impossible to pick an allocation in it; another possible objection to the core is similar to the one considered above for Pareto Optimality: Quite often, the core turns out to be a very large set and there is no obvious way of picking an appropriate selection from it because there seems to be no single universal solution which would satisfy everyone's sense of fairness. This naturally leads

to the question of finding relevant situations in which there is a suitable one-point selection process.

Another property considered in the literature as being desirable is "cost monotonicity," i.e., if the publicly owned technology gets better, then no agent should be worse off. In the case of just one public good, H. Moulin ([5]) has characterized the egalitarian-equivalent solution, proposed originally by E. A. Pazner and D. Schmeidler [10], by the core property together with cost monotonicity. Unfortunately, in the setting of several public goods, these two properties taken together are not always compatible. As we show in Section 3, if agents have utility functions which are quasilinear in the private good, then there is a mechanism satisfying the two properties above if and only if they have the same ordinal (but not necessarily cardinal) preferences on public goods. Furthermore, when such a mechanism exists, it must pick an egalitarian equivalent allocation and all the possible mechanisms give the same utility profile to the agents. Thus, the results in [5] cover, essentially, all the cases for which a cost monotonic core selection mechanism is possible.

The difference between just one and several public goods is that, in the first case, there is no conflict of interests: everybody likes more of the public good. Nevertheless, with more than one public good to choose from, different agents might differ in their opinions about which ones of them should be given priority over the others creating, thus, a possible source of conflict. Clearly, under the equal ordering property, these discrepancies in priority do not arise.

One chance of getting away from the above impossibility result, is to restrict the domain of the mechanisms considered. Indeed, in Section 4 we show that there is an egalitarian equivalent mechanism characterized by cost monotonicity and core selection provided one looks only at the reduced class of "separable economies."

2   THE MODEL

We consider economies with one private good and, possibly, more than one public good. The set $\mathbb{R}^m_+$ will be the space of public goods. These are produced at a cost which is financed by the members of the society. The technology, which is jointly owned by all the agents, satisfies, in addition, the following hypothesis.

Assumption 2.1 *The cost function $c : \mathbb{R}^m_+ \longrightarrow \mathbb{R}$ is continuous, non-decreasing and $c(0) = 0$.*

For each $y \in \mathbb{R}^m_+$, $c(y)$ is interpreted as the cost of producing the bundle y. We assume that only one bundle of public goods is eventually produced. There are a finite number of agents in the economy, represented by $N = \{1, \ldots, n\}$. They have preferences on public and private goods, described by utility functions defined on a common consumption set, $\mathbb{R}^m_+ \times \mathbb{R}$. Since, we do not require the private good to be positive, in principle, private transfers of money are allowed.

Assumption 2.2 *The preference relation of agent $i \in N$, is described by a quasilinear utility function $u_i : \mathbb{R}^m_+ \times \mathbb{R} \longrightarrow \mathbb{R}$*

$$u_i(y, x) = b_i(y) + x$$

where $b_i : \mathbb{R}^m_+ \longrightarrow \mathbb{R}$ is assumed to be continuous, nondecreasing and $b_i(0) = 0$.

An economy consists of a triple $e = (N, (u_i)_{i \in N}, c)$, where N is the set of agents with utilities $(u_i)_{i \in N}$ and c is the cost function. The space of public goods and the set of consumers along with their utilities will be fixed through out this work and only the cost function will be allowed to vary.

Given an economy, $e = (N, (u_i)_{i \in N}, c)$, and a coalition $S \subset N$, a point $(y; (t_i)_{i \in S}) \in \mathbb{R}^m_+ \times \mathbb{R}^{|S|}$ is said to be an allocation for the coalition S. It is feasible for S whenever $c(y) \leq \sum_{i \in S} t_i$.

An allocation $(y; t_1, \ldots, t_n)$ is in core(e), the core of the economy e, if it is feasible for N and there is no other coalition $S \subset N$ and allocation $(z, (r_k)_{k \in S}) \in \mathbb{R}_+^m \times \mathbb{R}^{|S|}$, feasible for S, such that for all $k \in S$, $u_k(z, r_k) \geq u_k(y, t_k)$ with at least one inequality strict. The set of utilities sustainable by an allocation in core(e) is defined to be

$$U(e) = \{(b_1(y) - t_1, \ldots, b_n(y) - t_n) : (y; t_1, \ldots, t_n) \in \text{core}(e)\}$$

Given a non empty coalition $S \subset N$ and a point $y \in \mathbb{R}^m$ we define

$$b_S(y) = \sum_{i \in S} b_i(y)$$

and let

$$v(S) = \sup\{b_S(y) - c(y) : y \geq 0\} \tag{2.1}$$

denote the solution to the surplus maximization problem of coalition S. In general, the supremum in Equation 2.1 is not necessarily finite or even achieved at some point of $\mathbb{R}_+^m$. However, we will restrict our attention to economies with nonempty core. Under this hypothesis, the set

$$\arg \text{Max}\{b_N(y) - c(y) : y \geq 0\}$$

is non-empty. Otherwise, given any feasible allocation $(y; t_1, \ldots, t_n)$ we can find $z \in \mathbb{R}_+^m$ such that

$$b_N(z) - c(z) > b_N(y) - c(y) = b_N(y) - \sum_{i \in N} t_i$$

and one can take $r_1, \ldots, r_n$ such that $c(z) = \sum_{i \in N} r_i$ and at the same time, $b_i(z) - r_i > b_i(y) - t_i$ for every $i \in N$. This shows that the grand coalition can improve upon any allocation, and the core must be empty.

Thus, we will use the notation y^N to denote some point

$$y^N \in \arg \text{Max}\{b_N(y) - c(y) : y \geq 0\}$$

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In the following, the results are independent of which particular point of $\arg \operatorname{Max}\{b_N(y) - c(y) : y \geq 0\}$ we choose. Clearly, $v(N) = b_N(y^N) - c(y^N)$ is finite and so is $v(S) \leq v(N)$ for each coalition $S \subset N$.

Note that $v(N)$ maximizes total surplus and is the value to distribute among the players. The value $v(S)$ is interpreted here as the maximum net benefit coalition S can achieve by itself. A vector of payoffs $(v_1, \ldots, v_n)$ is in the core of the game v if for each coalition $S \subset N$ we have that

$$\sum_{i \in S} v_i \geq v(S).$$

The following lemma shows that

$$U(e) = \{(v_1, \ldots, v_n) : \text{For each } \emptyset \neq S \subsetneq N, \; \sum_{i \in S} v_i \geq v(S) \text{ and } \sum_{i \in N} v_i = v(N)\}$$

i.e., the core of the economy and the core of the game are essentially equivalent.

Lemma 2.3 *Let $e = (N, (u_i)_{i \in N}, c)$ be an economy satisfying Assumptions 2.1 and 2.2. A feasible allocation $(z; t_1, \ldots, t_n)$ is in core(e) if and only if for each non empty coalition $S \subseteq N$*

$$\sum_{i \in S} (b_i(z) - t_i) \geq v(S).$$

Proof
Let $(z; t_1, \ldots, t_n)$ be in core(e) and suppose there is a nonempty coalition $S \subset N$ such that

$$\sum_{i \in S} (b_i(z) - t_i) < v(S).$$

Fix $y \in \mathbb{R}_+^m$ such that

$$b_S(y) - c(y) > \sum_{i \in S} (b_i(z) - t_i)$$

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and consider the sets

$$M = \{i \in S : b_i(y) \geq b_i(z) - t_i\}$$

and

$$L = \{i \in S : b_i(y) < b_i(z) - t_i\}.$$

Note that $M \neq \emptyset$. For, otherwise, $L = S$ so

$$\sum_{i \in S} b_i(y) - c(y) \leq \sum_{i \in S} b_i(y) < \sum_{i \in S} (b_i(z) - t_i)$$

which contradicts the election of y. Thus,

$$c(y) < \sum_{i \in S} (b_i(y) - b_i(z) + t_i) \leq \sum_{i \in M} (b_i(y) - b_i(z) + t_i)$$

so M can improve upon $(z; t_1, \ldots, t_n)$ by taking $0 \leq r_i < b_i(y) - b_i(z) + t_i$ such that $c(y) = \sum_{i \in M} r_i$, since, $b_i(y) - r_i > b_i(z) - t_i$ whenever $i \in M$. This contradicts that $(z; t_1, \ldots, t_n)$ is a core allocation.

To prove the other implication, let $(z; t_1, \ldots, t_n)$ be a feasible allocation such that for each coalition $S \subset N$

$$\sum_{i \in S} (b_i(z) - t_i) \geq v(S)$$

and suppose some coalition $S \subset N$ improves upon $(z; t_1, \ldots, t_n)$ via the allocation $(x; (r_i)_{i \in S})$. Then

$$v(S) \geq \sum_{i \in S} b_i(x) - c(x) = \sum_{i \in S} (b_i(x) - r_i) > \sum_{i \in S} (b_i(z) - t_i) \geq v(S)$$

a contradiction which shows that $(z; t_1, \ldots, t_n)$ is in core(e). $\qquad\square$

We give next two examples which illustrate some of the dissimilarities between the two different settings $m = 1$ and $m \geq 2$. If $m = 1$, then the game v defined above is convex ([7]) and hence the core is large ([12], [13]). Surprisingly, the game may fail to be convex when there is more than one public good, as the next example shows.

Example 2.4 The space of public goods is $\mathbb{R}^3_+$ and there are three consumers with the following utility functions on public goods

$$b_1(y) = \sqrt{y_1 + y_2}, \quad b_2(y) = 2\sqrt{y_2 + y_3}, \quad b_3(y) = 4\sqrt{y_1 + y_3}$$

and a technology described by the cost function

$$c(y) = y_1 + y_2 + y_3.$$

Let v denote the TU game defined by Equation 2.1. A computation shows that $v(\{1,2\}) = \frac{9}{4}$, $v(\{2,3\}) = 9$, $v(\{1,2,3\}) = \frac{21}{4} + 2\sqrt{5}$ and $v(\{2\}) = 1$. Hence, v cannot be a convex game since

$$v(\{1,2\}) + v(\{2,3\}) > v(\{1,2,3\}) + v(\{2\}).$$

In contrast with the very well known case $m = 1$, in which the core is nonempty under very mild assumptions, the second example ([1]) evinces that more conditions are necessary with more than one public good.

Example 2.5 Consider two public and two agents with preferences

$$u_1(y, x) = 2y_1 + x, \qquad u_2(y, x) = 2y_2 + x.$$

The (decreasing returns to scale) technology is represented by the cost function

$$c(y) = (y_1 + y_2)^2.$$

One checks easily that $v(\{1\}) = v(\{2\}) = v(\{1,2\}) = 1$. Hence the core is void.

3   Cost Monotonic Mechanisms

Recall that the set of agents N and their utilities $(u_i)_{i \in N}$ satisfying Assumption 2.2 are fixed. Consider the set of economies with non-empty core, i.e.,

$$E = \{e = (N, (u_i)_{i \in N}, c) : c \text{ satisfies Assumption 2.1 and core}(e) \neq \emptyset\}$$

A mechanism is a mapping

$$R : E \longrightarrow \mathbb{R}_+^m \times \mathbb{R}^n$$

assigning to each economy e in E a feasible allocation. By abuse of notation, we will identify economies $e = (N, (u_i)_{i \in N}, c)$ in E with the cost function c. Accordingly, we will rather write $R(c)$ than $R(e)$ to denote the image of the mapping R. Likewise, if $R(c) = (y; t_1, \dots, t_n)$, then $u_i(R(c))$ will be used to denote $u_i(y, t_i)$.

Consider the set

$$\mathrm{Eg}(c) = \Big\{ z \in \mathbb{R}_+^m : \text{ There is } (y; t_1, \dots, t_n) \text{ such that } c(y) = \sum_{i=1}^n t_i$$
$$\text{and for each } i = 1, \dots, n, \quad b_i(z) = b_i(y) - t_i \Big\}. \quad (3.1)$$

In the one dimensional context ($m = 1$) the egalitarian equivalent level of public good ([5]) is defined as <u>the</u> maximum element of this set. The problem when $m \geq 2$ is that the supremum of that set may not be a maximum. The best one may hope for, is to select <u>a</u> maximal element which, of course, may not be unique.

Definition 3.1 ([5]) *A bundle $y \in \mathbb{R}_+^m$ is an egalitarian equivalent level of public good if it is a maximal element of the set defined by Equation 3.1. Let z be an egalitarian equivalent level of public good, a feasible allocation $(y; t_1, \dots, t_n)$ satisfying $b_i(z) = b_i(y) - t_i$ for each $i = 1, \dots, n$ is called egalitarian-equivalent. The set of all egalitarian equivalent allocations of $c \in E$ will be denoted by $EE(c)$.*

We let

$$\begin{aligned} U_{EE}(c) &= \{(b_1(y) - t_1, \dots, b_n(y) - t_n) \in \mathbb{R}_+^n : (y; t_1, \dots, t_n) \in EE\} \\ &= \{(b_1(z), \dots, b_n(z)) \in \mathbb{R}_+^n : b_N(z) = v(N)\} \end{aligned}$$

be the set of egalitarian equivalent utilities.

When preferences are quasi-linear, the egalitarian equivalent allocations are characterized by the following procedure: The efficient bundle of public goods produced is y^N and any vector $y^* \in \mathbb{R}^m_+$ satisfying $\sum_{i=1}^n b_i(y^*) = v(N)$ is an egalitarian level. The proposed cost share agent $i \in N$ has to provide for financing y^N is $t_i = b_i(y^N) - b_i(y^*)$. Note also that the egalitarian equivalent allocations are Pareto optimal ([10]).

A very natural question arising in this context is how much of the one dimensional theory can we recover? In the case of one public good, the egalitarian-equivalent solution enjoys some appealing properties. In particular, it is shown in [5] that it is the only cost monotonic core selection.

Definition 3.2 *A mechanism R is said to be cost monotonic if given two cost functions c_1 and c_2, such that $c_1(y) \geq c_2(y)$ for all y, it assigns allocations $R(c_j)$, with $j = 1, 2$, such that $u_i(R(c_1)) \leq u_i(R(c_2))$ for all $i = 1, \ldots, n$.*

One difficulty which arises in the setting of several public goods is that in most economies there is a continuum of egalitarian-equivalent allocations yielding different utilities to the agents.

Example 3.3 The economy e consists of two public goods and two consumers with quasi-linear preferences in money given by the utility functions

$$u_1(y, t_1) = b_1(y) + t_1 = 2y_1 + 2y_2 + t_1, \qquad u_2(y, t_2) = b_2(y) + t_2 = 2y_1 + t_2$$

where $y = (y_1, y_2) \in \mathbb{R}^2_+$. The cost of producing the bundle $y \in \mathbb{R}^2_+$ of public goods is

$$c(y) = y_1^2 + y_2^2.$$

It is easy to compute that $v(N) = 5$, $v(\{1\}) = 2$, $v(\{2\}) = 1$. The set of egalitarian levels is

$$\{y \in \mathbb{R}^2_+ : b_1(y) + b_2(y) = V(N)\} = \{y \in \mathbb{R}^2_+ : 4y_1 + 2y_2 = 5\}$$

All the egalitarian equivalent allocations are Pareto optimal. In contrast, only a strict subset of them are in the core. The set of utilities given by egalitarian equivalent allocations in the core is

$$U_{EE}(c) \cap \text{core}(v) = \{(v_1, v_2) : v_1 + v_2 = 5, v_1 \geq v_2, v_1 \geq 2, v_2 \geq 1\}.$$

Hence, not all the egalitarian levels provide an allocation in the core. Furthermore, there are several distributions of utilities in $U_{EE}(c) \cap \text{core}(v)$. □

The next Proposition shows that given an allocation $(y; t_1, \ldots, t_n)$ in $\text{core}(c) \cap EE(c)$, it is possible to improve the technology to obtain a new economy, say c^1, in which there is only one utility profile in $\text{core}(c^1)$. This profile of utilities corresponds to the egalitarian equivalent allocation chosen in the economy c. This result plays a key role in Theorem 3.8 below.

Proposition 3.4 *Let $e = (N, (u_i)_{i \in N}, c)$ be an economy in E and let $(x; t_1, \ldots, t_n)$ be an egalitarian equivalent allocation in $\text{core}(c)$ with associated egalitarian equivalent level z. Then there exists $c^1 \leq c$ such that in the economy $e^1 = (N, (u_i)_{i \in N}, c^1)$,*

$$U(c^1) = \{(b_1(z), \ldots, b_n(z))\}.$$

Proof

Let $(x; t_1, \ldots, t_n)$ be an egalitarian equivalent allocation in $\text{core}(c)$ and z an associated egalitarian level. Then $b_i(z) = b_i(x) - t_i$ for each $i \in N$ and $(b_1(z), \ldots, b_n(z)) \in U(c)$. Given a non empty coalition $S \subset N$, define

$$c_S(y) = \max\{0, b_S(y) - b_S(z)\}$$

and let

$$c^1(y) = \max_{\emptyset \neq S \subset N} c_S(y).$$

Clearly, $c^1(0) = 0$ and $c^1(y)$ is an increasing and continuous function.

Consider the economy $e^1 = (N, (u_i)_{i \in N}, c^1)$. We claim that $v_{c^1}(S) = b_S(z)$ for all $S \subset N$. Indeed, for each $y \geq 0$ and $S \subset N$ we have

$$c^1(y) \geq b_S(y) - b_S(z).$$

So, for each $y \geq 0$ and $S \subset N$, $b_S(z) \geq b_S(y) - c^1(y)$. Therefore, $b_S(z) \geq v_{c^1}(S)$ for every $S \subset N$. But, $c^1(z) = 0$ so

$$v_{c^1}(S) \geq b_S(z) - c^1(z) \geq v_{c^1}(S)$$

and we conclude that $v_{c^1}(S) = b_S(z)$. It follows that $\text{core}(c^1) \neq \emptyset$ and $U(c^1) = \{(b_1(z), \ldots, b_n(z))\}$.

We prove that $c^1(y) \leq c(y)$ for every $y \geq 0$. Suppose not, then there exists $y^* \geq 0$ such that
$$c^1(y^*) > c(y^*) \geq 0.$$
Hence, there exists $S \subset N$ such that

$$b_S(y^*) - b_S(z) = c^1(y^*) > c(y^*).$$

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Thus,
$$b_S(z) < b_S(y^*) - c(y^*) \le v_c(S).$$
which contradicts that $(b_1(z), \dots, b_n(z)) \in U(c)$. $\square$

In the context of quasilinear utility functions, the following definition seems appropriate to indicate that the agents have the same indifferent curves. Note that, in the present setting, different utility functions, even when they have the same indifferent curves in $\mathbb{R}_+^m \times \mathbb{R}$, are considered to represent genuinely different preferences on consumption bundles.

Definition 3.5 *We say that the agents of the economy order the bundle of public goods equally (or that the economy satisfies the equal ordering property) whenever for each $i, j \in N$ and $y, z \in \mathbb{R}_+^m$ if $b_i(y) > b_i(z)$ then $b_j(y) \ge b_j(z)$.*

That is the equal ordering property is fulfilled whenever given $y, z \in \mathbb{R}_+^m$ either $b_i(y) \ge b_j(z)$ for every $i, j \in N$ or $b_j(y) \ge b_j(z)$ for every $i, j \in N$. In other words, if a consumer likes the bundle of public goods z rather than the bundle y, then so do the other agents. This property eliminates the possible sources of disagreement among players. It, clearly, holds in the case of one public good. It is very restrictive, however, in the context of various public goods. The next result provides the justification to consider this definition: It is the only instance in which there is hope for a cost monotonic core selection to exist.

Proposition 3.6 *Let R be a cost monotonic core selection. Then all the agents order the bundles of public goods in the same way.*

Proof
Define
$$c_1(y) = \sup\{b_S(y) : S \subsetneq N\} = \sup\{b_S(y) : |S| = |N| - 1\}$$
and
$$d(y) = b_N(y) - c_1(y) = \inf\{b_i(y) : i = 1, \dots, |N|\}.$$
Note that d, being the infimum of continuous, increasing functions is also continuous and increasing. In addition, it verifies $d(0) = 0$. Given t in the image of the map d, the level set
$$E_t = \{y \in \mathbb{R}_+^m : d(y) = t\}$$

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is nonempty and divides $\mathbb{R}_+^m \setminus E_t$ into two connected components.

Fix now $\alpha \in \mathbb{R}_{++}$ and choose a continuous function $g : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ such that: (i) $g(t) = t$ for $0 \le t \le \alpha$, (ii) $g(t)$ is decreasing in $[\alpha, \alpha + 1]$ and (iii) $g(t) = 0$ for $t \ge \alpha + 1$. Remark that $0 \le g(t) \le \min\{t, \alpha\}$. Define $h : \mathbb{R}_+^m \longrightarrow \mathbb{R}$ by $h(y) = g(d(y))$ and consider the following cost function

$$c_\alpha(y) = b_N(y) - h(y).$$

Clearly, c_α is continuous. For each $y \in E_t$ we have that $c_\alpha(y) = c_1(y)$ if $t \le \alpha$, whereas $c_\alpha(y) = b_N(y)$ if $t \ge \alpha + 1$. In particular, $c_\alpha(0) = 0$.

We show next that c_α is increasing. Let $x \in E_r$, $y \in E_s$ $z \in E_t$ such that $x \le y \le z$. Since d is increasing, $r \le s \le t$. We consider different possibilities.

Suppose, first, that $r \le s \le \alpha$. Since c_1 is increasing, we have $c_\alpha(x) = c_1(x) \le c_1(y) = c_\alpha(y)$. Likewise, if $\alpha + 1 \le r \le s$, then $c_\alpha(x) = b_N(x) \le b_N(y) = c_\alpha(y)$, for b_N is increasing.

Assume now that $\alpha \le r \le s \le \alpha + 1$. Since g is decreasing along that interval, we have that $c_\alpha(x) = b_N(x) - h(x) = b_N(x) - g(r) \le b_N(y) - g(s) = b_N(y) - h(y) = c_\alpha(y)$.

Finally, if $r \le \alpha \le s \le \alpha + 1 \le t$, then $c_\alpha(x) = c_1(x) \le c_1(y) = b_N(y) - s \le b_N(y) - g(s) = b_N(y) - h(y) = c_\alpha(y) \le b_N(y) \le b_N(z) = c_\alpha(z)$.

Consider, now, the economy $e_\alpha = ((u_i)_{i \in N}, c_\alpha)$. We compute the game $v : 2^N \longrightarrow \mathbb{R}$ associated with e_α. Let $y \in E_t$. Then

$$b_N(y) - c_\alpha(y) = h(y) = g(t) \le \alpha$$

with equality only if $t = \alpha$. On the other hand, if $S \underset{\ne}{\subset} N$,

$$c_\alpha(y) = b_N(y) - h(y) = b_N(y) - g(t) \ge b_N(y) - t = c_1(y) \ge b_S(y)$$

Hence, $v(N) = \alpha$, and $v(S) = 0$ if $S \underset{\ne}{\subset} N$. It follows that $\text{core}(e) \ne \emptyset$.

Let now
$$F(\alpha) = \{y \in \mathbb{R}_+^m : b_N(y) = \alpha\}$$
be the set of egalitarian levels of the economy e_α. Since $v(S) = 0$ for $S \underset{\ne}{\subset} N$, we have that $\{(b_1(y), \ldots, b_n(y)) : y \in F(\alpha)\} \subset U(e_\alpha)$. Pick $y \in F(\alpha)$. From

Proposition 3.4, there is $c \le c_\alpha$ such that the economy $e = (\mathbb{R}^m, N, (u_i)_{i \in N}, c)$ satisfies

$$U(e) = \{(b_1(y), \dots, b_n(y))\}$$

Since, R is a core selection, we must have that $u_i(R(e)) = b_i(y)$, for each $i \in N$ and since R is also cost monotonic, $u_i(R(e)) \ge u_i(R(e_\alpha))$ for each $i \in N$. But,

$$\sum_{i \in N} u_i(R(e)) = v(N) = \sum_{i \in N} u_i(R(e_\alpha))$$

so $u_i(R(e)) = b_i(y) = u_i(R(e_\alpha))$ for each $i \in N$. Note now that $u_i(R(e_\alpha))$ does not depend on $y \in F(\alpha)$. Hence, $b_i(y) = u_i(R(e_\alpha))$ for every $y \in F(\alpha)$, that is, the mappings $b_1, \dots, b_n$ are constant on $F(\alpha)$.

Since $\alpha \in \mathbb{R}_+$ was arbitrary, we conclude that for any $i = 1, \dots, n$ and any $\alpha \in \mathbb{R}_+$, the function b_i is constant on $F(\alpha)$. It follows that for any economy $e \in E$

$$U_{EE}(e) = \{(b_1(z), \dots, b_n(z)) \in \mathbb{R}_+^n : z \in F(v(N))\}$$

consists of a single point.

Let now $y \in F(t)$, $z \in F(s)$ and suppose, for concreteness, that $t \ge s$. Then $b_N(y) \ge b_N(z)$. Since b_N is continuous and increasing, there is $\lambda \ge 1$ such that $x = \lambda z \in F(t)$. In particular, $x \ge z$, so $b_i(x) \ge b_i(z)$ for each $i = 1, \dots, n$. But the mappings b_i are constant on $F(t)$, so we must have that $b_i(y) = b_i(x) \ge b_i(z)$, for each $i = 1, \dots, n$. That is, all the agents order the bundles of public goods in the same way. $\qquad \square$

It follows, from the proof of Proposition 3.6, that whenever there is a core selection R which is cost monotonic, all the egalitarian equivalent allocations in the core give the same distribution profile of utilities to the agents. The next result constitutes the converse of Proposition 3.6. A mapping

$$R : E \longrightarrow \mathbb{R}_+^m$$

satisfying $R(c) \in EE(c)$, will be called an egalitarian equivalent mechanism.

Proposition 3.7 *Let N be a set of agents for whom Assumption 2.2 holds and who order equally the bundles of public goods. Then,*

(i) For any economy e in E, $EE(c) \subset core(c)$ and $|U_{EE}(c)| = 1$.

(ii) Any mapping

$$R : E \longrightarrow \mathbb{R}_+^m$$

satisfying $R(c) \in EE(c)$ is a cost monotonic core selection mechanism.

(iii) If T is a cost monotonic core selection, then $(u_1(T(c)), \ldots, u_1(T(c))) \in U_{EE}(c)$ for every $e \in E$.

Proof

Let $e = (N, (u_i)_{i \in N}, c)$ be an economy in E and choose $y^N \in \mathbb{R}_+^m$ such that $b_N(y^N) - c(y^N) = v(N)$. Fix a bundle of public goods $y^* \in \mathbb{R}_+^m$ such that $b_N(y^*) = v(N)$ and an agent $i \in N$. We claim that $b_i(y^*) \leq b_i(y^N)$. Otherwise, by the Equal Ordering Property, $b_k(y^*) \geq b_k(y^N)$ for every $k \in N$ so

$$v(N) = b_N(y^*) > b_N(y^N) \geq v(N)$$

which is a contradiction. It follows that $t_i = b_i(y^N) - b_i(y^*) \geq 0$, for each $i = 1, \ldots, n$.

We claim that $(y^N; t_1, \ldots, t_n)$ is in the core of the economy e. Suppose not, then there is a coalition S and an allocation $(z, (x_i)_{i \in S})$, feasible for S, such that for every $i \in S$,

$$b_i(z) - x_i > b_i(y^N) - t_i = b_i(y^*).$$

Hence,

$$\sum_{i \in S} b_i(z) - c(z) > \sum_{i \in S} b_i(y^*) \tag{3.2}$$

Since, $\sum_{i \in S} x_i = c(z) \geq 0$, there is some $k \in S$ such that $x_k \geq 0$. Thus, $b_k(z) \geq b_k(z) - x_k > b_k(y^*)$ and by the equal ordering property, $b_l(z) > b_l(y^*)$ for every $l \in N$. Adding, these inequalities for $l \in N \setminus S$ to Equation 3.2 we obtain the contradiction

$$v(N) \geq \sum_{i \in N} b_i(z) - c(z) > \sum_{i \in N} b_i(y^*) = v(N).$$

We conclude that $(y^N; t_1, \ldots, t_n)$ must belong to core(c). The utility obtained by agent $i \in N$ with the allocation $(y^N; t_1, \ldots, t_n)$ is $b_i(y^*)$. Thus, $EE(c) \subset$ core(c).

Let now z be another egalitarian equivalent level, so that $b_N(z) = v(N) = b_N(y^*)$. Since, either $b_k(z) \geq b_k(y^*)$ for every $k \in N$ or $b_k(z) \leq b_k(y^*)$ for every $k \in N$ we must have that $b_k(z) = b_k(y^*)$ for every $k \in N$. Hence, $|U_{EE}(c)| = 1$.

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Let now R be a mechanism such that $R(c) \in EE(c)$ and let $c_1 \leq c_2$ be two cost functions defined on $\mathbb{R}_+^m$ and with an associated economy having a non empty core. Let v_{c_i}, where $i = 1, 2$ denote the game associated to the economy $((u_i)_{i \in N}, c_i)$. Since, $c_1 \leq c_2$, we have $v_{c_1}(N) \geq v_{c_2}(N)$.

Suppose y (resp. z) is the egalitarian level of public good associated with the egalitarian equivalent allocation $R(c_1)$ (resp. $R(c_2)$). In particular, for each $i \in N$, $b_i(y) = u_i(R(c_1))$ and $b_i(z) = u_i(R(c_2))$. Then, $b_N(y) = v_{c_1}(N) \geq v_{c_2}(N) = b_N(z)$. And, since agents order the bundles of public goods in the same way, we must have $b_i(y) \geq b_i(z)$ for each $i \in N$. Thus R is cost monotonic.

Finally, let $T : E \longrightarrow \mathbb{R}_+^m$ be another cost monotonic core selection mechanism. Choose an economy $e = ((u_i)_{i \in N}, c)$ in E and an allocation $z \in EE(c)$. By Proposition 3.4, there is another economy $e^1 = (N, (u_i)_{i \in N}, c^1)$ with $c_1 \leq c$ and such that

$$U(c^1) = \{(b_1(z), \ldots, b_n(z))\}.$$

Since T is cost monotonic, $b_i(z) = u_i(T(e_1)) \geq u_i(T(c))$ for each $i \in N$. But

$$\sum_{i=1}^n u_i(T(c)) = v(N) = \sum_{i=1}^n b_i(z).$$

Hence, $b_i(z) = u_i(T(e_1)) = u_i(T(c))$ for each $i \in N$. $\qquad\square$

Putting toghether the above results we obtain the following conclusion.

Theorem 3.8 *Let N be a fixed set of agents, with quasilinear utility functions satisfying Assumption 2.2.*

(i) *There is a cost monotonic core selection mechanism if and only if the utility functions of the agents satisfy the equal ordering property.*

(ii) *If (i) holds, then $|U_{EE}(e)| = 1$ for any economy e in E. Furthermore, any mechanism is a cost monotonic core selection if and only if it is an egalitarian equivalent mechanism.*

4 MECHANISMS UNDER NON EQUAL ORDERING

In this section, we exhibit a particular class of economies for which the above definition of egalitarian equivalent allocations provides a core selection which is cost monotonic. As before, we use $\mathbb{R}_+^m$ and $\mathbb{R}$, respectively to denote the spaces of public and private goods.

A utility function $u : \mathbb{R}_+^m \times \mathbb{R} \longrightarrow \mathbb{R}$ is said to be separable if there are m increasing functions $b_j : \mathbb{R} \longrightarrow \mathbb{R}$, for $j = 1 \ldots, m$ such that

$$u(y, x) = \sum_{j=1}^{m} b_j(y_j) + x$$

where $y = (y_1, \ldots, y_m) \in \mathbb{R}_+^m$. Similarly, a cost function, $c : \mathbb{R}_+^m \longrightarrow \mathbb{R}$ is separable if there are m increasing functions $c_j : \mathbb{R} \longrightarrow \mathbb{R}$ where $j = 1, \ldots, m$ such that

$$c(y) = \sum_{j=1}^{m} c_j(y_j).$$

In the present context, we restrict our attention to the class of separable economies

$$E^s = \{e = (N, \{u_i\}_{i \in N}, c) : (u_i)_{i=1,\ldots,|N|}, c \text{ are separable and } core(e) \neq \emptyset\}.$$

Thus, each consumer $i \in N$ is characterized by a quasi-linear utility function

$$u_i(y, x) = b_i(y) + x = \sum_{j=1}^{m} b_{ij}(y_j) + x \tag{4.1}$$

where $y = (y_1, \ldots, y_m) \in \mathbb{R}_+^m$, $x \in \mathbb{R}$ and $b_{ij} : \mathbb{R}_+ \longrightarrow \mathbb{R}$ is nondecreasing.

We define next an egalitarian-equivalent mechanism, R, on the class of economies E^s. Fix an economy e in E^s. For each public good $j = 1, \ldots, m$, let y_j^N some point in

$$\arg \operatorname*{Max}_{z_j \geq 0} \sum_{i=1}^{n} b_{ij}(z_j) - c_j(z_j)$$

and choose $y_j^* \in \mathbb{R}$ such that

$$\sum_{i=1}^{n} b_{ij}(y_j^N) - c_j(y_j^N) = \sum_{i=1}^{n} b_{ij}(y_j^*).$$

Now let

$$y^N = (y_1^N, \ldots, y_m^N)$$

and

$$y^* = (y_1^*, \ldots, y_m^*).$$

There may be more than one y_j^* but all of them give the same utility to each consumer since the functions b_{ij} are non decreasing.

The amount of money consumer $i \in N$ has to pay to finance the public goods y^N is dictated to be $t_i = b_i(y^N) - b_i(y^*) \geq 0$. All these quantities are positive (because $y^N \geq y^*$) and the mechanism R is defined by $R(e) = (y^N; t_1, \ldots, t_n)$. Clearly, $R(e)$ is Pareto efficient since it maximizes the sum of the utilities.

Note that for each $j = 1, \ldots, m$, the allocation

$$(y_j^N; b_{1j}(y_j^N) - b_{1j}(y_j^*), \ldots, b_{nj}(y_j^N) - b_{nj}(y_j^*))$$

is the usual egalitarian equivalent allocation for one public good as in [5]. Thus, it follows from the same property of the egalitarian equivalent selection in the one public good case that R is cost monotonic.

We will prove that R is the only cost monotonic core selection mechanism. In particular, this will show that not all Egalitarian Equivalent mechanisms are cost monotonic.

Proposition 4.1 *The allocation* $(y^N; t_1, \ldots, t_n)$ *is in* $core(e)$.

Proof
Suppose not, then there exists a coalition S and a feasible allocation $(y', t_i')_{i \in S}$ such that

$$c(y') = \sum_{i \in S} t_i' \tag{4.2}$$

$$b_i(y') - t_i' > b_i(y^*) \quad \text{for every } i \in S. \tag{4.3}$$

Note, first, that $y' \geq y^*$ is not possible. Otherwise, $b_k(y') \geq b_k(y^*)$ for every $k \in N$. In particular, for every $k \notin S$. But this contradicts that $(y^N; t_1, \ldots, t_n)$ is a Pareto efficient allocation. Similarly, if $y' \leq y^*$, then $b_i(y') \leq b_i(y^*)$ for every $i \in S$, which contradicts Equation 4.3.

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We conclude that y' and y^* are not comparable. Then, the sets $M = \{j \in \{1, \ldots, m\} : y'_j \geq y^*_j\}$ and $L = \{j \in \{1, \ldots, m\} : y'_j < y^*_j\}$ are non-empty. Adding in Equation 4.3 for $i \in S$, we obtain,

$$\sum_{j=1}^{m} \left(\sum_{i \in S} b_{ij}(y'_j) - c_j(y'_j) \right) \geq \sum_{j=1}^{m} \sum_{i \in S} b_{ij}(y^*_j).$$

We split this sum into two parts

$$\sum_{j \in M} \left(\sum_{i \in S} b_{ij}(y'_j) - c_j(y'_j) \right) + \sum_{j \in L} \left(\sum_{i \in S} b_{ij}(y'_j) - c_j(y'_j) \right)$$
$$> \sum_{j \in M} \sum_{i \in S} b_{ij}(y^*_j) + \sum_{j \in L} \sum_{i \in S} b_{ij}(y^*_j). \tag{4.4}$$

If $j \in L$, then $b_{ij}(y'_j) \leq b_{ij}(y^*_j)$ for every $i \in N$. Hence,

$$\sum_{i \in S} \left(\sum_{j \in M} b_{ij}(y'_j) - b_{ij}(y^*_j) \right) > \sum_{j \in M} c_j(y'_j).$$

Thus, we can find $(x_i)_{i \in S}$ such that

$$\sum_{i \in S} x_i = \sum_{j \in M} c_j(y'_j)$$
$$x_i < \sum_{j \in M} (b_{ij}(y'_j) - b_{ij}(y^*_j)) \quad \text{for every } i \in S.$$

In addition, from the definition of M,

$$\sum_{j \in M} b_{kj}(y'_j) \geq \sum_{j \in M} b_{kj}(y^*_j) \quad \text{for every } k \notin S.$$

Consider the allocation $((y'_j)_{j \in M}, (x'_i)_{i \in N})$, where $x'_i = x_i$ if $i \in S$ and $x'_i = 0$ if $i \notin S$. Then,

$$\sum_{i \in S} x_i' = \sum_{j \in M} c_j(y_j')$$

$$\sum_{j \in M} b_{ij}(y_j') - x_i' > \sum_{j \in M} b_{ij}(y_j^*) \text{ for every } i \in S \qquad (4.5)$$

$$\sum_{j \in M} b_{ij}(y_j') \geq \sum_{j \in M} b_{ij}(y_j^*) \text{ for every } i \notin S$$

Construct a new economy as follows: Let $\mathbb{R}^p$, with $p = |M|$, be the new space of public goods. The utilities of the agents are

$$b_i(y) = \sum_{j \in M} b_{ij}(y_j)$$

for $i = 1, \ldots, n$ and the technology is

$$c(y) = \sum_{j \in M} c_j(y_j).$$

Denote by y_M and y_M^* the projections of y^N and y^* onto the space $\mathbb{R}^p$ and let

$$q_i = \sum_{j \in M} (b_{ij}(y_j^N) - b_{ij}(y_j^*)) \text{ for } i = 1, \ldots, n.$$

The allocation $(y_M, (q_1, \ldots, q_n))$ is Pareto efficient in this new economy since it maximizes the sum of utilities of consumers. The utility obtained from this allocation by each consumer is $b_i(y_M) - q_i = b_i(y_M^*)$. But Equations 4.5 show that, in the restricted economy, all consumers are no worse off with the allocation $((y_j')_{j \in M}, (x_i')_{i \in N})$ and the ones who belong to S are strictly better off. This contradicts that $(y_M, (q_1, \ldots, q_n))$ is Pareto efficient. □

Finally, we obtain the main result in this section. The only assertion left to prove in the following result is the uniqueness of the mechanism defined above.

Theorem 4.2 *The mechanism R is cost monotonic and for each $e \in E^s$, $R(e)$ is in core(e). Furthermore, if T is another cost monotonic core selection*

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mechanism, then $u_i(R(e)) = u_i(T(e))$ for each economy e in E^s and each agent $i \in N$.

Proof

Let $e = (N, (u_i)_{i \in N}, c)$ be an economy in E^s. We know that R is a cost monotonic mechanism which always yields an allocation in the core. Let y^* be the egalitarian level of the allocation $R(c)$, so that $v(N) = b_N(y^*)$. For each $S \in N$ and $j = 1, \ldots, m$ let

$$b_{jS}(y) = \sum_{i \in S} b_{ij}(y)$$

and take a point

$$c_{jS}(y) = \max\{b_{jS}(y) - b_{jS}(y^*), 0\}$$

and

$$c_j^1(y) = \max_{S \subset N}\{c_{jS}(y)\}.$$

Construct now the cost function

$$c^1(y) = \sum_{i=1}^{n} c_j^1(y),$$

and consider the economy $e^1 = (N, (u_i)_{i \in N}, c^1)$. One can prove as in Proposition 3.4 that $c_j^1 \leq c_j$ for each $j = 1, \ldots, m$ and $v^1(S) = b_S(y^*)$, for each $S \subset N$. It follows that $U_{e^1} = \{(b_1(y^*), \ldots, b_n(y^*))\}$. Hence, if T is a cost monotonic core selection mechanism, we have that

$$u_i(R(c)) = b_i(y^*) = u_i(T(c^1)) \geq u_i(T(c))$$

whereas $u_i(T(c^1)) = b_i(y^*)$ for each $i \in N$. So by Lemma 2.3

$$\sum_{i=1}^{n} u_i(T(c)) = v(N) = b_N(y^*) = \sum_{i \in N}^{n} u_i(R(c))$$

and we conclude that $u_i(T(c)) = u_i(R(c))$, for any $i \in N$. $\qquad\square$

5   CONCLUDING COMMENTS

The problem of finding suitable "principles" which might be used in deciding the optimal production and financing scheme for the provision of a bundle of public goods, is still unresolved.

Our results indicate that there seems to be little hope for the cost monotonicity axiom. Only under very restrictive assumptions on the preferences of the consumers does there exist a core selection with this property. On the other hand, whenever these restrictions are met the egalitarian equivalent mechanisms are the only ones which are cost monotonic. Furthermore, all such mechanisms are equivalent, since they all yield the same distribution of utilities to the agents.

There is a related literature, in the context of monotonicity with respect to changes in resources ([14], [9]). The conclusion there is that Pareto Optimality and Resource Monotonicity are incompatible with other normative properties such as Individual Rationality from Equal Division or Envy-Free.

We have concentrated our attention in cost monotonic core selections. One would like to find other normative axioms which might take us beyond the impossibility results presented above. One obvious candidate is to weaken either of the properties and require, for example, a Pareto efficient selection not necessarily in the core. Or, perhaps, to formulate a less stringent version of cost monotonicity. Of course, a third alternative, would be to look for different substitutes of either (or even both) of the axioms considered here.

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