

NUMERICAL REPRESENTATION OF PARTIAL ORDERINGS*

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WP-AD 92-03

* This paper is based on Chapter 2 in Subiza (1992). Thanks are due to J.E. Peris, D. Schmeidler, A. Villar and the participants in the Seminar on Nonconventional Preference Modelling held in Alicante, February 1992, for helpful suggestions. Financial support from the Spanish D.G.C.Y.T., under project PS89-0066 is gratefully acknowledged.

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**Editor: Instituto Valenciano de
Investigaciones Económicas, S.A.**
Primera Edición Julio 1992.
ISBN: 84-7890-909-5
Depósito Legal: V-2502-1992
Impreso por KEY, S.A., Valencia.
Cardenal Benlloch, 69, 46021-Valencia.
Printed in Spain.

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A B S T R A C T

In this paper a numerical representation of preferences by means of subsets of the real line is proposed. This representation turns out to be natural for partial orderings. Some results on this representation, extending those for utility functions and pairwise representations, are provided for this case.

1.- INTRODUCTION

Consider an agent who has to choose an element within a possibility set X , according to a binary relation \succ defined on X , such that $x \succ y$ means that x is strictly preferred to y . Relation \succ is called a *preference relation* whenever it is *asymmetric*, in the sense that

$$x \succ y \text{ entails } \text{not}(y \succ x) \quad x, y \in X.$$

A *partial ordering* is a *transitive* preference relation, in the sense that $x \succ y$, and $y \succ z$ implies $x \succ z$ $x, y, z \in X$.

If a preference relation \succ is defined on a set X , two other binary relations on X are defined in a standard way: the *preference-indifference relation*, \succeq , such that $x \succeq y$ if and only if $\text{not}(y \succ x)$, (which turns out to be *reflexive* and *complete*), and the *indifference relation*, \sim , such that $x \sim y$ if and only if $x \succeq y$, and $y \succeq x$, simultaneously (which is *reflexive* and *symmetric*).

It is quite frequent to assume that the agent's preferences are representable by means of a real function $u: X \rightarrow \mathbb{R}$, in such a way that $x \succ y$ if and only if $u(x) > u(y)$. Function u is referred to as a *utility function*. Assuming that the agent's preferences are representable by a utility function ensures that both \succ and \sim are *transitive* [i.e., if $x \succ y$, and $y \succ z$, then $x \succ z$, and if $x \sim y$, $y \sim z$, then $x \sim z$]. In this case the preference relation is called a *preorder*.

It is obvious then that, in general, a partial ordering cannot be represented by means of a utility function. Nonetheless, the following case has been considered for representing partial orderings: Suppose that a function $u: X \rightarrow \mathbb{R}$ exists such that $x \succ y$ entails $u(x) > u(y)$. Let us call *weak utility function* the aforementioned function u [cf. Fishburn (1970), Peleg (1970), Richter (1971), Majumdar & Sen (1976)]. The existence of a weak utility function implies that \succ is acyclic, a less restrictive condition than being a partial ordering [confront Bridges (1983b)].

Notice that when having a weak utility function, from $u(x) > u(y)$ it does not follow that $x \succ y$. In consequence, the representation provided by a weak utility function is of a different nature from that one provided by a utility function.

Looking for a representation of preferences less restrictive than preorders, Fishburn (1973) studies the case in which the preference relation \succ on X is representable by means of two real functions, $u, v: X \rightarrow \mathbb{R}$, $v(x) \geq u(x) \forall x \in X$, in such a way that $x \succ y$ if and only if $u(x) > v(y)$. The existence of the previous pairwise numerical representation implies that the preference relation is *pseudotransitive*, that is,

$$\text{if } x \succ y, y \sim z, z \succ t, \text{ then } x \succ t$$

Notice that, under pseudotransitivity, the indifference relation \sim is not necessarily transitive, but the strict preference relation \succ is transitive. A pseudotransitive preference relation is usually called an

interval order. Thus, interval orders constitute an intermediate case between preorders and partial orderings.

The aim of this paper is to provide numerical representations for partial orderings in a similar way to those representations provided for preorders and interval orders by means of utility functions and pairwise representations. In so doing, we introduce, in Section 2, the so called *set-representation*, in which we associate a subset $S(x)$ of \mathbb{R} to every element x in the possibility set, in such a way that whenever $x \succ y$, then $S(x) \supset S(y)$. If a preference relation admits a set-representation, then it is a partial ordering. Section 3 is devoted to the case of a countable possibility set X . In this case, a preference relation admits a set-representation if and only if it is a partial ordering. Section 4 introduces additional conditions under which a partial ordering admits a set-representation in the uncountable case. Some final comments and remarks, in Section 5, close the paper.

2.- PARTIAL ORDERINGS AND SET-REPRESENTATION

A preference relation \succ on a set X is called a *Partial Ordering* if whenever $x \succ y$; $y \succ z$, then $x \succ z$. It is worth noticing that both preorders and interval-orders are partial orderings. An example of a partial ordering which is not an interval-order is the Pareto relation in $X = \mathbb{R}^n$.

For a given set A , let $X = 2^A$ the possibility set, and consider the following preference relation: given $B, C \in X$ [$B, C \subseteq A$], $B \succ C$ iff $B \supset C$. Obviously, this preference relation is a partial ordering. This example motivates the following definition:

Definition 1.- Let \succ be a preference relation on a set X . A set-valued mapping $S: X \rightarrow \mathbb{R}$ is a *set-representation* of \succ , if

- (i) $\forall x \in X, S(x) \neq \emptyset, S(x)$ is a bounded set.
- (ii) $x \succ y \iff S(y) \subset S(x), \sup S(y) < \sup S(x)$.

Observe that, when the preference relation admits a representation by means of a utility function, then it also admits a set-representation, by taking $S(x) = [-u(x), u(x)]$, $u: S \rightarrow \mathbb{R}$ being a utility function with positive values (notice that if the initial utility function has no positive values, we can take a monotone transformation with positive values, by taking $v(x) = \exp u(x)$).

Similarly, if a preference relation admits a numerical representation by means of two real functions, $u, v: X \rightarrow \mathbb{R}$ [which, as before can be taken with positive values], then a set-representation is easily constructed by taking $S(x) = (-u(x), u(x)) \cup \{v(x)\}$.

The existence of a set-representation has an immediate consequence:

Proposition 1.- If a preference relation \succsim over a set X admits a set-representation, $S: X \rightarrow \mathbb{R}$, then \succsim is a partial ordering.

Furthermore, a weak utility function for \succsim can be defined by considering $u: X \rightarrow \mathbb{R}$, $u(x) = \sup S(x)$.

3.- THE COUNTABLE CASE

In the case X is a finite or countable set, the converse of the first part in Proposition 1 also holds true. In order to get this result, let us start by the following lemma, which ensures the existence in this case of a weak utility function with additional properties:

Lemma 1.- Let X be a countable set. If \succ is a partial ordering on X , there exists an injective function $u: X \rightarrow \mathbb{R}_+$ such that $x \succ y$ implies $u(x) > u(y)$.

Proof:

Let $X = \{x_i, i \in \mathbb{N}\}$, and define $D(x_j) = \{k \in \mathbb{N} \mid x_k \prec x_j\}$. Let now define $u(x_j) = \sum_{k \in D(x_j)} \frac{1}{5^k} + \frac{1}{5^j}$. Notice that if $x_j \succ x_r$, then $D(x_j) \supset D(x_r)$.

Moreover, $j \notin D(x_r)$, but $r \in D(x_j)$. Therefore, $u(x_j) > u(x_r)$.

Additionally, u is trivially injective, since in the case that $u(x_j) = u(x_r)$, with $r \neq j$, we will have, simultaneously, $x_r \succ x_j$ and $x_j \succ x_r$, which is a contradiction. ■

Then, we get the following result:

Proposition 2.- Let X be a countable set, and \succ a preference relation on

X . Then, the following conditions are equivalent:

- (i) \succ is a partial ordering
- (ii) \succ admits a set-representation.

Proof:

We only have to prove (i) \Rightarrow (ii). By lemma 1, there exists $u: X \rightarrow \mathbb{R}$ such that u is injective and $x \succ y \Rightarrow u(x) > u(y)$. Define then $S: X \rightarrow \mathbb{R}$ in the following way: $S(x) = \{ u(z) \mid z \prec x \} \cup \{u(x)\}$.

Since $u(x) \in S(x) \forall x \in X$, $S(x) \neq \emptyset$. By construction, $S(x) \subset \mathbb{R}_+$, and $u(x) = \sup S(x)$. Therefore, $S(x)$ is bounded.

Suppose now that $x \succ y$. Then, $S(x) \supset S(y)$. Additionally, $u(x) \in S(x)$, but $u(x) \notin S(y)$, and therefore $S(y) \neq S(x)$. Moreover,

$$u(y) = \sup S(y) < \sup S(x) = u(x).$$

Conversely, suppose that $S(x) \supset S(y)$, $\sup S(x) > \sup S(y)$. Then, we cannot have $y \succ x$. Since $S(y) \subset S(x)$, then, $u(y) \in S(x)$. This only can hold true if $y \prec x$, since $u(y) \neq u(x)$, and therefore, S is a set-representation of \succ . ■

Since utility functions and pairwise representation give rise to set-representations, Proposition 2 can be seen as the extension of the analogous representation results, in the countable case, for preorders (see Debreu (1954)) and interval-orders (see Fishburn (1970) or Bridges (1983b), Theorem 2).

4.- THE UNCOUNTABLE CASE

In the case the opportunity set X is uncountable, it is not possible, in general, to obtain a set-representation *for any partial ordering*. To see this, consider the lexicographical order on $\mathbb{R} \times (0,1)$, which is a partial preordering, and suppose a set-representation $S: \mathbb{R} \times (0,1) \rightarrow \mathbb{R}$ for this preference relation. Since there are no indifferent elements, the weak utility function associated to S turns out to be a utility function, which does not exist in this case, as it is well known. So, additional conditions are required in order to get set-representations for uncountable opportunity sets.

In order to get numerical representations (utility functions, pairwise representations or weak utility functions) in the uncountable case, separability conditions on the preference relation are required. Furthermore, it is usual to assume some cardinality or topological properties on the opportunity set [see Debreu (1954) (1964), Fishburn (1970) (1983) or Monteiro (1987) for the preorder case, and Bridges (1983a) (1985) or Chateauneuf (1987), for the interval-order case]. In a similar way, we will consider two different separability conditions on the preference relation, which, combined with cardinality or topological requirements on the opportunity set convey to the existence of a set-representation in this case.

Adopting Herden's terminology, we will say that a preference relation \succsim on X is *weakly separable* if a countable subset $G \subseteq X$ exists such that,

whenever $x, y \in X$ are such that $x \succ y$, then two elements $g_1, g_2 \in G$ can be found such that $x \succ g_1 \succ g_2 \succ y$. A preference relation \succ on X is *separable* if a countable subset $G \subseteq X$ exists such that, whenever $x, y \in X$ are such that $x \succ y$, an element $g \in G$ can be found in such a way that $x \succ g \succ y$. Obviously, if \succ is separable, then it is weakly separable (see Herden (1989a,b)).

Remark: The concept of weak separability was introduced by Chateauneuf (1987), but he used the term *strong separability*.

Let χ_1 denote the cardinal of \mathbb{R} . By combining weak separability of the preference relation with a cardinality condition on the opportunity set, the following result is obtained:

Proposition 3.- Let \succ be a weakly separable preference relation on a set X , such that $\text{card}(X) \leq \chi_1$. Then, \succ admits a set representation if and only if it is a partial ordering.

Proof:

Since $\text{card}(X) \leq \chi_1$, an injective function $v: X \rightarrow (0,1/2)$ can be defined.

Let us now to construct a weak utility function, by using the weak separability condition. Since \succ is weakly separable, a countable subset G of X exists, such that whenever $x \succ y$, $g_1, g_2 \in G$ can be found such that

$$x \succ g_1 \succ g_2 \succ y$$

Let now define $E(x) = \{g_1 \in G \mid x \succ g_1\}$, and consider the following mapping:

$$u: X \rightarrow \mathbb{R}, \quad u(x) = \begin{cases} 1 & \text{if } E(x) = \emptyset \\ 1 + \sum_{g_i \in E(x)} \frac{1}{5^i} & \text{otherwise} \end{cases}$$

It is easy to see that u is a weak utility function for \succ : If $x \succ y$, then $E(x) \supseteq E(y)$. Moreover, there exist g_k, g_r such that $x \succ g_r \succ g_k \succ y$, and therefore $g_r \in E(x), g_r \notin E(y)$ and $u(x) > u(y)$.

Now, construct $S: X \rightarrow \mathbb{R}$ as follows:

$$S(x) = \{u(z), v(z) \mid x \succ z\} \cup \{u(x), v(x)\}$$

$S(x)$ is nonempty and bounded for any $x \in X$. Furthermore, $\sup S(x) = u(x)$.

Let us now to check that S is a set-representation of \succ .

If $x \succ y$, $S(x) \supseteq S(y)$. Moreover, $v(x) \in S(x), v(x) \notin S(y)$, and thus, $S(x) \supset S(y)$. Additionally, $u(x) > u(y)$, since u is a weak utility function.

Conversely, if $S(x) \supset S(y)$, and $\sup S(x) > \sup S(y)$, we get $x \succ y$. Furthermore, since $v(y) \in S(x)$, and v is injective, only two possibilities are open: $x \succ y$, or $x = y$. But this last equality is not possible, since $S(x) \neq S(y)$. So, $x \succ y$, and S is a set-representation of \succ . ■

As a corollary of Proposition 3, we get:

Proposition 4.- Let \succ be a weakly separable preference relation on a separable topological space X . Then, \succ admits a set-representation if and only if it is a partial ordering.

Proof:

It is enough to check that if X is separable, then $\text{card}(X) \leq \aleph_1$.

The separability of X indicates that a countable subset $D \subseteq X$ exists such that $\text{cl}(D) = X$, $\text{cl}(D)$ being the topological closure of D . Thus, for every $x \in X$, a sequence $\{d_{n(x)}\} \subseteq D$ can be chosen such that $\lim_{n(x)} d_{n(x)} = x$, and therefore we can associate with x the sequence of natural numbers $\{n(x)\}$. In consequence the cardinality of X is less or equal than the cardinality of the set of sequences of natural numbers, that is, \aleph_1 . ■

Peleg (1970) studies a case under which a continuous weak utility function can be defined for a partial ordering on a topological space. We shall say that a preference relation \succ on a topological space X is *continuous* if $L(x) = \{y \in X \mid x \succ y\}$ is open, $\forall x \in X$. If we consider the case of a continuous partial ordering, by properly redefining the weak utility function u in Proposition 3, in order to use Peleg's representation, we get the following result:

Proposition 5.- Let X be a topological space, such that $\text{card}(X) \leq \aleph_1$, and

let \succ be a preference relation on X such that:

- (i) \succ is a continuous and separable partial ordering.
- (ii) Whenever $x \succ y$, then $\text{Cl}\{z \in X \mid y \succ z\} \subseteq \{z \in X \mid x \succ z\}$

Then, \succ admits a set-representation $S: X \rightarrow \mathbb{R}$, such that $u: X \rightarrow \mathbb{R}$, $u(x) = \sup S(x)$ is continuous.

5.- FINAL REMARKS

In this paper a new type of numerical representation of preferences (by means of subsets of the real line) has been defined, as the natural extension of previous numerical representations (for preorders and interval-orders) to partial orderings. Similar results to those at hand for utility functions and pairwise representations have been obtained, as well as the relationship between the proposed set-representation and the (only one way) weak utility function.

It is worth mentioning that, whenever a set-representation $S: X \rightarrow \mathbb{R}$ can be defined for a preference relation \succ over X , any monotone transformation of S does also the work (that is, if we consider a monotone function $m: \mathbb{R} \rightarrow \mathbb{R}$, by considering $T: X \rightarrow \mathbb{R}$, $T(x) = m \circ S(x)$, T turns out a set-representation of \succ).

As it has been pointed out, a weak utility function $u: X \rightarrow \mathbb{R}$, such that $x \succ y \Rightarrow u(x) > u(y)$ does not characterize the preference relation \succ . We constructed the set-representation by properly adding up some elements to the weak representation, in order to get a characterization result. Nevertheless, it is interesting to observe that, as $\sup S$ keeps being a weak utility function, no information is lost, and moreover, we obtained a full characterization property.

It may be interesting to illustrate the role of $\sup S(y) < \sup S(x)$ in definition 1, by means of the following example: consider the

lexicographical ordering on $]0,1[\times \{0,1\}$, and the set-valued mapping $S:]0,1[\times \{0,1\} \rightarrow \mathbb{R}$, $S(x,0) = (0,x)$; $S(x,1) = (0,x]$. It is clear that

$$(x,s) \succ_L (x',s') \iff S(x,s) \supset S(x',s')$$

but nevertheless, the condition on $\text{Sup}(x,s)$ fails to hold. Obviously, as it was mentioned before, there is no any set-representation for the lexicographical ordering on $]0,1[\times \{0,1\}$, since it does not admit any weak utility function.

Finally notice that, in the case whereby the preference relation \succ (on X) admits a set-representation $S: X \rightarrow \mathbb{R}$ then an element $x^* \in X$ is maximal if and only if $\sup S(x^*) \geq \sup S(x)$, $\forall x \in X$ such that $S(x^*) \subset S(x)$. Therefore, we may use function $\sup S$ in order to get maximal elements of \succ .

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