BARGAINING WITH REFERENCE POINTS - BARGAINING WITH CLAIMS: EGALITARIAN SOLUTIONS REEXAMINED*

Carmen Herrero**

WP-AD 94-15

* Financial support from the DGICYT, under project PB92-0342, is gratefully acknowledged.

** Instituto Valenciano de Investigaciones Económicas and University of Alicante.

Editor: **Instituto Valenciano de Investigaciones Económicas, S.A.**
Primera Edición Septiembre 1994.
ISBN: 84-482-0696-7
Depósito Legal: V-3156-1994
Impreso por Copisteria Sanchis, S.L.,
Quart, 121-bajo, 46008-Valencia.
Printed in Spain.

BARGAINING WITH REFERENCE POINTS-BARGAINING WITH CLAIMS: EGALITARIAN SOLUTIONS REEXAMINED

Carmen Herrero

A B S T R A C T

A unified approach to the problems of bargaining with a reference point and of bargaining with claims is addressed. Then, two solutions are reexamined from this perspective: the lexicographic egalitarian and the lexicographic claim-egalitarian. New characterization results are also provided.

Keywords: Bargaining Claims; Endogenous Reference Points.

1. INTRODUCTION

The axiomatic approach to the *Bargaining Problem*, initiated by Nash's seminal paper of 1950 has been the object of an increasing literature. Several extensions and modifications to the *pure bargaining problem* have also been the object of study during the last years, most significantly those devoted to *coalitional bargaining* [see Shapley (1969), Aumann (1985), Harsanyi (1959) (1963), Hart (1985) and Kalai & Samet (1985)], and the *noncooperative approach* [see Binmore & Dasgupta (1987), Rubinstein (1982), Sutton (1985), Osborne & Rubinstein (1990)].

Two particular modifications of the pure bargaining problem are the *bargaining with a reference outcome* [Gupta & Livne (1988)], and the *bargaining with claims* [Chun & Thomson (1992)]. These two extensions of the pure bargaining problem share the idea of adding, to the feasible set S, and the disagreement point d, a third element, in one case the *reference point*, in the feasible set, and in the other, the *claims point*, outside the feasible set. Several interpretations of both the reference point and the claims point have been provided by the quoted authors, most significantly that of *rights, expectations or previous promises*, for the claims point, and *minimal equitable agreements*, in the case of the reference point.

Both in Gupta & Livne (1988) and in Chun & Thomson (1992), solutions for their respectively modified problem were proposed and axiomatically characterized. Interestingly, in both papers a similar solution appears as the "most natural" in order to solve the aforementioned problems: the proportional solution, which, in both cases turns out to be a modification of the Kalai-Smorodinski solution for the pure bargaining problem. Thus, in both papers, the new point added to the traditional case [the reference point and the claims point, respectively], played the role of the *utopia point* in constructing the proposed solution.

Herrero (1993), proposed a suitable modification of the proportional solution for the problem of bargaining with claims, by introducing a "natural" *reference point*, in a paralell to the way *adjusted solutions* associate a reference point to any bankruptcy problem [see Auman & Maschler (1988), Curiel, Maschler & Tijs (1988) and Dagan & Volij (1993). Thus, by starting with a bargaining with claims problem, we can move to a bargaining problem with reference point in a natural way. It has to be stressed that the rationale of the *natural reference point* goes back to the idea of unanimously agreeded "minimal concessions", and if we take the natural reference point as the new disagreement level, the claims point is not the utopia point of this problem [except in the two person case].

In this paper the relationship between claims and reference point is pursued in a more precise and complete way. We can reinterpret any bargainig problem with reference point as a problem of bargaining with claims, by associating a *natural claims point* to our problem, in a consistent way, i.e., the natural reference point associated to the new bargaining with claims problem is the original one, and vice-versa, if we start with a bargaining with claims problem, and construct the associated natural reference point, we also can go the other way around, *whenever we confine ourselves to the relevant claims.*

By interpreting the reference point as a minimally equitably agreement, two solutions are reexamined: the lexicographic egalitarian solution and the lexicographic extended claim-egalitarian solution. This two solutions are chosen from the perspective of the reinterpretation of the bargaining problem when "reference points" are taken into account. Both solutions share the idea of *equalization* as far as it is possible, but respecting Pareto Optimality.

Section 2 presents the way of associating a "reference" point to any bargaining with claims problem, and the way of associating a "claims" point to any bargaining problem with a reference outcome, and presents the proposed solutions. Section 3 is devoted to the axiomatic characterization of the solutions. Section 4, with some final comments, closes the paper.

2. BARGAINING WITH CLAIMS-BARGAINING WITH A REFERENCE POINT

An *n-person bargaining problem with claims* [see Chun & Thomson (1992)], is a triple (S,d,c), where S is a subset of $\mathbb{R}^n$, d and c are points in $\mathbb{R}^n$, such that:

(i) $d \in S$, $c \notin S$; S is closed, convex and comprehensive[(1)]

(ii) there exist $p \in \mathbb{R}^2_{++}$ and $r \in \mathbb{R}$ such that for all $x \in S$, $\sum p_i x_i \leq r$.

(iii) there exists $x \in S$, $x >> d$

S is the *feasible set.* Each point x of S is a *feasible alternative.* Points d and c are the *disagreement point* and the *claims point,* respectively. The intended interpretation of (S,d,c) is as follows: the agents can achieve any point x of S if they unanimously agree on it. The coordinates of x are the utility values, measured in some Von Neumann-Morgenstern scales, attained by the agents through the choice of some joint action. Point d is the alternative at which the agents end up in the case of no agreement. Finally, each coordinate of the claims point may represent a promise made to the corresponding agent. If $c \notin S$, then the promises made to the agents are impossible to comply. In this case, we face a problem, and the only way of solving it is by choosing some compromise.

---

(1) Vector inequalities: given $x, y \in \mathbb{R}^2$, $x \geq y$, $x > y$, $x >> y$.
$\mathbb{R}^2_{++} \equiv \{ x \in \mathbb{R}^2 | \ x >> 0 \}$. S is comprehensive if for all $x \in S$, for all $y \in \mathbb{R}^2$, if $y \leq x$, then $y \in S$. In Herrero (1994), convexity of S is not assumed.

Let us call $IR(S,d) = \{x \in S \mid x \geq d\}$, the set of individually rational outcomes. Taking into account the interpretation of the disagreement point, the set $IR(S,d)$ plays a central role in obtaining any solution proposal not subject to individual veto. For $x \in \mathbb{R}^n$, $\alpha \in \mathbb{R}$, let us call (α,x_{-i}) that element in $\mathbb{R}^n$ such that $(\alpha,x_{-i})_i = \alpha$, $(\alpha,x_{-i})_j = x_j$, for $j \neq i$. The *utopia point* of the problem (S,d), $a(S,d)$, is defined in the following way: $a_i(S,d) = \max \{ \alpha \in \mathbb{R} \mid (\alpha,d_{-i}) \in IR(S,d) \}$. If we consider a bargaining with claims problem, (S,d,c), the *relevant claims point*, c^* is defined as follows: $c^*_i = \min \{ c_i, a_i(S,d) \}$.

Associated with a problem (S,d,c), a natural *reference point* can be defined [see Herrero (1993)].

$$r_i(S,d,c) = \begin{cases} \max \alpha \mid (\alpha, c_{-i}) \in IR(S,d), & \text{if } \{\alpha \mid (\alpha,c_{-i}) \in IR(S,d)\} \neq \emptyset \\ d_i & \text{otherwise} \end{cases}$$

The reference point $r = (r_1,\ldots,r_n)$ represents a "no conflict minimum point", since the utility for agent i represented by r_i is not claimed by the coalition of the (n-1) agents $N \setminus \{i\}$. So, we well may assume that this "minimal utility level" represents a "natural concession" from coalition $N \setminus \{i\}$ to agent i. Notice that $r(S,d,c) = r(S,d,c^*)$.

A *bargaining problem with a reference point* [see Gupta & Livne (1988)], is a triple (S,d,r), where S is a subset of $\mathbb{R}^n$, d and r are points in $\mathbb{R}^n$, such that:

(i) $d, r \in S$; $d \leq r$, S is closed, convex and comprehensive

(ii) there exist $p \in \mathbb{R}^2_{++}$ and $\lambda \in \mathbb{R}$ such that for all $x \in S$, $\sum p_i x_i \leq \lambda$.

(iii) there exists $x \in S$, $x \gg d$

As before, S is the *feasible set.* Each point x of S is a *feasible alternative.* Points d and r are the *disagreement point* and the *reference point,* respectively. The intended interpretation of (S,d,r) is as follows: the agents can achieve any point x of S if they unanimously agree on it. The coordinates of x are the utility values attained by the agents through the choice of some joint action. Point d is the alternative at which the agents end up in the case of no agreement. Finally, each coordinate of the reference point may represent a minimally equitable expectation the corresponding agent may have, which actually move his absolutely minimal expectations from d_i to r_i. If $r \in PO(S)$, then r is viewed as a *fair* solution of the problem. If $r \notin PO(S)$, a different solution has to be proposed, but in obtaining it, it seems fair to take r into account.

Associated with a problem (S,d,r), a natural *claims point* can be defined in the following way: Let $U_i = \{ x \in \mathbb{R}^n : (r_i, x_{-i}) \in IR(S,d) \cap PO(S)\}$, $i = 1,\ldots,n$. Then $c(S,d,r) = \bigcap_{i=1}^{n} U_i$. In the transferable-utility case, $\bigcap_{i=1}^{n} U_i$ is the intersection of n hyperplanes, and it is a point. In the strictly convex case, it is also a unique point. In general, it is the intersection of n hypersurfaces.

The interpretation of c(S,d,r) is as follows. We can construct a bargaining with claims problem [S,d,c(S,d,r)], associated to (S,d,r) for which r turns out to be the 'natural' reference point, that is, r[S,d,c(S,d,r)] = r. Thus, c(S,d,r) represents the "maximal fair claims" agents can have in the problem (S,d,r).

Because of previous constructions, we can think of any bargaining with claims problem as a problem of bargaining with a reference point and vice-versa. So, we can consider both types of problems under a unique setting.

It is interesting to notice that, for more than two agents, the natural claims point associated to the reference point d is not the ideal point. For instance, consider S = CoCom {(10,0,0),(0,5,0),(0,0,10/3)}, d = (0,0,0). Then, c(S,d) = (5,5/2,5/3). Moreover, for NTU problems, c(S,d) and the ideal point are not proportional in general [take the problem of distributing 100 units of money between three agents, whose utility functions are $u_1(x) = x$, $u_2(x) = x^{1/2}$, $u_3(x) = L(1+x)$. $S = \{(a,b,c) \in \mathbb{R}^3 \mid a = u_1(x),\ b = u_2(y),\ c = u_3(z),\ x+y+z \leq 100\}$, and d = (0,0,0). In this case, a(S,d) = (100,10,L101), and $c(S,d) = (50, 5.2^{1/2}, L51)$]. Previous fact has some implications in the way of defining some solution concepts from this perspective, since we do not always recover traditional solutions [most apparent are proportional solutions and Kalai-Smorodinski]. Which is important, then, is to decide if the natural claims point is more or less appealling than the utopia point, and in the case it is decided that it is more appealling for some problems, then to discuss their implications in the way the solutions may behave.

Thus, from now on, we consider *general bargaining problems.*

Definition: A general bargaining problem is a tuple (S,d,r,c), where $S \subset \mathbb{R}^n$, d,c and $r \in \mathbb{R}^n$, where

(i) $d, r \in S$, $r \geq d$, S is closed, convex and comprehensive

(ii) $c \notin S$, $c \leq a(S,d)$

(iii) $c = c(S,d,r)$ and $r = r(S,d,c)$

Obviously, traditional bargaining problems are a particular case of general bargaining problems, for which $d = r$, and $c = c(S,d)$; also bargaining with claims and bargaining with a reference point are particular cases of general bargaining problems.

Let Σ^n be the class of general bargaining problems defined before. Let us also call $\Sigma = \bigcup_{n \in N} \Sigma^n$.

A *solution in* Σ^n (resp. in Σ), is a function $F:\Sigma^n \longrightarrow \mathbb{R}^n$ (respectively, a functional on Σ), such that $F(S,d,r,c) \in S$.

Let us consider the following sets:

$IR(S,d) = \{ x \in S \mid x \geq d \}$, the set of individually rational points

$ME(S,r) = \{ x \in S \mid x \geq r \}$, the set of minimally equitable points

$PO(S) = \{ x \in S \mid \text{if } y > x, \text{ then } y \notin S \}$, the set of Pareto optimal points.

We think any sensible solution concept must satisfy three minimum requirements, viz., individual rationality, minimal equity and Pareto optimality, so we are looking for solutions of the problem in the intersection $IR(S,d) \cap ME(S,r) \cap PO(S)$. Moreover, we are interested in solutions satisfying minimum equity requirements. We confine ourselves to solutions in which we admit interpersonal comparability of the utility functions of the individuals, and therefore, only changes in the origin of measurement of utilities are allowed. That is, we are interested in solutions satisfying the following assumption:

Translation Invariance (TI): $\forall$ (S,d,r) $\in \sum^n$, $\forall$ a $\in \mathbb{R}^n$,

$$F(a+S,a+d,a+r) = a + F(S,d,r).$$

In order to simplify notation, from now on let us consider d = 0. Let us call this class $\sum_0^n$. (respectively, $\sum_0$). A generic element of $\sum_0^n$ takes the form (S,r,c).

Whithin the solutions satisfying these requirements, we shall consider the following solutions:

Definition: The *Lexicographic Egalitarian Solution*, L,

$$L(S,r,c) = x^* \text{ , such that } x_i^* = \max\{r_i,\lambda\},\ x^* \in PO(S).$$

So, L can be think of as a modification of the traditional lexicographic egalitarian solution for bargaining games, which also respects minimal equity ($x^* \geq r$).

Definition: The *Lexicographic Extended Claim-Egalitarian Solution*, E,

is the lexicographic extension of F(S,d,r,c) = x^*, such that

$$c_i - x_i^* = \max\{r_i,\lambda\},\ x^* \in PO(S).$$

Now, E can be looked at as a modification of the lexicographic extended claim-egalitarian solution for bargaining problems with claims [see Marco (1994)], which also respects minimal equity.

3. CHARACTERIZATION RESULTS

Consider the following properties:

Pareto Optimality (PO): $\forall$ (S,r) $\in \Sigma_0^n$, F(S,r) $\in$ PO(S).

Minimal Equity (ME): $\forall$ (S,r) $\in \Sigma_0^n$, F(S,r) $\in$ ME(S,r).

Anonimity (A): $\forall$ (S,r) $\in \Sigma_0^n$, $\forall$ permutation π: N --> N, F(πS,πr) = πF(S,r).

Contraction Consistency (CC): $\forall$ (S,r), (T,r') $\in \Sigma_0^n$, with ME(T,r') $\subseteq$ ME(S,r), if F(S,r) $\in$ ME(T,r'), then F(S,r) = F(T,r').

Consider now the following concept: For a given problem (S,r) $\in \Sigma_0^n$, let z $\in$ PO(S)$\cap$ME(S,r). We shall say that z is *interior* if it is in the relative interior of PO(S)$\cap$ME(S,r). For M $\subseteq$ N, $|M| > 1$, we shall say that z is *M-interior* if $\forall$ Q $\subset$ M, Q $\neq \emptyset$, there exist x,y $\in$ PO(S)$\cap$ME(S,r) with $x_Q \gg z_Q$ and $y_Q \ll z_Q$.

Interior Equitable Monotonicity (IEMON): $\forall$ M $\subset$ N, $|M| > 1$ and all (S,r), (T,r) with S $\subset$ T where

(i) F(S,r) $\in$ PO(S)$\cap$ME(S,r), is an M-interior point,

(ii) F_i(S,r) = F_i(T,r) for all i $\notin$ M

Then, $F_M(T,r) \geq F_M(S,r)$

The interpretation of all the axioms is straigtforward. As for IEMON, it says that if F(S,r) is a PO and ME point which is M-interior, and if in the 'larger' game every player outside M receives the same utility as in S, so that the 'additional resources' represented by T can be divided among the players in M, then nobody in M loses.

Obviously, L satisfies PO, ME, A and CC. Let see that it also satisfies IEMON:

Lemma 1: L satisfies IEMON

Proof: Let S, T such that they are in the hypotheses of IEMON. First notice that if $i,j \in M$, then $L_i(S,r) = L_j(S,r)$. Otherwise, since L(S,r) is an M-interior point, its smallest coordinate can be increased at the expenses of someone larger, without leaving $PO(S) \cap ME(S,r)$. But in such a case, L(S,r) would not be the lexicographic egalitarian solution. Furthermore, L(T,r) must dominate L(S,r) in terms of the lexicographic ordering. Since by assumption $L_i(S,r) = L_i(T,r)$ for $i \notin M$ then $\min_{i \in M} L_i(T,r) \geq L_i(S,r)$ for $i \in M$, that is, $F_i(T,r) \geq F_i(S,r)$ for all $i \in M$.■

Proposition 1.- L is the unique solution in $\sum_0^n$ satisfying PO, ME, A, CC and IEMON.

PROOF: Let us consider a solution F satisfying all the properties. We shall show that L = F.

Let $(S,r) \in \sum_0^n$, and let $L(S,r) = z$. By Anonymity, we can rename the indices in such a way that $z_1 \leq z_2 \leq \ldots \leq z_n$. Among the previous numbers, there are only k $(0 \leq k \leq n)$ different, $a_1,\ldots,a_k$. Call $N_j = \{ i \in N \mid a_j \geq z_i \}$. Thus, $N_1 \subset N_2 \subset \ldots \subset N_k = N$.

Let now call z^1 such that $z_i^1 = a_1$, for all i. Now we face two possibilities:

(1) $r << z^1$, and (2) for some i, $r_i \geq z_i^1$

Assume (1). Then, consider a small $\varepsilon > 0$ and take $x^1(\varepsilon)$ be such that $x_i^1(\varepsilon) = a_1 - \varepsilon$, $x_j^i = a_1$, for $j \neq i$, and let $S^1(\varepsilon) =$ CoCom $\{x^1(\varepsilon),\ldots,x^n(\varepsilon)\}$, [$\varepsilon$ has to be taken in such a way that $PO[S^1(\varepsilon)] \subset ME(S,d)$] and let us consider the problem $[S^1(\varepsilon),b]$, where $b_i = b_j$ for all i, j and is such that $b \leq 0$, $b \leq r$. By A and PO, $F_i[S^1(\varepsilon),b] = (a_1 - \varepsilon/n)$ for all $i \in N$, By (CC), $F[S^1(\varepsilon),r] = F[S^1(\varepsilon),b]$. Now, $S^1(\varepsilon) \subset S$, and $F[S^1(\varepsilon),d] \in PO[S^1(\varepsilon)] \cap ME[S^1(\varepsilon),r]$ and it is an N-interior point (moreover, (ii) in IEMON is satisfied trivially). Then, $F(S,r) \geq F[S^1(\varepsilon),r]$, for all small enough $\varepsilon > 0$. Now, let $\varepsilon \to 0$, an we obtain that $F(S,r) \geq z^1$.

Let us now check that $F_i(S,d) = z_i^1$, for all $i \in N_1$. Suppose not, then, for some $i \in N_1$, $F_i(S,d) > L_i(S,d) = a_1 = z_i^1$. Now, for $0 < \lambda < 1$, the point $\lambda F(S,r) + (1-\lambda)L(S,r)$ would dominate $L(S,r)$ in the lexicographic ordering, against the construction of $L(S,r)$. Thus, $F_i(S,d) = a_i$ for $i \in N_1$.

Construct now z^2 such that $z_i^2 = a_1$, for $i \in N_1$; $z_j^2 = a_2$, for $j \notin N_1$. Let now choose an small $\varepsilon > o$ and take, for every $j \notin N_1$, $y^j(\varepsilon)$ such that $y_j^j(\varepsilon) = a_2 - \varepsilon$; $y_k^j(\varepsilon) = z_k^2$, for $k \neq j$. Let now $S^2(\varepsilon) =$ CoCom $\{ y^j(\varepsilon),\ j \notin N_1 \}$. By A and PO, $F_i[S^2(\varepsilon),b] = a_1$ for $i \in N_1$, $F_j[S^2(\varepsilon),b] = (a_2 - \varepsilon/m)$ for $j \notin N_1$, where $m = n - n_1$, $n_1 = |N_1|$. By CC, $F[S^2(\varepsilon),r] = F[S^2(\varepsilon),b]$. $S^2(\varepsilon) \subset S$,

$F[S^2(\varepsilon),r] \in PO[S^2(\varepsilon)] \cap ME[S^2(\varepsilon),r]$ and it is an $N \backslash N_1$-interior point. Moreover, by construction, $F_i[S^2(\varepsilon),r] = a_1 = F_i(S,d)$, for $i \in N_1$. Thus, by IEMON, $F_j(S,d) \geq F_j[S^2(\varepsilon),r]$ for $j \notin N_1$, and for all $\varepsilon > 0$, small enough. Letting $\varepsilon \to 0$, $F_j(S,d) \geq a_2$ for $j \notin N_1$. In a similar way as before, we conclude that, if $j \in N_2$, then $F_j(S,d) = a_2$.

By repeating this procedure a finite number of times, we reach to F = L.

Let us now assume (2). Then we can find $r' \in S$ such that $r' \ll z^1$ and $L(S,r) = L(S,r') = F(S,r')$ (by (1)). We have to see that $F(S,r') = F(S,r)$. Bu it is the case, since $F(S,r') = L(S,r) \in ME(S,r)$, then by CC, $F(S,r') = F(S,r)$. $\blacksquare$

For $(S,r) \in \sum_0^n$, let us call $e_i(S,r) = \max \{x_i \mid (x_1,\ldots,x_n) \in ME(S,r)\}$, $i=1,\ldots,n$. That is, $e_i(S,r)$ is the maximum utility level agent i can achieve within the set of minimal equitable allocations.

Consider the following property:

Individual Equitable Monotonicity (IND.EMON): Let (S,r) and $(T,r) \in \sum_0^n$ be such that $T \subset S$, and for some agent i, $e_i(S,r) > e_i(T,r)$, whereas $e_j(S,r) = e_j(T,r)$ for all $j \neq i$. Then, $F_i(S,r) \geq F_i(T,r)$.

A solution satisfies individual equitable monotonicity whenever the feasible set expands in such a way that the maximum equitable aspirations of one agent increases while those of the rest of agents remain the same, then the first agent benefits.

We will allow now for a variable number of agents. Solutions on games with a fixed number of players extend in the obvious way to solutions for a variable population. If F is a solution on $\sum$ [or on $\sum_0$], we shall denote by F_N the solution restricted to games with n agents, where $n = |N|$ [that is, $F(S,d,r) = F_N(S,d,r)$, if $(S,d,r) \in \sum^n$].

Let F be a solution on $\sum_0$, and let $(S,r) \in \sum_0^n$, and F(S,r). Let us call $(S,r)_{M,F}$ the problem in $\sum_0^m$ obtained from (S,r) in which all the agents outside M received $F_i(S,r)$ and leave.

Let us now consider the following property:

Stability (STAB): For all N,M such that $M \subset N$, for all $(S,r) \in \sum_0^n$, $(T,s) \in \sum_0^m$, such that if $(T,s) = (S,r)_{M,F}$. Then, $F_M(S,r) = F(T,s)$.

A solution F satisfies stability whenever we face the reduced problem $(S,r)_M$, the utility levels attached to those agents in M by F are exactly the same they were when we were facing the bigger problem (S,r).

Then we obtain the following result:

Proposition 2.- The lexicographic egalitarian solution L is the unique solution in $\sum_o$ satisfying PO, ME, A, CC, IND.EMON and STAB.

PROOF: Obviously, L satisfies all the properties. Let us now consider a solution F satisfying the properties. We shall show that F = L.

Let $(S,r) \in \sum_0^n$, and let $L(S,r) = z$. By Anonymity, we can rename the indices in such a way that $z_1 \leq z_2 \leq \ldots \leq z_n$. Among the previous numbers, there are only k $(0 \leq k \leq n)$ different, $a_1,\ldots,a_k$. Call $N_j = \{ i \in N \mid a_j \geq z_i \}$. Thus, $N_1 \subset N_2 \subset .. \subset N_k = N$.

First, let us see that $F_i(S,r) = a_1$ for $i \in N_1$. If $F_i(S,r) = b < a_i$ for some $i \in N_1$, we can construct $T \subset S$ in such a way that $F(S,r) \in ME(T)$, $b_j(T,r) = b_j(S,r)$ for all $j \neq i$ and $b_i(T,r) = b$. By CC, $F(T,r) = F(S,r)$, contradicting IN.EMON. So, $F_i(S,r) \geq a_1 \ \forall \ i \in N_1$. If now $F_i(S,r) > a_1$ for some $i \in N_1$, for $0 < \lambda < 1$, $\lambda F(S,) + (1-\lambda)L(S,r)$ would lexicographically dominate $L(S,r)$. So, $F_i(S,r) = a_1$ for all $i \in N_1$.

Now, we can delete those agents in N_1, and consider the remaining problem $(S,r)_{N\setminus N_1,F}$. By STAB we know $F_{N\setminus N_1}(S,r) = F[(S,r)_{N\setminus N_1,F}]$. For this problem, y since L satisfies STAB, we know that the smallest value of the $(S,r)_{N\setminus N_1,F}$ utilities in $L[(S,r)_{N\setminus N_1,F}]$ is a_2, and $L_i[(S,r)_{N\setminus N_1,F}] = a_2$ for $i \in N_2\setminus N_1$. Now we apply a similar argument as before in order to obtain that for $i \in N_2\setminus N_1$, $F_i(S,r) = F_i[(S,r)_{N\setminus N_1,F}] = a_2$. In a finite number of steps, we obtain $F(S,r) = L(S,r)$.■

Proposition 3: The lexicographic extended claim egalitarian solution coincides with the traditional lexicographic extended claim egalitarian solution.

PROOF: It is enough to notice that the traditional extended claim egalitarian solution [see Bossert (1993)], satisfies minimal equity.

The traditional extended claim egalitarian solution, E^* is defined as follows: $F(S,c) = x^*$, such that $c_i - x_i^* = \max\{0,\lambda\}$, with $x^* \in WPO(S)$.

Now, suppose $x_i^* < r_i$ for some i. $x_i^* < c_i$ for all i. Thus, we shall have that $x^* \ll (r_i, c_{-i})$, against the hypothesis of $x^* \in WPO(S)$. Since $E(S,c) \geq x^*$, then $E(S,c) \in ME(S,r)$.■

Consider the following properties:

Independence of Individually Irrational Alternatives (IIIA): If (S,c), (T,c) are such that IR(S) = IR(T), then F(S,c) = F(T,c).

Weak Contraction Consistency (WCC): If (S,c), (T,c) are such that $S \subset T$, $F(T,c) \in IR(S)$, then F(S,c) = F(T,c).

Weak Monotonicity (W.Mon): For (S,c), (S',c'), if $S \subset S'$, and $S_{-i} = S'_{-i}$ for all $i \in N$, then $F(S',c') \geq F(S,c)$. [where $S_{-i} := cl\{x_{-i},\ x \in S,\ x \leq c\}$, x_{-i} being the vector in $\mathbb{R}^{n-1}$ obtained by deleting the ith component of x].

The following characterization result comes from Proposition 3 and Marco (1994).

Proposition 4.- The Lexicographic Extended Claim-Egalitarian solution is the unique solution in $\sum_0^n$ satisfying PO, IIIA, A, W.MON. and WCC.

4. FINAL REMARKS

The characterization results in Propositions 1 and 2 are closely related to those characterization results of the lexicographic egalitarian solution in the traditional context by Chun & Peters (1988) and Thomson & Lensberg (1989), respectively.

In Herrero (1993) the modification of the proportional solution actually moves the disagreement point to the reference point, following the idea of the "adjusted" solutions in the bankruptcy problem. The selection of the starting and final points for the proportional solutions provides with solutions which are not extensions of the proportional and the Kalai-Smorodinski, respectively, in this general setting. Nevertheless, it is interesting to observe that both the proportional solution in the bargaining with claims case, and the KS solution in the traditional bargaining case, satisfy ME. Moreover, only the extended claim-egalitarian solution and its lexicographic extension coincide with the corresponding "adjusted solutions", in any case.

REFERENCES

AUMANN, R., (1985), An Axiomatization of the Nontransferable Utility Value, *Econometrica*, 53: 599-612.

AUMANN, R. & MASCHLER, M., (1988), Game Theoretic Analysis of a Bankruptcy Problem from the Talmud, *Journal of Economic Theory*, 36: 195-213.

BINMORE, K. & DASGUPTA, P. (eds), (1987), *The Economics of Bargaining*, Basil Blackwell, Oxford.

BOSSERT, W., (1993), An alternative solution for bargaining problems with claims, *Mathematical Social Sciences*, 25: 205-220.

CURIEL, I.J., MASCHLER, M. & TIJS, S.H., (1988), Bankruptcy Games, *Z. Op.Research*, 31: A143-A159.

CHUN, Y. & PETERS, H. (1988), The Lexicographic Egalitarian Solution, *Cahiers du CERO,* 30: 149-156.

CHUN, Y. & THOMSON, W., (1992), Bargaining Problems with Claims, *Mathematical Social Sciences*, 24, 19-13.

DAGAN, N. & VOLIJ, O., (1993), The Bankruptcy Poblem: A Cooperative Bargaining Approach, *Mathematical Social Sciences*, 26: 287-297.

GUPTA,S. & LIVNE, Z.A., (1988), Resolving a conflict situation with a reference outcome: An Axiomatic Model, *Management Science*, 34, No. 11, 1303-1314.

HARSANYI, J.C., (1959), A Bargaining Model for the n-person cooperative Game, *Annals of Mathematics Studies*, Princeton U. Press, Princeton, 40: 325-355.

HARSANYI, J.C., (1963), A simplified bargaining model for the n-person cooperative game, *International Economic Review*, 4: 194-220.

HART, S., (1985), An Axiomatization of Harsanyi's nontransferable Utility Solution, *Econometrica*, 53: 1295-1313.

HERRERO, C., (1993), Endogenous reference points and the adjusted proportional solution for bargaining problems with claims, MIMEO, Universidad de Alicante.

KALAI, E. & SAMET, D. (1985), Monotonic Solutions to general cooperative games, *Econometrica*, 53: 307-327.

MARCO, M.C., (1994), Efficient solutions for bargaining problems with claims, MIMEO, Universidad de Alicante.

NASH, J.F. (1950), The bargaining problem, *Econometrica*, 18: 155-162.

OSBORNE, M.J. & RUBINSTEIN, A. (1990), *Bargaining and Markets*, Academic Press, San Diego.

RUBINSTEIN, A. (1982), Perfect equilibrium in a bargaining model, *Econometrica*, 50: 97-109.

SHAPLEY, L.S., (1969), Utility comparisons and the theory of games, in Guibauld (ed), *La Decision*, Editions du CNRS, Paris.

SUTTON, J. (1985), Noncooperative bargaining theory: an introduction, *Review of Economic Studies*, 53: 709-724.

THOMSON, W., & LENSBERG, T. (1989), *Axiomatic Theory of bargaining with a variable number of agents*, Cambridge U. Press, Cambridge.

PUBLISHED ISSUES

WP-AD 90-01 "Vector Mappings with Diagonal Images"
C. Herrero, A.Villar. December 1990.

WP-AD 90-02 "Langrangean Conditions for General Optimization Problems with Applications to Consumer Problems"
J.M. Gutierrez, C. Herrero. December 1990.

WP-AD 90-03 "Doubly Implementing the Ratio Correspondence with a 'Natural' Mechanism"
L.C. Corchón, S. Wilkie. December 1990.

WP-AD 90-04 "Monopoly Experimentation"
L. Samuelson, L.S. Mirman, A. Urbano. December 1990.

WP-AD 90-05 "Monopolistic Competition: Equilibrium and Optimality"
L.C. Corchón. December 1990.

WP-AD 91-01 "A Characterization of Acyclic Preferences on Countable Sets"
C. Herrero, B. Subiza. May 1991.

WP-AD 91-02 "First-Best, Second-Best and Principal-Agent Problems"
J. Lopez-Cuñat, J.A. Silva. May 1991.

WP-AD 91-03 "Market Equilibrium with Nonconvex Technologies"
A. Villar. May 1991.

WP-AD 91-04 "A Note on Tax Evasion"
L.C. Corchón. June 1991.

WP-AD 91-05 "Oligopolistic Competition Among Groups"
L.C. Corchón. June 1991.

WP-AD 91-06 "Mixed Pricing in Oligopoly with Consumer Switching Costs"
A.J. Padilla. June 1991.

WP-AD 91-07 "Duopoly Experimentation: Cournot and Bertrand Competition"
M.D. Alepuz, A. Urbano. December 1991.

WP-AD 91-08 "Competition and Culture in the Evolution of Economic Behavior: A Simple Example"
F. Vega-Redondo. December 1991.

WP-AD 91-09 "Fixed Price and Quality Signals"
L.C. Corchón. December 1991.

WP-AD 91-10 "Technological Change and Market Structure: An Evolutionary Approach"
F. Vega-Redondo. December 1991.

WP-AD 91-11 "A 'Classical' General Equilibrium Model"
A. Villar. December 1991.

WP-AD 91-12 "Robust Implementation under Alternative Information Structures"
L.C. Corchón, I. Ortuño. December 1991.

WP-AD 92-01 "Inspections in Models of Adverse Selection"
I. Ortuño. May 1992.

WP-AD 92-02 "A Note on the Equal-Loss Principle for Bargaining Problems"
C. Herrero, M.C. Marco. May 1992.

WP-AD 92-03 "Numerical Representation of Partial Orderings"
C. Herrero, B. Subiza. July 1992.

WP-AD 92-04 "Differentiability of the Value Function in Stochastic Models"
A.M. Gallego. July 1992.

WP-AD 92-05 "Individually Rational Equal Loss Principle for Bargaining Problems"
C. Herrero, M.C. Marco. November 1992.

WP-AD 92-06 "On the Non-Cooperative Foundations of Cooperative Bargaining"
L.C. Corchón, K. Ritzberger. November 1992.

WP-AD 92-07 "Maximal Elements of Non Necessarily Acyclic Binary Relations"
J.E. Peris, B. Subiza. December 1992.

WP-AD 92-08 "Non-Bayesian Learning Under Imprecise Perceptions"
F. Vega-Redondo. December 1992.

WP-AD 92-09 "Distribution of Income and Aggregation of Demand"
F. Marhuenda. December 1992.

WP-AD 92-10 "Multilevel Evolution in Games"
J. Canals, F. Vega-Redondo. December 1992.

WP-AD 93-01 "Introspection and Equilibrium Selection in 2x2 Matrix Games"
G. Olcina, A. Urbano. May 1993.

WP-AD 93-02 "Credible Implementation"
B. Chakravorti, L. Corchón, S. Wilkie. May 1993.

WP-AD 93-03 "A Characterization of the Extended Claim-Egalitarian Solution"
M.C. Marco. May 1993.

WP-AD 93-04 "Industrial Dynamics, Path-Dependence and Technological Change"
F. Vega-Redondo. July 1993.

WP-AD 93-05 "Shaping Long-Run Expectations in Problems of Coordination"
F. Vega-Redondo. July 1993.

WP-AD 93-06 "On the Generic Impossibility of Truthful Behavior: A Simple Approach"
C. Beviá, L.C. Corchón. July 1993.

WP-AD 93-07 "Cournot Oligopoly with 'Almost' Identical Convex Costs"
N.S. Kukushkin. July 1993.

WP-AD 93-08 "Comparative Statics for Market Games: The Strong Concavity Case"
L.C. Corchón. July 1993.

WP-AD 93-09 "Numerical Representation of Acyclic Preferences"
B. Subiza. October 1993.

WP-AD 93-10 "Dual Approaches to Utility"
M. Browning. October 1993.

WP-AD 93-11 "On the Evolution of Cooperation in General Games of Common Interest"
F. Vega-Redondo. December 1993.

WP-AD 93-12 "Divisionalization in Markets with Heterogeneous Goods"
M. González-Maestre. December 1993.

WP-AD 93-13 "Endogenous Reference Points and the Adjusted Proportional Solution for Bargaining Problems with Claims"
C. Herrero. December 1993.

WP-AD 94-01 "Equal Split Guarantee Solution in Economies with Indivisible Goods Consistency and Population Monotonicity"
C. Beviá. March 1994.

WP-AD 94-02 "Expectations, Drift and Volatility in Evolutionary Games"
F. Vega-Redondo. March 1994.

WP-AD 94-03 "Expectations, Institutions and Growth"
F. Vega-Redondo. March 1994.

WP-AD 94-04 "A Demand Function for Pseudotransitive Preferences"
J.E. Peris, B. Subiza. March 1994.

WP-AD 94-05 "Fair Allocation in a General Model with Indivisible Goods"
C. Beviá. May 1994.

WP-AD 94-06 "Honesty Versus Progressiveness in Income Tax Enforcement Problems"
F. Marhuenda, I. Ortuño-Ortín. May 1994.

WP-AD 94-07 "Existence and Efficiency of Equilibrium in Economies with Increasing Returns to Scale: An Exposition"
A. Villar. May 1994.

WP-AD 94-08 "Stability of Mixed Equilibria in Interactions Between Two Populations"
A. Vasin. May 1994.

WP-AD 94-09 "Imperfectly Competitive Markets, Trade Unions and Inflation: Do Imperfectly Competitive Markets Transmit More Inflation Than Perfectly Competitive Ones? A Theoretical Appraisal"
L. Corchón. June 1994.

WP-AD 94-10 "On the Competitive Effects of Divisionalization"
L. Corchón, M. González-Maestre. June 1994.

WP-AD 94-11 "Efficient Solutions for Bargaining Problems with Claims"
M.C. Marco-Gil. June 1994.

WP-AD 94-12 "Existence and Optimality of Social Equilibrium with Many Convex and Nonconvex Firms"
A. Villar. July 1994.

WP-AD 94-13 "Revealed Preference Axioms for Rational Choice on Nonfinite Sets"
J.E. Peris, M.C. Sánchez, B. Subiza. July 1994.

WP-AD 94-14 "Market Learning and Price-Dispersion"
M.D. Alepuz, A. Urbano. July 1994.

WP-AD 94-15 "Bargaining with Reference Points - Bargaining with Claims: Egalitarian Solutions Reexamined"
C. Herrero. September 1994.

WP-AD 94-16 "The Importance of Fixed Costs in the Design of Trade Policies: An Exercise in the Theory of Second Best"
L. Corchón, M. González-Maestre. September 1994.