

**ROBUST IMPLEMENTATION UNDER ALTERNATIVE  
INFORMATION STRUCTURES\***

**Luis Corchón and Ignacio Ortuno\*\***

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# ROBUST IMPLEMENTATION UNDER ALTERNATIVE INFORMATION STRUCTURES

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## A B S T R A C T

*In this paper we consider a model in which agents have complete information about their neighbours and, possibly, incomplete information about the rest of the economy. We consider two different informational frameworks. In the first, agents do not have priors about what is going on in the rest of the economy. In the second, agents are supposed to have priors about the unknown characteristics. We present a mechanism which implements any social choice correspondence satisfying monotonicity and no veto power in both informational settings for every possible prior thus requiring little knowledge from the point of view of the designer of the information possessed by agents about the economy.*



## 1: INTRODUCTION

The Theory of Implementation studies the feasibility of achieving social goals when taking properly into account agents' incentives, i.e. the possibility of reconciling utopia and self-interested behavior. Usually, an agent's behavior is modeled according to some game-theoretical concept which is suited to an informational framework. Social goals are described by a mapping, sometimes called Social Choice Correspondence (SCC in the sequel), which associates a feasible allocation which is assumed to be optimal to each economy in a certain class. A Mechanism (sometimes called a Game Form) is a description of the language in which agents communicate and the consequences of the messages (strategies) they send. A mechanism is said to implement a given SCC if, for any economy in the domain of this correspondence, there are equilibrium messages (in some game-theoretical sense) and the consequences of these messages coincide with the allocation prescribed by the SCC.

A basic assumption in the Theory of Implementation is that the mechanism should be designed prior to the environment being known, but the designer is allowed to know how agents would behave, i.e. the equilibrium concept. The purpose of this paper is to investigate the consequences of assuming that the mechanism should work, at least partially, regardless of the structure of information in the economy and therefore, it should implement the SCC for some equilibrium concepts rather than for a single one.

A typical result in the Theory of Implementation says that, given a kind of rational behavior (implied by the structure of information), some SCC can

or can not be implemented in a certain range of environments. Thus, if agents use dominant strategies, implementation of acceptable SCC is essentially impossible if the domain of SCC is large enough (Hurwicz (1972), Gibbard (1973), Satterthwaite (1975), see also Ledyard-Roberts (1974)). However, implementation of acceptable SCC is possible, under certain conditions, if the equilibrium concept is Nash (Maskin (1977), see also Repullo (1987)). Moreover these conditions can be adapted to the case in which agents are assumed to be Bayesian (see Palfrey-Srivastava (1989) and Jackson (1991)). All these results suggest a trade-off between the information possessed by an agent about other agents -none in the case of dominant strategies, a common prior in the Bayesian framework and complete information in the case of Nash- and the possibility of implementing a satisfactory SCC.

In this paper we will consider the implementation problem when the information possessed by agents is variable. In order to give structure to the problem we will make an additional assumption: throughout the paper we will assume that agents have complete information about a part of the environment. In other words, the economy is composed of islands, with a population of at least three agents each. Every agent has complete knowledge about the preferences of other people on the same island. This is also common knowledge to the agents and the designer. Examples of islands include families, groups of friends, firms (individuals) in the same industry (task force), departments in a university, etc. With respect to the information about the rest of the economy, we will assume two different settings.

1. *Ignorance.* Agents only know the set of possible types of agents outside their islands, but not their actual types. Moreover, priors on types are

meaningless. In this framework, the equilibrium strategy for an agent must be the best reply to what the rest of the agents on the island play, and to any possible message sent by agents outside her island when they follow their equilibrium strategies. The last requirement resembles dominant strategies, the difference being that a strategy is dominant if it is a best reply regardless of how other players *behave*, and in our case, the strategy must be a best reply no matter what other players (outside the island) *are like*. This equilibrium concept has been used in the literature (without our assumption about islands) under the name of Uniform Nash Equilibrium (see d'Aspremont -Gerard-Varet (1979). See also Matsushima (1988) for a similar attempt).

2. *Risk*. In this framework, our equilibrium concept is the usual Bayesian Equilibrium introduced by Harsanyi (1967), with the above condition that players have complete information inside islands. The difference here from the usual notion of Bayesian implementation is that we require a mechanism to implement a SCC for any possible prior and when the designer does not know the exact prior. This amounts to requiring the mechanism to be designed before the planner has any knowledge of the agent's information. Therefore our results here are, in this sense, stronger than the usual ones. Thus, the same mechanism implements a SCC under complete and incomplete information.

We define a mechanism as *Robust* relative to a SCC, if it implements this correspondence in Uniform Nash Equilibrium and Bayesian Equilibrium for any possible prior with the designer being ignorant of the actual prior. This notion attempts to capture, at least partially, the fact that the mechanism should be flexible enough to cope with different informational settings and that changes in the structure of information do not therefore imply a change

in the mechanism. A more satisfactory concept of robustness would allow for the possibility that some islands might be populated by Bayesian agents and others populated by agents acting under ignorance. Unfortunately we have been unable to make progress in this direction.

Our main result is that there is a Robust mechanism which implements any SCC satisfying the well-known conditions of Monotonicity and Non-Veto Power introduced by Maskin in the framework of Nash Implementation<sup>(1)</sup>. Thus, given our assumption about islands, the fact that information is or is not complete makes no essential difference from the point of view of Implementation. Therefore our results can be understood as a generalization of those obtained by Maskin under complete information. The cost of this generalization is the common knowledge assumption on islands.

An alternative motivation for our approach is that the theory of Nash implementation has been criticised for the following reason. On the one hand, the outcome function is not continuous on messages (at least when the general implementation problem is studied). On the other hand, exact complete information is required. Therefore a slight mistake on other people's characteristics might entail allocations far from those selected by the SCC. Our mechanism only requires exact complete information on a usually small subset of agents. Thus, our mechanism can be regarded as introducing some robustness on Nash implementation.

(2) It is not difficult to show that in our model -private values and non-exclusive information- monotonicity implies Bayesian monotonicity.



It should be remarked that the paper does not make any progress in other important topics such as the reduction of strategy spaces (even though our strategy spaces are not much larger than those in Williams (1986), Saijo (1988) and McKelvey (1989)), the avoidance of modulo games (see Jackson (1988)), the consideration of continuous and "realistic" mechanisms and the possibility of coalitions. All these points are left for future research.

The rest of the paper goes as follows. The next Section explains the basic model and our main assumptions. Section 3 is devoted to proving our results and, finally, Section 4 gathers our conclusions.

## 2: THE MODEL

In this Section we present our basic framework. Let  $N = \{1, \dots, n\}$  be the finite set of agents. Let  $T_i$  be the (finite but greater than one) set of *attributes* of player  $i$  and  $t_i$  an element of  $T_i$ . We assume that  $t_i$  embodies all relevant information about agent  $i$ . Let  $T = \prod_{i=1}^n T_i$  be the set of states. A *state*  $t \in T$  provides a complete description of information and preferences of all agents. Let  $\mathcal{A}$  stand for the set of social alternatives and let  $\mathcal{F} : T \rightarrow \mathcal{A}$  be the social choice correspondence (SCC in the sequel). This mapping is assumed to embed the social objectives.

We now describe the informational framework. We will assume throughout the paper that there is a partition of  $N$ ,  $G = (G_1, \dots, G_s)$  with  $\# G_i > 2$ ,  $i = 1, \dots, s$  such that each agent in a given element of  $G$  has complete information about all the characteristics of any agent in this element of the partition,

and this is common knowledge for all agents in the partition. We may think of the economy as being composed of islands in the terminology introduced by Lucas (1972) (but notice that here, any island is composed of at least three agents). Therefore in our model information is *not exclusive*. Also, each agent is assumed to know the sets  $N$ ,  $T$ ,  $\mathcal{A}$ , and the SCC  $\mathcal{F}$ . This information is assumed to be common knowledge. Let  $G_k$  be a typical element of  $G$  and  $i$  a typical element of  $G_k$ . Let  $t_{(i)}$  be the information of  $i \in G_k$  about the attributes of agents on her island, i.e.  $t_{(i)} = (t_j, \dots, t_p)$  where  $j, \dots, p$  are all the agents in  $G_k$ . We write  $T_{(i)} = T_j \times \dots \times T_p$ . Also let us denote the attributes of all agents outside  $G_k$  by  $t_{-(i)}$  and  $T_{-(i)} = \prod_{j \notin G_k} T_j$ . We will say that  $t_{(i)}$  is the *type* of agent  $i$ .

With respect to the information possessed by an agent about other islands, we will consider two set-ups. In the first one, agents will be assumed to act under ignorance, i.e. they do not assign probabilities to the occurrence of states of the world (which are the types of agents living on other islands). In the second one, agents are considered to be Bayesian, i.e. they assign probabilities to the states of the world.

Each agent has a state dependent utility function  $u_i: \mathcal{A} \times T_i \rightarrow \mathbb{R}$ , i.e. we assume *private values* (it is not difficult to extend our results to the case in which the utility function depends on the *type* of the agent). Let us denote by  $L(a, t_i)$  the elements of  $\mathcal{A}$  which are not preferred to  $a \in \mathcal{A}$  according to the utility function  $u_i(\cdot, t_i)$ , i.e. the lower contour set of agent  $i$  relative to her utility function. Let us define  $a_i^M(t_i) \subseteq \mathcal{A}$  as the set of maximal elements of  $\mathcal{A}$  according to  $u_i(\cdot, t_i)$ . It is assumed that this set is

not empty for all possible attributes (see Jackson (1991) p. 469 and Palfrey -Srivastava (1991) p. 482 for a similar assumption). This assumption holds if, for instance, preferences are continuous, and the feasible set is compact.

We now define the strategic elements. A Mechanism (Game Form) is a pair  $(M, g)$  where  $g : M \longrightarrow \mathcal{A}$  and  $M = \prod_{i=1}^n M_i$ .  $g$  is the outcome function and  $M_i$  is the message space of agent  $i$ . A strategy for agent  $i$  is a function  $s_i : T_{\{i\}} \longrightarrow M_i$ . We write  $s$  as the vector of strategies for all agents, i.e.  $s = (s_1, \dots, s_n)$ . Also  $s_{\{i\}} = (s_1, \dots, s_p)$  is the tuple of strategies of all agents inside  $G_k$ , and  $s_{-\{i\}}$  the tuple of strategies of all agents outside  $G_k$ .

In the first informational setting, we assume that agents act in complete ignorance of the characteristics of any agent outside her group. Therefore each agent will play "Nash" against agents within her group and uniform Nash against any agent outside her group. Formally

**Definition 1.-**  $s^* = (s_1^*, \dots, s_n^*)$  is a Uniform Nash Equilibrium with Complete Local Information for the state  $t = (t_1, \dots, t_n)$  if  $\forall i = 1, \dots, n$

$$u_i(g(s_{\{i\}}^*(t_{\{i\}}), s_{-\{i\}}^*(t_{-\{i\}})), t_i) \geq u_i(g(m_i, s_{\{i\}/i}^*(t_{\{i\}}), s_{-\{i\}}^*(t_{-\{i\}})), t_i)$$

$$\forall t_{-\{i\}} \in T_{-\{i\}}, \quad \forall m_i \in M_i.$$

Notice that the equilibrium strategy of any agent must maximize her utility for any possible message recommended by the mapping  $s_{-\{i\}}^*(T_{-\{i\}})$  for all agents outside  $G_k$ . Let  $UNE(t, M, g)$  be the set of Uniform Nash Equilibria

with Complete Local Information (or Uniform Nash Equilibria for short) for the Game Form  $(M, g)$  when the state is  $t$ .

**Definition 2.**  $(M, g)$  implements the SCC  $\mathcal{F}$  in UNE if  $\forall t \in T$

- a)  $\forall a \in \mathcal{F}(t), \exists s^* \in \text{UNE}(t, M, g)$  such that  $a = g(s^*)$ .
- b) If  $s^* \in \text{UNE}(t, M, g)$ , then  $g(s^*) \in \mathcal{F}(t)$ .

We will say that  $\mathcal{F}$  is implementable in *Uniform Nash Equilibrium* if there is a mechanism implementing  $\mathcal{F}$  when agents behave according to Definition 1.

Our definition of a Uniform Nash equilibrium does not pay attention to the case in which agents have priors about the occurrence of states of the world. In order to deal with this case let us introduce a new informational setting. Agents have a common prior distribution  $p(t)$  which is assumed to be common knowledge. Let  $P$  be the set of all admissible common prior distributions  $p$  on  $T$ . In accordance with the previous ideas, we can define the conditional probability distribution for agents on island  $G_k, q_{(i)}(t_{-(i)} | t_{(i)})$  by

$$q_{(i)}(t_{-(i)} | t_{(i)}) = \frac{p(t_{-(i)}, t_{(i)})}{p_{(i)}(t_{(i)})}$$

where  $p_{(i)}(t_{(i)}) = \sum_{t_{-(i)} \in T_{-(i)}} p(t_{-(i)}, t_{(i)})$

We also assume that agent  $i$ 's preferences on risky outcomes can be represented by a von Neumann-Morgenstern utility function.

**Definition 3.**  $s^* \equiv (s_1^*, \dots, s_n^*)$  is a *Bayesian Equilibrium with Complete Local Information* if, for a given state of the world  $t$  and common prior distribution  $p(\cdot)$  and for all  $i \in N$ , we have

$$\sum_{t_{-(i)} \in T_{-(i)}} q_{(i)}(t_{-(i)} | t_{(i)}) u_i(g(s_{(i)}^*(t_{(i)}), s_{-(i)}^*(t_{-(i)})), t_i) \geq$$

$$\sum_{t_{-(i)} \in T_{-(i)}} q_{(i)}(t_{-(i)} | t_{(i)}) u_i(g(m_i, s_{(i)\setminus i}^*(t_{(i)}), s_{-(i)}^*(t_{-(i)})), t_i)$$

$$\forall m_i \in M_i.$$

This is the usual definition of Bayesian equilibrium for the information structure given above. Let  $B(M, g, p, t)$  be the set of Bayesian Equilibria with Complete Local Information (Bayesian Equilibrium for short) for the Game form  $(M, g)$  when the state of the world is  $t$  and the common prior distribution is  $p$ .

We remind the reader that the standard Bayesian implementation approach assumes that the designer of the mechanism knows the common prior distribution. In our model, the center does not need to know  $p$ . However, it must know the partition  $G$  which seems to us to be a less demanding informational requirement, at least in some cases.

**Definition 4.** *The Game Form  $(M, g)$  implements the SCC  $\mathcal{F}$  in Bayesian Equilibrium with Complete Local Information if for all  $t \in T$ ,  $p \in P$*

- 1)  $\forall a \in \mathcal{F}(t)$ ,  $\exists s^* \in B(M, g, p, t)$  such that  $g(s^*) = a$
- 2) If  $s^* \in B(M, g, p, t)$ , then  $g(s^*(t)) \in \mathcal{F}(t)$ .

Notice that the complete information model is a particular case of the Bayesian framework where  $q$ 's are either zero or one. Finally, we come to the main notion of the paper.

**Definition 5.** *Let  $\mathcal{F}$  be a SCC. The mechanism  $(M, g)$  is Robust relative to  $\mathcal{F}$  if it implements  $\mathcal{F}$  in Uniform Nash Equilibrium and Bayesian Equilibrium*

The idea behind Definition 5 is that a mechanism is robust relative to  $\mathcal{F}$  (in short, robust) if it implements  $\mathcal{F}$  irrespective of the information on the economy. Therefore the designer does not need to have much knowledge of the information that agents possess on the economy. Moreover, if agents acquire more information or change their priors, the proposed mechanism implements  $\mathcal{F}$ .

Before we present our main result we will define two concepts which will play an important role in our paper. Both are standard in the theory of Nash implementation.

A SCC  $\mathcal{F}$  satisfies monotonicity if for all  $t, t' \in T$  and  $a \in \mathcal{F}(t')$  we have  $L_i(a, t') \subset L_i(a, t)$  for all  $i \in N$  only if  $a \in \mathcal{F}(t)$ .

A SCC  $\mathcal{F}$  satisfies no veto power if for all  $i \in N, t \in T$  and  $a \in A$  we have  $L_i(a, t_j) = A$  for all  $j \neq i$  only if  $a \in \mathcal{F}(t)$ .

Since monotonicity is a necessary condition of Nash Implementation, it is also a necessary condition of Robust Implementation. Our main result is that monotonicity (plus no veto power) is sufficient for Robust Implementation.

**Theorem 1.** *Any SCC which satisfies Monotonicity and No Veto Power can be implemented by a robust mechanism relative to  $\mathcal{F}$ .*

First, we will present the mechanism and the next Section will be devoted to proving that this mechanism implements in Uniform Nash and Bayesian Equilibria any SCC satisfying monotonicity and no veto power.

Let the message space for agent  $i$  be given by

$$\mathcal{M}_i = T_{\{i\}} \times F \times \mathcal{A} \times \mathcal{F}(T) \times \mathbb{N} \times \mathcal{A}$$

where  $F$  is the set of all selections of  $\mathcal{F}$ ,  $\mathcal{F}(T)$  is the range of the SCC  $\mathcal{F}$  and  $\mathbb{N}$  is the set of natural numbers. We write an element of  $\mathcal{M}_i$  in the following way

$$m_i = (t_i^1, t_i^2, \dots, t_i^p, f_i, a_i, c_i, n_i, b_i)$$

Thus,  $t_i^j$  will be the "report" on agent  $j$ 's attributes given by agent  $i$ . We will write  $t_i^{\langle i \rangle} = (t_i^1, t_i^2, \dots, t_i^p)$ , i.e.  $t_i^{\langle i \rangle}$  is the information provided by  $i$  about the attributes of the people on her island. We now define the outcome function by way of the following three rules

**RULE 1:** If  $m = (m_1, \dots, m_n)$  is such that  $t_i^j = t_j^j$  for all  $i, j \in G_k$  and  $k=1, \dots, s$ , and there exists  $f \in F$  such that  $f_i = f$  for all  $i$ , then  $g(m) = f(t_1^1, \dots, t_n^n)$ .

**RULE 2:** There exists a unique group  $k$  and there is a selection  $f \in F$  such that for all  $G_m \neq G_k$  we have  $t_s^r = t_r^r$  and  $f_s = f_r = f$  for all  $s, r \in G_m$  and for the group  $G_k$  there exists a unique  $i \in G_k$  such that  $(t_i^{\langle i \rangle}, f_i) \neq (t_j^{\langle i \rangle}, f_j)$  with  $(t_j^{\langle i \rangle}, f_j) = (t_h^{\langle i \rangle}, f_h)$  and  $f_j = f \forall j, h \in G_k, j, h \neq i$ . Then

$$g(m) = \begin{cases} a_i, & \text{if } a_i \in L(c_i, t_j^i) \text{ and } c_i = f(t_1^1, \dots, t_j^i, t_j^j, \dots, t_n^n), \text{ where } j \in G_k \\ f(t_1^1, \dots, t_j^i, t_j^j, \dots, t_n^n), & \text{otherwise.} \end{cases}$$

**RULE 3:** If Rules 1 or 2 do not apply, then  $g(m) = b_i$

where  $i = \max \{j : n_j \geq n_i, \text{ for all } i \in N\}$

When Rule 1 applies we say that there exists complete agreement on the reports. Whenever Rule 2 applies we say that there exists a unique disagreement and agent  $i$  is the dissident. If Rule 3 applies it is said that there are at least two dissidents.

Rule 1 says that if there is total consistency on the reports on states and the selection of  $\mathcal{F}$ , the mechanism chooses the allocation given by the selection  $f$  using the reported states.

Rule 2 takes care of the case whereby all agents but one send consistent reports on states. In this case the mechanism will choose the allocation  $a_i$  given by the "dissident" whenever 1)  $c_i$  coincides with the allocation recommended by the chosen selection  $f$  and the states reported by all other agents 2) this allocation belongs to the lower contour set of  $c_i$  using the states reported by all other agents on  $i$ . If  $a_i$  does not satisfy 1) and 2), the mechanism chooses the allocation recommended by the selection  $f$  when states are those reported by all agents but the "dissident".

In all other cases, i.e. when there is more than one "dissident" Rule 3 (the roulette) applies. These rules allow for some possible interpretation of  $a_i$ ,  $c_i$  and  $b_i$  as a choice, a guess and a best alternative in  $\mathcal{A}$  respectively.



### 3: THE PROOF OF THE THEOREM

This Section will be devoted to proving Theorem 1. We will divide the proof into two Propositions. These Propositions will show that the proposed mechanism implements a SCC in Bayesian and Uniform Nash Equilibria respectively.

**Proposition 1.-** *If the Social Choice Function  $\mathcal{F}$  satisfies Monotonicity and No Veto Power, then the above Game Form  $(M, g)$  implements  $\mathcal{F}$  in Bayesian Equilibrium.*

**Proof:** a) *Firstly, we show that part 1 of Definition 4 is satisfied by  $(M, g)$ , i.e.,  $\forall t \in T, \forall p \in P$ , and for any  $a \in \mathcal{F}(t)$  there exists  $s^* \in B(M, g, p, t)$  such that  $g(s^*(t)) = a$ . Choose any selection  $f \in F$  such that  $f(t) = a$  and consider for any agent  $i$  the "truth-telling" strategy  $s_i^*: T_{\{i\}} \longrightarrow M$  given by  $s_i^*(t_{\{i\}}) = (t_{\{i\}}, f, a_i, c_i, n_i, b_i)$  where  $a_i, c_i, n_i, b_i$  are arbitrary choices and  $t_{\{i\}}$  is the true type of agent  $i$ . We now show that  $s^* \in B(M, g, p, t)$ . If all agents follow these strategies, Rule 1 applies and the outcome is  $g(s^*(t)) = f(t)$ . Suppose that agent  $i$  chooses  $m_i = (t_i^{\{i\}}, f_i', a_i', c_i', n_i', b_i')$  instead of  $s_i^*(t_{\{i\}})$ , with  $(t_i^{\{i\}}, f_i') \neq (t_{\{i\}}, f)$ . Thus, Rule 2 applies, and the outcome is either  $a_i'$  or  $f(t)$ . In both cases the outcome cannot be preferred to  $g(s^*(t))$ .*

b) *Secondly, we show that  $(M, g)$  satisfies (2) in Definition 4. Now three cases must be considered.*

i)  *$g(s^*(t))$  is given by Rule 1, i.e.,  $s^*(t)$  yields a complete agreement. In this case any agent  $i$  could have chosen a message  $m_i = (\bar{t}_i^{\{i\}}, \bar{f}_i, \bar{a}_i, \bar{c}_i, \bar{n}_i,$*

$\bar{b}_i$ ) instead of  $s^*_i(t_{(i)}) = (t^{(i)}_i, f_i, a_i, c_i, n_i, b_i)$ , where  $(t^{(i)}_i, f_i) \neq (t^{(i)}_i, f_i)$ ,  $\bar{a}_i \in L(c_i, t^i_i)$ ,  $\bar{n}_i = \hat{n}(s^*)$ ,  $\bar{b}_i \in a^M_i(t_i)$  and  $c_i = f_i(t^*)$ , where  $t^*$  are the reports on the states associated to  $s^*(t)$  and  $\hat{n}(s^*)$  is a natural number greater than any number recommended by the equilibrium strategy for any agent and any state of the world ( $\hat{n}(s^*)$  is well defined because the finiteness of  $N$  and  $T$ ). We want to know the reason why agent  $i$  did not choose message  $m_i$ . For all  $\bar{t}_{-(i)}$  such that  $s^*(t_{(i)}, \bar{t}_{-(i)})$  produces at least one dissident, the announcing of  $m_i$  will take the mechanism to Rule 3 and the outcome will be  $g(s^*_{-i}(t), m_i) \in a^M_i(t_i)$ . For all the cases  $t_{-(i)}$  such that  $s^*(t_{(i)}, t_{-(i)})$  produces complete agreement, the announcing of  $m_i$  yields the outcome  $g(s^*_{-(i)}(t_{-(i)}), s^*_{(i)}(t_{(i)}))$  whenever  $c_i \neq f_i(t^1_1, \dots, t^i_j, t^j_j, \dots, t^n_n)$  where  $(t^1_1, \dots, t^i_j, t^j_j, \dots, t^n_n)$  are the report on types associated to  $s^*(t_{-(i)}, t_{(i)})$ , i.e. the reports on types given by all agents but agent  $i$ , and  $a_i$  whenever  $c_i = f_i(t^1_1, \dots, t^i_j, t^j_j, \dots, t^n_n)$ . Thus, the reason for which she did not choose  $m_i$  must be that

$$\bar{a}_i \in L(c_i, t^i_i) \Rightarrow \bar{a}_i \in L(c_i, t_i).$$

This implication holds for all agents. Therefore by monotonicity of  $\mathcal{F}$  and since  $c_i = f_i(t^*)$  for all  $i$ , then  $f(t^*) \in \mathcal{F}(t)$  and  $g(s^*(t)) \in \mathcal{F}(t)$ .

ii)  $g(s^*(t))$  is given by Rule 2, i.e.  $g(s^*(t))$  yields a unique dissident. Let agent  $i \in G_k$  be the dissident. In this case, it is clear that for all  $j \in G_k$ ,  $j \neq i$ , we have  $g(s^*(t)) \in a^M_j(t_j)$ . To see this note that agent  $j$  can always choose message  $m_j = (t^{(j)}_j, f_j, a_j, c_j, n_j, b_j)$  such that  $(t^{(j)}_j, f_j) \neq (t^{(i)}_i, f_i)$ ,  $(t^{(j)}_j, f_j) \neq (t^{(l)}_l, f_l)$ ,  $l \in G_k$ ,  $l \neq i$ ,  $b_j \in a^M_j(t_j)$  and  $n_j = \hat{n}(s^*)$ . By the choice of  $m_j$  agent  $j$  takes the outcome from Rule 2 to Rule 3, and in this case she can get her best alternative. Now we show that the same must be true for

agents outside  $G_k$ , i.e.,  $g(s^*(t)) \in a_u^M(t_u) \forall u \notin G_k$ . Take an agent  $u \in G_r$ . Suppose that  $g(s^*(t)) \notin a_u^M(t_u)$ . Agent  $u$  could have chosen, instead of the message  $s_u^*(t_{\{u\}}) = (t_u^{(u)}, f_u, a_u, c_u, n_u, b_u)$ , the message  $m_u = (\bar{t}_u^{(u)}, \bar{f}_u, \bar{a}_u, \bar{c}_u, \bar{n}_u, \bar{b}_u)$  where  $(\bar{t}_u^{(u)}, \bar{f}_u) \neq (t_u^{(u)}, f_u)$ ,  $\bar{a}_u = c_u$ ,  $\bar{n}_u = \hat{n}(s^*)$  and  $\bar{b}_u \in a_u^M(t_u)$ . For all  $t_{-\{u\}}$  such that  $s^*(t_{\{u\}}, t_{-\{u\}})$  is given by rule 1,  $m_u$  and  $s_u^*(t_{\{u\}})$  yield the same outcome. To see this, observe that for these cases, by the choice of  $m_u$ , agent  $u$  can force the mechanism to go to Rule 2 (and she becomes the dissident). Since  $\bar{a}_u = \bar{c}_u$  his dissidence is, however, a vacuous one.

For all  $t_{-\{u\}}$  such that  $s^*(t_{\{u\}}, t_{-\{u\}})$  is given by Rules 2 or 3 message  $m_u$  is such that  $g(s_{-u}^*(t), m_u)$  is always given by Rule 3 and since  $\bar{n}_u = \hat{n}(s^*)$  we have  $g(s_{-u}^*(t), m_u) = \bar{b}_u \in a_u^M(t_u)$ . Therefore for some  $t_{-\{u\}}$ , the message  $m_u$  and  $s_u^*(t_{\{u\}})$  yield the same outcome and for the rest of  $t_{-\{u\}}$ , message  $m_u$  yields an outcome which is strictly preferred by agent  $u$  to the outcome achieved by announcing  $s_u^*(t_{\{u\}})$  [observe that since we are in the case where  $g(s^*(t))$  is given by Rule 2, there always exists such a  $t_{-\{u\}}$  for which  $m_u$  is strictly preferred to  $s_u^*$ ]. Thus, regardless of the conditional probability function,  $m_u$  is strictly preferred to  $s_u^*(t_{\{u\}})$ , which is a contradiction. Therefore,  $g(s^*(t)) \in a_u^M(t_u)$  for all  $u \notin G_k$ . Thus, we have that  $g(s^*(t)) \in a_j^M(t_j), \forall j \neq i$ , and by No Veto Power of  $\mathcal{F}$  we have that  $g(s^*(t)) \in \mathcal{F}(t)$ .

iii)  $g(s^*(t))$  is given by Rule 3, i.e. there are at least two dissidents. In this case it is clear that all agents obtain the best outcome, i.e.,  $g(s^*(t)) \in a_i^M(t_i)$ . Then by No Veto Power of  $\mathcal{F}$ ,  $g(s^*(t)) \in \mathcal{F}(t)$ . ■

**Proposition 2.-** *If the Social Choice Correspondence  $\mathcal{F}$  satisfies Monotonicity and No veto Power, then the above Game form  $(M, g)$  implements  $\mathcal{F}$  in Uniform Nash Equilibrium with Complete Local Information.*

**Proof:** *The first part of this Proposition (i.e.  $\forall a \in \mathcal{F}(t), \exists s^* \in UNE(t, M, g)$  such that  $a = g(s^*)$ ) can be done in the same way as in Proposition 1, using "truth-telling" strategies. The second part of the proof (i.e. If  $s^* \in UNE(t, M, g)$ , then  $g(s^*) \in \mathcal{F}(t)$ ), can be proved as follows. It is clear that  $UNE(t, M, g) \subseteq B(M, g, p, t) \forall p, t$ . Therefore  $g(UNE(t, M, g)) \subseteq g(B(M, g, p, t))$ . By Proposition 1,  $g(B(M, g, p, t)) \subseteq \mathcal{F}(t)$  and this proves the claim. ■*

#### 4: CONCLUSIONS

In the literature on Implementation, it is usual to assume a given equilibrium concept. An exception to this, is the work on Double Implementation (see Maskin (1985), Schmeidler (1980) and Corchon-Wilkie (1991)) which takes care of the problem of coalition formation requiring a mechanism to implement a given SCC in both Nash and Strong equilibria. The idea behind this, is that when designing a mechanism, the planner does not know whether agents will form coalitions or play non-cooperatively. Therefore the mechanism should be robust to the different possibilities that might arise. Our concept of Robust Implementation is conceived in the same spirit, but applied to the problem of information. We require a mechanism to implement a SCC under two different informational structures: ignorance and risk (and

since implementation in our sense is for every possible prior, it is also implemented under complete information). Therefore, we might say that this mechanism doubly implements a SCC<sup>(2)</sup>.

We have shown that with some assumptions -including private values and that a most preferred element in the social choice set  $\mathcal{A}$  exists for every agent- the existence of complete information inside islands guarantees the existence of a Robust Mechanism relative to any SCC satisfying the well-known conditions of monotonicity and no-veto power. Therefore, under our assumptions, the sufficient conditions for implementation in Nash Equilibrium turn out to be sufficient for implementation under Uniform Nash and Bayesian Equilibria as well. In other words, we can achieve the same allocations under complete and incomplete information. This is basically due to the fact that in our case information is not exclusive (see Palfrey-Srivastava (1986) and Blume-Easley (1990)). This result contrasts with that obtained by Ledyard (see Ledyard (1978)) about the impossibility of implementing with Bayesian Equilibrium, for every prior, a large class of SCC, and with the so called "incentive compatibility approach to organization design" in which the choice of the mechanism depends crucially on prior beliefs (see Ledyard (1987) p. 149 and the discussion therein) and -usually- first best allocations can not be

(3) The reader may wonder if implementation under some kind of Dominant-Nash strategies (instead of Uniform-Nash) can occur in our informational setting. However it is easy to find an example disproving this conjecture.

achieved (see Harris-Townsend (1981) pp. 36-7, 53-4 and 58 for a discussion on ex-ante vs ex-post efficiency and prior-free mechanisms).

We finally remark on an interesting feature of our mechanism. If  $\mathcal{F}$  is a function and the set of maximal elements is always a singleton, it can be shown that the strategies which are a Bayesian Equilibrium are (ex post) a Nash Equilibrium. Therefore no agent will ever regret her choice.

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