REVEALED PREFERENCE AXIOMS FOR RATIONAL CHOICE ON NONFINITE SETS*

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ABSTRACT

Following the work of Bandyopadhyay and Sengupta [2], we analyze the rationalization of a choice function in terms of the revealed preference but in a more general context: choice sets with a continuum of alternatives. Firstly it is proved that some results which are verified in the finite case are not true in this context and new conditions to characterize the different kinds of rationality are stated. Furthermore, we analyze the continuity of the revealed preference and a characterization of open revealed preferences in terms of the hemicontinuity of the choice function is obtained.

Keywords: Revealed Preference on Nonfinite Sets; Rational Choice Function.

1.- INTRODUCTION

Almost every human act, individual or collective, could be interpreted as a choice between feasible alternatives which can be analyzed as a generalization of the pure theory of consumers' behavior. If the set of feasible alternatives changes (because of changes in the consumer's income or in goods' prices) the choice will be changed accordingly, so that a functional relationship between the *choice set* -the set of all chosen alternatives-, and the feasible set can be established. This functional relationship, usually called *choice function*, bears a clear resemblance to the demand function, which is one of the main concepts of the theory of consumers' behavior.

Historically, the "revealed preference" was introduced by Samuelson [6] as a way of describing the rationality of consumer's behavior in terms of the demand function. He studied those situations in which the demand function can be induced by a utility function. Uzawa [9] and Arrow [1] analyzed the same problem in a more general framework: they consider a choice function as a rule that selects a nonempty subset of alternatives for each and every set of alternatives presented for choice. They then study under which conditions these choice functions can be generated by a preference relation. This is the "rationality problem of a choice function": to find a preference relation whose maximal elements on any subset of alternatives determines the choice set.

Arrow [1] characterizes the rationalization of a choice function by a transitive preference relation, by means of an axiom that explains the choice function behavior when the set of alternatives expands or contracts (Arrow's condition). In fact, this axiom is equivalent to the Weak Axiom of Revealed Preference introduced by Samuelson.

The question which arises from this result is why ask for transitivity in the rationalization process. Indeed, several important papers have been written discussing and relaxing the assumption of transitivity on the preference relation (in particular the transitivity of indifference). All of them are based on the notion of "imperfect discrimination". As Luce [5] pointed out, in many situations individuals, due to their lack of sensitivity, can not distinguish between alternatives which are "close together" (sweetness, temperatures, ...). When the set of alternatives is a continuum, this imperfect discrimination is especially relevant, since we can find alternatives which are as "close together" as we want them to be. Thus in this context, requiring transitivity on the relation which rationalizes the choice function is too restrictive.

Since Arrow's work, much research has attempted to find characterizations of rationality of the choice function requiring weaker conditions than transitivity on the preference relation (pseudotransitive, quasitransitive or acyclic rationalization). These characterizations, following Arrow's axiomatic approach, involved properties about the expansion or contraction of the feasible alternative set and needed several axioms to characterize each class of rationalizable choice functions (Sen [7], Fishburn [4]).

None of these weak rationalization concepts were characterized in terms of the properties of the revealed preference until the work of Bandyopadhyay and Sengupta [2], which offers a characterization of rationalizable choice functions by means of a preorder, an interval-order, a quasiorder or an acyclic relation. Furthermore, they use an unique axiom established in terms of the revealed preference in each case, although their work only covers choices within finite sets of alternatives.

Our aim is to establish the analogous results but in a more general context. First, in Section 1, we introduce the main definitions and analyze whether the axioms introduced by Bandyopadhyay and Sengupta can be used to characterize rational choice over nonfinite sets. In Section 2 we modify those that do not characterize the rationalization in the non finite case. The new axioms introduced are equivalent to those of Bandyopadhyay and Sengupta in the finite case , but not in general. Furthermore, when the alternative set is a continuum, it is usual to require the preference relation to be continuous. Section 3 is therefore devoted to analyzing the relationship between the continuity of the revealed preference and the continuity of the choice function.

2.- PRELIMINARIES

Let X be the set of alternatives (not necessarily finite). A choice function F is a correspondence that assigns a nonempty subset of elements *(choice set)* for any set within a particular domain of nonempty subsets of X. In this work it will be assumed that this domain, namely $\mathcal{D}(X)$, consists of a family of subsets of X closed under finite unions which includes all unit sets. Whenever X is a finite set, it is usual to consider $\mathcal{D}(X)$ as the family of nonempty subsets of X ($\mathcal{P}(X)$), (see Suzumura, [8]).

Definition 1.1.

A *choice function* is a correspondence F: $\mathcal{D}(X) \longrightarrow X$ which specifies non empty subsets of X:

$$\forall A \in \mathcal{D}(X), \qquad F(A) \subset A, \qquad F(A) \neq \emptyset$$

Now some definitions of binary relations which will be used are given. Henceforth R defined on X will be considered a complete and reflexive binary preference relation. The strict preference relation P and the indifference relation I are defined in the usual way:

$$xPy \Leftrightarrow xRy \text{ and } \text{not}(yRx); \qquad xIy \Leftrightarrow xRy \text{ and } yRx.$$

Definition 1.2.

A binary relation R defined over X is:

a).- *acyclic* iff for all $x_1, x_2, .., x_n \in X$,

$$x_1 \; P \; x_2 \; P \; \ldots \; P \; x_n \quad \text{implies} \quad x_1 \; R \; x_n.$$

b).- *quasitransitive (quasiorder)* iff for all $x,y,z \in X$

$$x \; P \; y \; P \; z \quad \text{implies} \quad x \; P \; z.$$

c).- *pseudotransitive (interval order)* iff for all $x,y,z,t \in X$

$$x \; P \; y \; R \; z \; P \; t \quad \text{implies} \quad x \; P \; t.$$

d).- *transitive (preorder)* iff for all $x,y,z \in X$

$$x \; R \; y \; R \; z \quad \text{implies} \quad x \; R \; z.$$

From a binary relation R on X, we define the set of *maximal elements* for this relation on any subset $A \subset X$ as follows:

$$M(A,R) = \{ \; a \in A: \; a \; R \; z \quad \forall z \in A \; \}.$$

Another set which will be used throughout the work is the set of alternatives which are not necessarily maximal elements but are indifferent to every maximal element. Formally we define

$$IM(A,R) = \{ \; y \in A \mid \; y \; I \; a \quad \forall a \in M(A,R) \; \}$$

Notice that in general $M(A,R) \subset IM(A,R)$, although in the case of transitive binary relations or in the case of finite sets with quasitransitive binary relations, it is verified that $IM(A,R) = M(A,R)$.

The idea of *rationality* for a choice function is to find a binary relation whose maximal elements define the choice set for any subset of $\mathcal{D}(X)$. Formally we give the following definition:

Definition 1.3.

A choice function $F:\mathcal{D}(X) \longrightarrow X$ is said to be *rational* if a reflexive complete binary relation R exists such that

$$F(A) = M(A,R) \qquad \forall A \in \mathcal{D}(X)$$

In this case it is said that F is *rationalized* by R and the relation R is called a *rationalization* of F.

According to the properties that are required for the rationalization of F, it will be called *transitive-rational* (if the rationalization R is transitive), *quasitransitive-rational* (if R is quasitransitive), *pseudotransitive-rational* (if R is pseudotransitive) and *rational* (in the case where R is acyclic).

From a choice function F there always can be defined a binary relation based on it, which is called the *revealed preference.*

Definition 1.4.

Let $F: \mathcal{D}(X) \longrightarrow X$ be a choice function and x,y elements of X. It is said that x *is revealed preferred to* y , x R y, iff there exists $A \in \mathcal{D}(X)$ such that $x,y \in A$ and $x \in F(A)$.

In words we say that an alternative x is revealed preferred to another y if and only if there exists a set in $\mathcal{D}(X)$ such that both of them are available and x is chosen.

It is easy to prove that if there exists a binary relation R^* which rationalizes a choice function F, then it coincides with the revealed preference relation R. So in these cases, R^* can be described from the choice function.

Now we are interested in characterizing the rationality of the choice functions in terms of certain assumptions about the associated revealed preference. In this line Bandyopadhyay and Sengupta [2] characterize the rationalization, when X is a finite set, by means of an acyclic binary relation as well as quasitransitive, pseudotransitive and transitive binary relations. Furthermore they obtain these charaterizations by means of only one axiom in each case.

They define $\mathcal{D}(X)$ as the family of all nonempty subsets of X and introduce the following axioms:

Let $F: \mathcal{D}(X) \longrightarrow X$ be a choice function and let R be the associated revealed preference relation.

(A1). Axiom 1.- For any $A \in \mathcal{D}(X)$, if $x \in A-F(A)$, then there exists at least one $y \in A$ such that $y \; P \; x$.

(A2). Axiom 2.- For any $A \in \mathcal{D}(X)$, if $x \in A-F(A)$, then there exists at least one $y \in M(A,R)$ such that $y\ P\ x$.

(A3). Axiom 3.- For any $A \in \mathcal{D}(X)$, if $F(A) \neq A$, then there exists at least one $y \in M(A,R)$ such that $y\ P\ x$ for all $x \in A-F(A)$.

(A4). Axiom 4 (Weak Axiom of Revealed Preference) .- For any $A \in P(X)$, if $x \in A-F(A)$, then $y\ P\ x \quad \forall\ y \in M(A,R)$.

The results they obtain can be summarized in the following Theorem:

Theorem 1.5. *(Bandyopadhyay and Sengupta, [2])*

Let $F\colon \mathcal{D}(X) \longrightarrow X$ be a choice function, where X is a finite set of alternatives and $\mathcal{D}(X) = \mathcal{P}(X)$. Then this choice function is

i).- rational iff it satisfies Axiom 1.

ii).- quasitransitive-rational iff it satisfies Axiom 2.

iii).- pseudotransitive-rational iff it satisfies Axiom 3.

iv).- transitive-rational iff it satisfies Axiom 4.

If we want to state these results in the context of non finite sets, some problems arise in the cases of quasitransitive or pseudotransitive rationalizations. This is illustrated by means of the following example.

Example 1.6.

Let X be the unit interval, [0,1], and consider the complete reflexive binary relation R such that:

$$x \; P \; y \quad \Leftrightarrow \quad x > y \qquad \forall x,y \in [0,1)$$

$$1 \; I \; x \qquad \forall x \in [0,1]$$

It is obvious that this is a pseudotransitive relation and is therefore quasitransitive.

If we define the choice function given by the maximization of this relation on $\mathcal{D}(X)$, with $\mathcal{D}(X)$ being the family of non empty compact subsets of X, we obtain that both Axioms 2 and 3 fail. It is enough to consider $A = [0,1]$, $F(A) = 1$ and yet not[$1 \; P \; x$] $\forall \; x \in A-F(A)$, since $1 \; I \; x$ $\forall \; x \in A-F(A)$.

3.- RATIONAL CHOICE WITH A CONTINUUM OF ALTERNATIVES

In the light of the problems that appeared in the case of a continuum of alternatives when using Bandyopadhyay and Sengupta's axioms, some new assumptions have to be introduced. The new assumptions are weaker than (A2) and (A3), although in the finite case they are equivalent. They are used to characterize the rationalization by pseudotransitive and quasitransitive relations. In the other cases (preorder and acyclic relation) the ones used by Bandyopadhyay and Sengupta provide the characterization, since in their proofs the finite character of the set of alternatives does not play any role.

Let F be a choice function, then the new axioms are as follows:

(A2'). Axiom 2'.- For any $A \in \mathcal{D}(X)$, if $x \in A-F(A)$, then there exists at least one $y \in IM(A,R)$ such that $y \; P \; x$.

(A3'). Axiom 3'.- For any $A \in \mathcal{D}(X)$, if $x \in A-F(A)$, then there exists at least one $y \in IM(A,R)$ such that $y \; P \; x$ and $y \; P \; z$ for all $z \in L_x - M(L_x,R)$, (where $L_x = \{ w \in X \mid x \; R \; w \}$).

Notice that the only difference between (A2') and (A2) is that in the new axiom, if an alternative x is not chosen, we can not ensure the existence of an alternative strictly revealed preferred to x within the choice set. We only now ask for the existence of an alternative strictly revealed preferred to x in A and, at the same time, indifferent to all of the maximal alternatives in the choice set.

In any case, these axioms are equivalent to the ones used by Bandyopadhyay and Sengupta in the finite case and it is verified that A4 implies A3', which implies A2' which, in turn, implies A1, and the converses are not true (see counterexamples in Bandypoadhyay and Sengupta, [2]). Moreover, Example 1.6 proves that, in general, the new assumptions (A2') and (A3') are weaker than (A2) and (A3), respectively.

Now the characterization results can be stated. Since the proofs of Theorem 2.1 and 2.4 run parallel to those of the finite case, they are not given.

Theorem 2.1.

A choice function $F: \mathcal{D}(X) \longrightarrow X$ is rationalized by an acyclic relation iff it satisfies Axiom 1.

Theorem 2.2.

A choice function $F: \mathcal{D}(X) \longrightarrow X$ is rationalized by a quasitransitive relation iff it satisfies Axiom 2'.

Proof:

Let F be a choice function and assume that R^* is a quasitransitive binary relation that rationalizes F. As we have pointed out, in this case $R^* = R$, so we can consider the choice set as $F(A) = \{a \in A: a \, R \, z \quad \forall z \in A\}$.

Because every quasitransitive relation is acyclic, we can apply Theorem 2.1 and obtain that this function satisfies Axiom 1.

Let x be an element of A-F(A), by Axiom 1 there exists an element $y \in A$ such that yPx. If $y \in IM(A,R)$, we obtain Axiom 2'. Alternatively if there exists an element $z \in F(A)$ such that zPy, since yPx by the quasitransitivity of R we obtain zPx and due to $z \in F(A)$, in particular $z \in IM(A,R)$. So Axiom 2' is satisfied.

Now we assume that F is a choice function which satisfies Axiom 2'. Since Axiom 2' implies Axiom 1, by Theorem 2.1 we know that there exists an acyclic binary relation R which rationalizes F. So it is enough to prove that under Axiom 2' this relation R is quasitransitive.

Suppose that $x,y,z \in X$ such that xPy and yPz and consider the set $A = \{x,y,z\}$, then $F(A) = \{x\} = IM(A,R)$. Since $z \notin F(A)$, by Axiom 2' we obtain that xPz which implies quasitransitivity.

□

Theorem 2.3.

A choice function $F: \mathcal{D}(X) \longrightarrow X$ is rationalized by a pseudotransitive relation iff it satisfies Axiom 3'.

Proof:

Let F be a choice function and assume that F is rationalized by a pseudotransitive binary relation R*. As R* rationalizes F we know that $R^* = R$ and therefore $F(A) = \{a \in A\text{: } a \; R \; z \quad \forall z \in A\} \quad \forall A \in \mathcal{D}(X)$.

Since a pseudotransitive relation is a quasitransitive one, by Theorem 2.2 we know that Axiom 2' is satisfied. We show that the pseudotransitivity implies Axiom 3'.

Let x be an element of A-F(A). By Axiom 2' there exists $y \in IM(A,R)$ such that yPx.

Let z be an element in Lx - M(Lx,R), then there exists $z' \in Lx$ such that z' P z. Since z' is in Lx we have y P x R z' P z and by pseudotransitivity y P z.

Now we show that a choice function satisfying Axiom 3' is necessarily rationalized by a pseudotransitive binary relation. Since Axiom 3' implies Axiom 2' it follows from Theorem 2.2 that there exists a quasitransitive relation R which rationalized F. We need to show that under Axiom 3' this relation R is also pseudotransitive.

Let x,y,z and t be elements of X such that x P y R z P t and consider the set A = {x,y,z,t}. Since $F(A) = \{a \in A: aRw \ \ \forall w \in A\}$, $y \notin F(A)$ and by Axiom 3' there exists an element w_1 in IM(A,R) such that w_1 P y and w_1 P t because of $t \in$ Ly - M(Ly,R). Since $IM(A,R) \subseteq \{x,z\}$, either $w_1 = x$ or $w_1 = z$, but in the last case z P y which is a contradiction. So it follows that x P t.

□

Theorem 2.4.

A choice function F: $\mathcal{D}(X) \longrightarrow X$ is rationalized by a preorder iff it satisfies Axiom 4.

4.- CONTINUOUS REVEALED PREFERENCES

When the set of alternatives is a continuum, it is usual to assume some continuity conditions. In this way, when the agent's preferences are defined on a topological space, the preference relation should be continuous in order to ensure the existence of a continuous numerical representation. These kind of requirements are also useful to guarantee the existence of maximal elements.

On the other hand, the hemicontinuity of the choice function is an important property since it implies that small changes in the set of alternatives produce small changes in the choice set. In this sense, there exist important results which prove the hemicontinuity of the demand function as Berge's theorem [3] (when the demand function is defined by maximizing a continuous utility function) or Walker's [10] (for demands generated by an open binary relation). Both of them impose continuity conditions on the binary relation which is used.

Therefore we are now interested in analyzing what conditions the choice function should satisfy in order to guarantee the continuity of the revealed preference when it rationalizes this function. And conversely, which conditions on the preference relation would be necessary to ensure the hemicontinuity of the choice function. In order to do this, we should know the characteristics of the sets of the domain of the choice function. Since whenever F is a choice function rationalized by the binary relation R, F(A) is the set of maximal elements of the revealed preference, it seems natural to restrict $\mathcal{D}(X)$ to the class of the compact subsets of the metric topological space X, because in this context the existence of maximal elements can be guaranteed.

So throughout this section we consider a choice function defined as follows:

$$F: \mathcal{C}(X) \longrightarrow X \quad \text{such that} \quad F(A) \neq \emptyset, \quad F(A) \subset A \quad \text{and} \quad F(A) \text{ closed}$$

where $\mathcal{C}(X)$ is the family of non empty compact subsets of X

First of all, the notions of continuity which are going to be used are given.

Definition 3.1.

Let X be a compact metric topological space and $F:X \longrightarrow X$ a correspondence with non empty closed images, then F is *upper hemicontinuous (u.h.c.)* iff

$$\left.\begin{array}{l} \forall \{x_n\} \subset X \text{ such that } x_n \longrightarrow x \\ \{y_n\} \text{ such that } y_n \in F(x_n) \end{array}\right\} \quad y_n \longrightarrow y \quad \text{implies} \quad y \in F(x)$$

Definition 3.2.

Let X be a metric topological space and R a binary preference relation defined over X; it is a continuous one iff the following sets

$$U(x) = \{ y \in X \mid y \; P \; x \} \quad \text{and} \quad L(x) = \{ y \in X \mid x \; P \; y \}$$

are open sets $\forall \; x \in X$.

Definition 3.3.

Let X be a metric topological space and R a binary preference relation defined over X; R is an open relation iff the graph of P is an open set, that is

$$G(P) = \{ (x,y) \mid x \; P \; y \} \text{ is open.}$$

To state the characterization result which has been obtained in this context , we use the Hausdorff Topology over $\mathcal{C}(X)$.

Theorem 3.4.

Let X be a metric topological space and F: $\mathcal{C}(X) \longrightarrow X$ a choice function, then

F satisfies Axiom 1 and is u.h.c. $\Leftrightarrow$ F is rationalized by an acyclic open binary relation R

Proof:

Let F be an u.h.c. choice function and assume it satisfies Axiom 1. By Theorem 2.1. the associated revealed preference relation rationalizes F and is acyclic. So it is enough to prove that in this case R is an open relation, namely we have to show that $G = \{ (x,y) \mid xPy \}$ is an open set. In fact we will prove that $X \times X - G = \{ (x,y) \mid yRx \}$ is closed.

Let $\{(x_n,y_n)\}$ be a sequence in $X \times X - G$ such that it converges to (x,y), and define the sets $B_n = \{y_n,x_n\}$. Since $(x_n,y_n) \notin G$, $y_n \; R \; x_n \quad \forall n \in \mathbb{N}$ so $y_n \in F(B_n) \; \forall n \in \mathbb{N}$ and it is easy to prove that $\{B_n\}$ converges to $B = \{y,x\}$ in the Hausdorff Topology. Finally by applying the u.h.c. of F we obtain that $y \in F(B)$ which implies that $y \; R \; x$, namely $(x,y) \in X \times X - G$.

Suppose now that F is a choice function rationalized by an acyclic open binary relation which is in fact the revealed preference relation R. Applying Theorem 2.1. we obtain that F satisfies Axiom 1, so we only have to show that under these conditions F is u.h.c.

Let $\{S_n\}$ be a sequence of subsets of $\mathscr{C}(X)$ that converges to S in the Hausdorff Topology and let $\{y_n\}$ be a sequence in $F(S_n)$ such that $\{y_n\}$ converges to y. If $y \notin F(S)$, by Axiom 1 there exists $z \in S$ such that $z \; P \; y$. Since $z \in S = \lim S_n$, there exists a sequence $\{z_n\}$ such that $z_n \in S_n$ $\forall \; n \in \mathbb{N}$ and $\{z_n\} \longrightarrow z$. Since $y_n \in F(S_n)$ and $z_n \in S_n$, then $y_n \; R \; z_n \quad \forall \; n \in \mathbb{N}$ and since P is open, $y \; R \; z$, which is a contradiction. So $y \in F(S)$.

□

As an immediate consequence of this result and those of Section 2 it is obtained that F is an u.h.c. choice function satisfying (A2') if and only if it is quasitransitive-rational by an open binary relation. Analogous results are obtained in the cases of pseudotransitive-rational and transitive-rational choice functions, and the axioms (A3') and (A4) respectively.

However in many contexts it is usual to impose on the binary relation the condition of being continuous instead of open. Since asking for the continuity of the binary relation is weaker than asking for it to be open, the next result is another immediate consequence of Theorem 3.4.

Corollary 3.5.

Let X be a metric topological space and F: $\mathscr{C}(X) \longrightarrow X$ an u.h.c. choice function and assume that it satisfies Axiom 1, then F is rationalized by a continuous acyclic binary relation.

Remark 3.6. If X is a connected metric topological space and R is a preorder defined on X, then the conditions of being open or continuous for a binary relation are equivalent. So in this particular case the result can be established as a characterization of choice functions rationalized by means of a *continuous* preorder.

Another framework where these two conditions are equivalent is when X is a connected metric space and R is a quasitransitive binary relation without *gaps* [namely whenever xPy there exists $z \in X$ such that xPzPy], although in general this equivalence is not true.

Remark 3.7. On the other hand, if X is a connected subset of $\mathbb{R}^n$, it is well known (see Debreu, 1959) that a continuous preorder is representable by means of a utility function $u: X \longrightarrow \mathbb{R}$ in such a way that $xRy \Leftrightarrow u(x) \geq u(y)$. So in this case Theorem 3.4 can be rewritten in the following way:

"Let $F: \mathscr{C}(X) \longrightarrow X$ be a choice function such that $X \subset \mathbb{R}^n$ is a connected set, then

$$F \text{ satisfies Axiom 4 and is u.h.c.} \Leftrightarrow \exists\ u: X \longrightarrow \mathbb{R} \text{ continuous such that } "$$
$$\forall A \subset \mathscr{C}(X), \quad F(A) = \arg\max_{x \in A} u(x)$$

Remark 3.8. The existence of a numerical representation for a preference relation, as the utility function, is an important tool in consumer theory. Therefore it could be interesting to know in which cases the revealed preference has an associated numerical representation. This fact is related to the separability of the binary relation, and as such, the problem is one of determining what kind of conditions must be imposed on the choice function in order to obtain separability properties. In these cases (under certain conditions) the choice function could be written as the maximization of a real-valued function.

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